Igor Menshov, KIAM RAS (Moscow)

Yoshihiro Kato, Toyota Central R&D Labs (Aichi)

SPLITTING METHOD FOR AEROACOUSTIC SIMULATIONS

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The issue to be considered is how to compute acoustic field generated from low velocity gas flow.

- Flow is nearly incompressible – elliptic type of equations ;
- Need to use compressible model – convergence, accuracy;
- Difficulties of treating the acoustic field – what represents sound in incompressible flow;
- Problems of extraction of the acoustic field from the flow field – how to split acoustic waves from the flow;

A common way to treat aeroacoustics is the uncoupled approach (a postprocessing procedure .

- Calculate the basic unsteady flow with some level of accuracy by means of DNS, LES, URANS.
- Extract fluctuation characteristics (pressure, velocity time histories) at points of a representative surface.
- Use Lighthill's wave equations (*) with extracted fluctuation characteristics and approximate analytical solutions to compute the acoustic field (Ffowcs Williams-Hawkings method).

Assumptions: the solid surface is small enough in comparison with the propagating acoustic wave length, and the observation point is far from sound sources.

(*) M. J. Lighthill. On Sound Generated Aerodynamically I. General Theory. Proceedings of the Royal Society of London A, v.211, 1952, pp.564-587.

Purpose: develop a numerical method for prediction near acoustic field from low Mach number flow (e.g., sound generated by an automobile rear-view mirror) when FWH method is not applicable.

The method is inspired by the study of Slimon et al (*) – ***splitting approach***.

- Compressible Navier-Stokes equations;
- Expansion of the solution in series with respect to a small parameter (squared Mach number).
- Leading terms in these expansions define the base flow field - incompressible Navier-Stokes equations + equation of density variation.
- Higher order terms - the acoustic field, governed by a reduced system that depends on parameters of the base flow.
- Acoustic equations are applied both near and far field.

The method has certain **advantages** over other methods that treat sound and flow fields in uncoupled manner.

- Different discrete models for the base flow and the acoustic field equations.
- Different numerical schemes to take into account basic features of these equations.
- Flow equations are unsplit from the acoustic equations, i.e., the generated
- sound does not affect the flow field.

(*) S.A. Slimon, et al. Development of Computational Aeroacoustics Equations for Subsonic Flows Using a Mach Number Expansion Approach. JCP, v.159, 2000, pp.377-406.

Compressible N.-S. equations (calorically perfect gas):

$$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{v}) = 0$$

$$\rho \dot{\mathbf{v}} = -grad(p) + div(\tau), \quad \tau = 2\mu\varepsilon - 2/3\mu \mathrm{tr}(\varepsilon)$$

$$\dot{p} - \frac{\gamma p}{\rho}\dot{\rho} = (\gamma - 1)\Phi, \quad \Phi = 2\mu\varepsilon : \varepsilon - \frac{2}{3}\mu[div(\mathbf{v})]^2 + div[k\,grad(\theta)\}$$

Non-dimensional flow parameters: ***U, P, and R -***

$$\mathbf{v} = u_0 \boldsymbol{U}, \quad p = p_0(1 + \gamma M_0^2 P), \quad \rho = \rho_0(1 + M_0^2 R), \quad M_0^2 = squared\ Mach\ number$$

$$M_0^2 \dot{R} + (1 + M_0^2 R) div(\boldsymbol{U}) = 0$$

$$(1 + M_0^2 R)\dot{\boldsymbol{U}} = -grad(P) + div(\frac{1}{\mathrm{Re}} T), \quad T = 2\mu E - 2/3\mu \mathrm{tr}(E)$$

$$\dot{P} - \frac{1 + \gamma M_0^2 P}{1 + M_0^2 R}\dot{R} = \frac{2(\gamma - 1)}{\mathrm{Re}} E : E - \frac{2(\gamma - 1)}{3\mathrm{Re}}[div(\boldsymbol{U})]^2 + div[\frac{\mathrm{Pr}}{\mathrm{Re}} grad(\frac{\gamma P - R}{1 + M_0^2 R})]$$

Asymptotic expansion:

$$P = P_0 + M_0^2 P_1 + O(M_0^4), \quad \boldsymbol{U} = \boldsymbol{U}_0 + M_0^2 \boldsymbol{U}_1 + O(M_0^4), \quad R = R_0 + M_0^2 R_1 + O(M_0^4)$$

Leading terms *O(1)*:

$$div(\boldsymbol{U}_0) = 0$$

$$\dot{\boldsymbol{U}}_0 = -grad(P_0) + div(\frac{1}{\mathrm{Re}} T_0), \quad T_0 = 2\mu E_0 \text{-} 2/3 \mu \,\mathrm{tr}(E_0)$$

$$\dot{P}_0 - \dot{R}_0 = \frac{2(\gamma - 1)}{\mathrm{Re}} E_0 : E_0 + div[\frac{\mathrm{Pr}}{\mathrm{Re}} grad(\gamma P_0 - R_0)]$$

System is split into 2 parts: incompressible N.-S. + equation for density variation. **Incompressible N.-S. model for low speed gas dynamics doesn't assume constant density: variation in density is of order of pressure variation**!

If neglect dissipative factors in the R.-H.S, $P_0 = R_0 \quad or \quad \rho - \rho_0 = c_0^2 (p - p_0)$

This is very similar to the **hydrodynamic density correction of Hardin and Pope** (*)

$$\rho - \rho_0 = \overline{c}_0^2 (p - \overline{p}), \quad \overline{p} = < p(t) >$$

By splitting $P_0 = \overline{P}_0 + P_0'$ so that $\dot{\overline{P}}_0 = \frac{2(\gamma - 1)}{\mathrm{Re}} E_0 : E_0 + div[\frac{\mathrm{Pr}}{\mathrm{Re}} grad(\gamma P_0 - R_0)]$

we get $R_0 = P_0'$ i.e., exactly the Hardin-Pope density correction, but with a different meaning of $\overline{P}_0$. **This correction must be computed on the base of incompressible solution $(\boldsymbol{U}_0, P_0)$** !

(*) Hardin,J. C., and Pope, S. D., "An Acoustic/Viscous Splitting Technique for Computational Aeroacoustics," Theoretical and Computational Fluid Dynamics, Vol. 6, No. 5–6, 1994, pp. 334–340

Model for **acoustic field** is derived on the base of incompressible solution denoted by subscript *. Acoustic parameters are treated as **counterpart to incompressible solution** in the flow parameters:

$$p = p_* + p', \quad \rho = \rho_* + \rho', \quad \boldsymbol{v} = \boldsymbol{v}_* + \boldsymbol{v}'$$

where ρ_* includes both **unperturbed constant density and density correction given by incompressible model**.

Substituting to the **compressible mass and momentum balance equations**, assume:
- Neglecting dissipative viscous and heat conduction effects;
- Isentropicity of the acoustic field, $s' = 0$;
- acoustic quantities are much smaller than quatities of the base flow.

With these assumptions, linearization of EOS yields a simple relation for p' and ρ' :

$$p_* + p' = P(\rho_* + \rho', s_* + s')$$

$$p' = c_*^2 \rho' \qquad\qquad p' = \frac{\gamma p_*}{\rho_*} \rho'$$

Mass and momentum balance equations result in the governing equations for acoustic variables:

$$\frac{\partial \rho'}{\partial t} + div(\boldsymbol{f}) = -\frac{D_*(\rho_*)}{Dt}, \qquad \boldsymbol{f} = (\rho_* + \rho')\boldsymbol{v}' + \rho' \boldsymbol{v}_*$$

$$\frac{\partial \boldsymbol{f}}{\partial t} + div(h) = -\frac{D_*(\rho_* - \rho_0)\boldsymbol{v}_*}{Dt}, \qquad h = \boldsymbol{f} \bullet (\boldsymbol{v}_* + \boldsymbol{v}') + \rho_* \boldsymbol{v}_* \bullet \boldsymbol{v}' + c_*^2 \rho' I$$

Hyperbolic system with r.-h.s. source

$$\lambda = v_* + v' \pm c_*$$

Mathematical model: solution of **two system of equations – for the base flow and for the acoustic field**. Specifics of these systems – **different grids and time steps:**

- Resolving wall boundary layer and structures of the sound wavelength scale in the far field;
- Resolving sound characteristics in a region of 1Khz to 2Khz – much smaller time step than that of the base flow (~4-5 times).

To adjust the **flow data to acoustics grid and time levels**:

- Cubic spline for the fluid time step;
- Tri-linear interpolation for remapping from flow grid onto acoustic grid.

Flow discrete model:

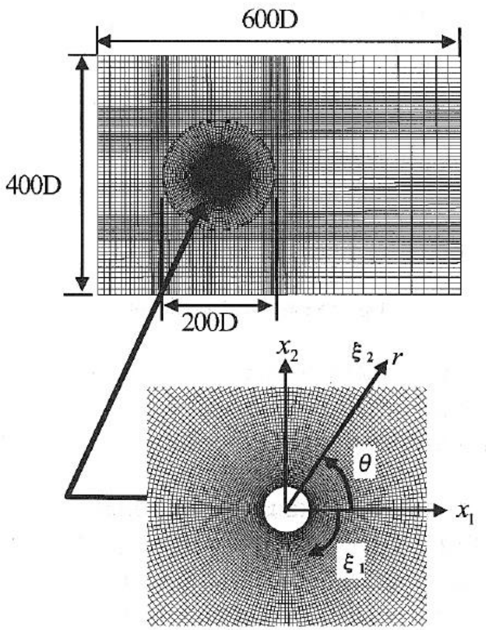
- LES method with the mixed-time-scale SGS model (*) for turbulence;
- FVM on colocated grid with 2nd order central difference for spatial derivatives;
- Implicit Crank-Nicolson time marching scheme;
- SMAC method for pressure field calculation.

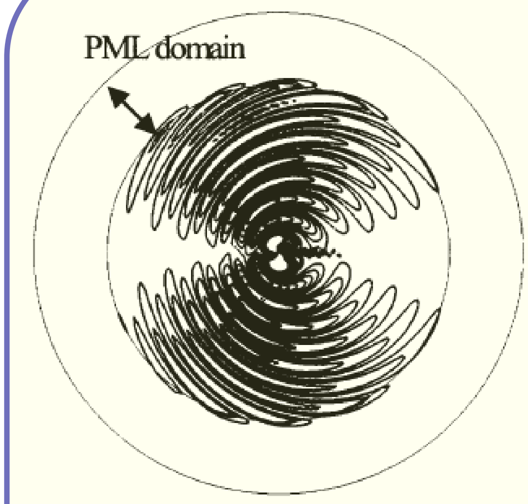
Acoustics discrete model:

- FVM with 4th order WENO cell interface interpolation;
- HLLE approximate Riemann solver for calculating numerical acoustic fluxes at cell interfaces;
- Explicit two-stage Runge-Kutta time marching scheme.

(*) Inagaki M et al "A mixed-time-scale SGS model with fixed model parameters for practical LES" Journal of Fluids Engineering, Vol. 127, No. 5–6, 2005, pp. 1–13

Acoustic field around a circular cylinder (Aeolian tones): Re=150, M=0.3, Pr=0.72

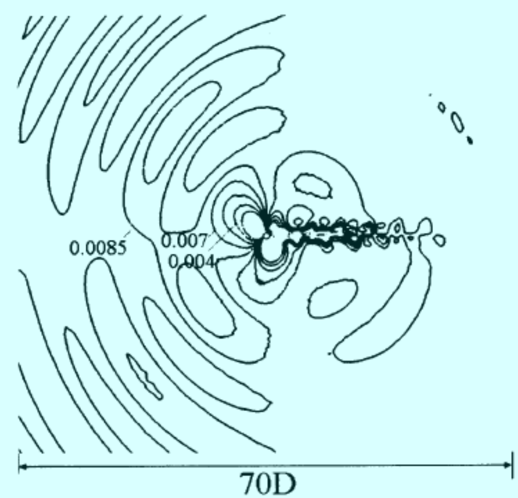




p' : -0.01 to 0.01 by 8 lines

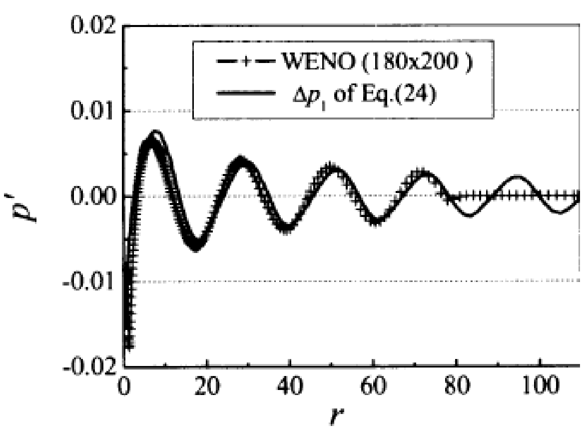
Flow calculation - Chimera-grid;
Acoustics – O-type grid with PML (Perf.Match.Layer) boundary condition.
Mesh: 180x200 cells (2D) ;
Comparison with theoretical predictions (Curle's dipole):

$$\Delta p_1 = \frac{1}{2^{1.5}\pi c^{0.5} r^{0.5}} \int_{-\infty}^{\tau} \frac{x_j}{\sqrt{\tau - t'}} \frac{\partial F_j(t')}{\partial t} dt',$$

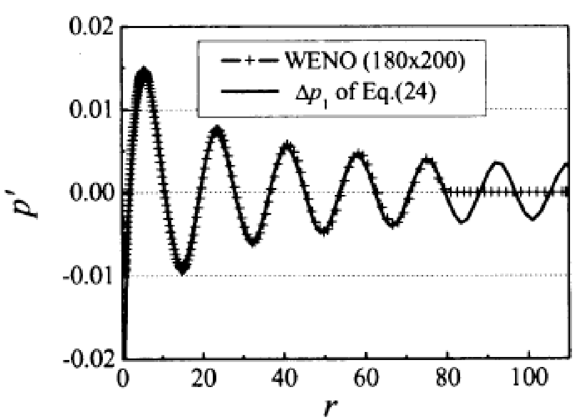


$|\, p' - \Delta p_1 \,|$,

$1.d-3$ to $8.5d-3$, 6lines



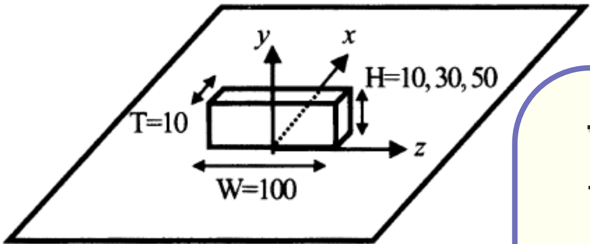
(a) $\theta = 45^\circ$



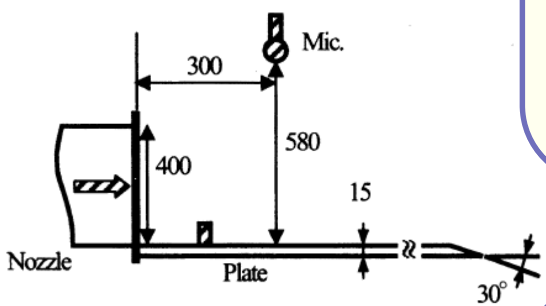
(b) $\theta = 90^\circ$

Acoustic field with a large solid surface: rectangular cylinder on the ground.

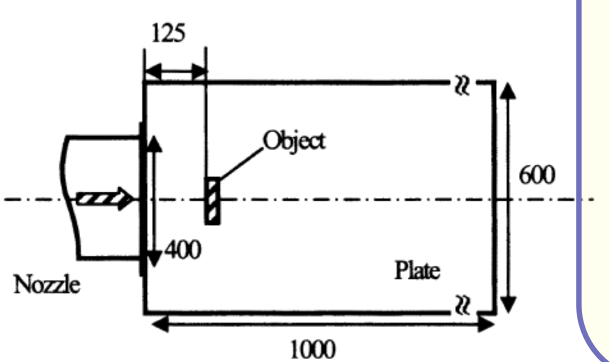
Experiment: $u_\infty = 44.7 m/s,\ \mathrm{Re} = 29000,\ M_\infty = 0.13$ ***Calculation:***



object on plane, mm



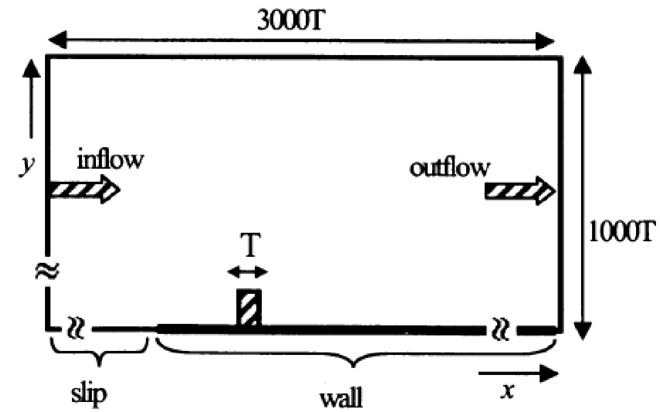
side view, mm

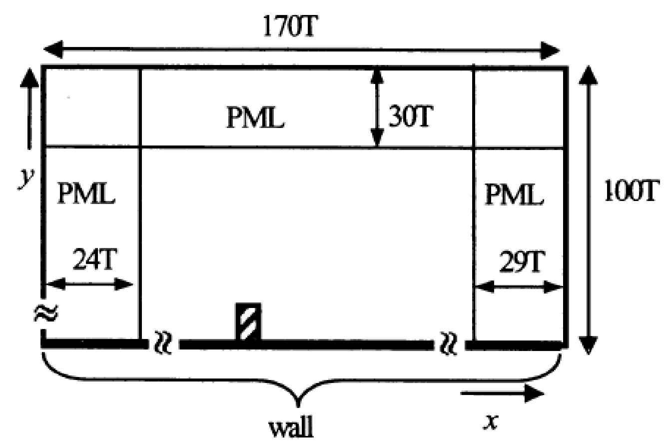


top view, mm

flow field:
- calculation domain (x-y plane),
- span – 2000T,
- grid – nonuniform 165x70x113
- min cell size 0.5 mm.

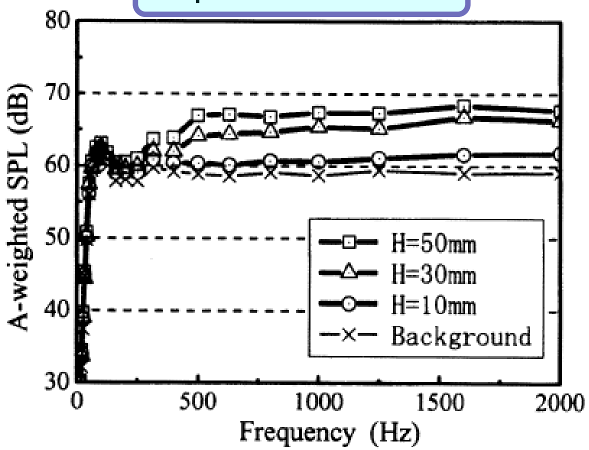
acoustic field:
- calculation domain (x-y plane),
- span – 200T,
- grid – nonuniform 125x70x130
- min cell size 2 mm



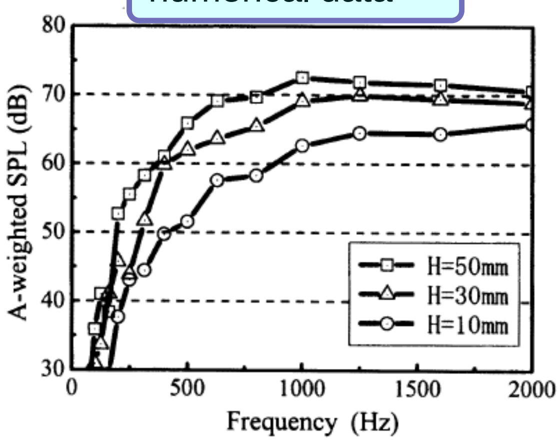


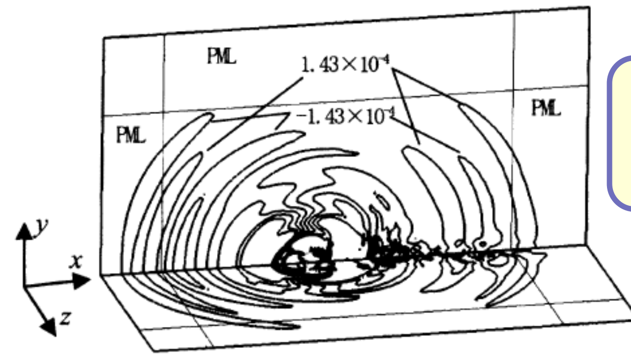
Comparison calculation with experiment

experimental data



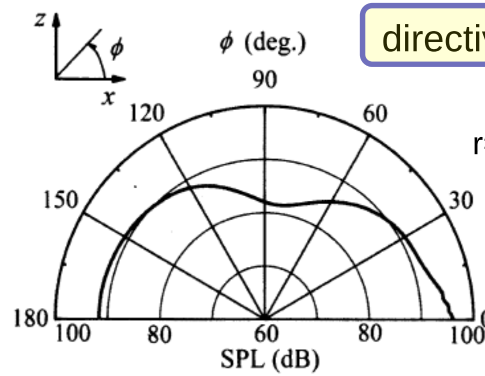
numerical data



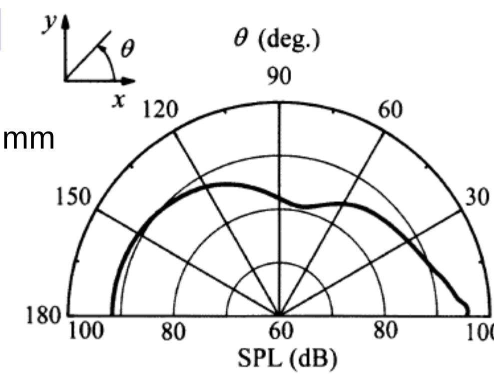


sound pressure *p'*:
-1.d-3 to 1.d-3,
8 lines

directivity

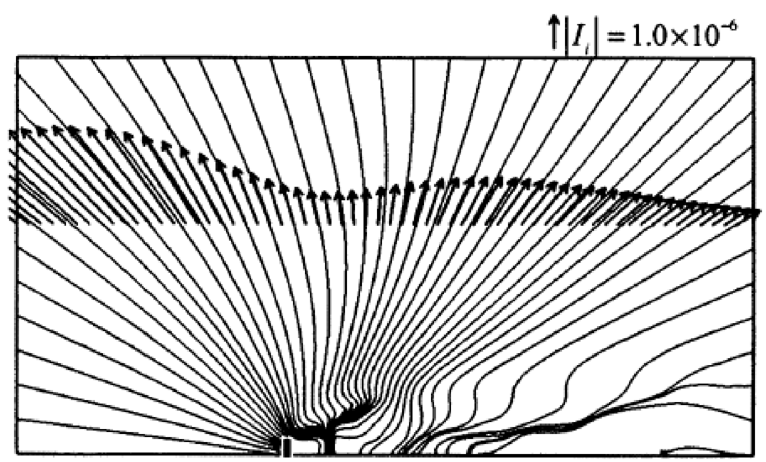


r=400 mm



SPL on the ground

SPL on the symmetric plane

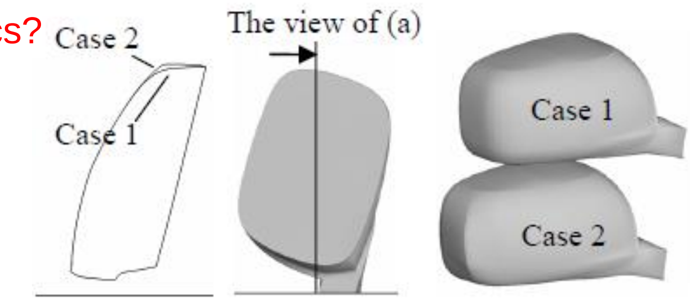


Acoustic intensity vectors and acoustic streamlines, z=0 plane

$$I_j = \overline{(p' - \overline{p'})(u' - \overline{u'})}$$

Application: automobile rear-view mirror

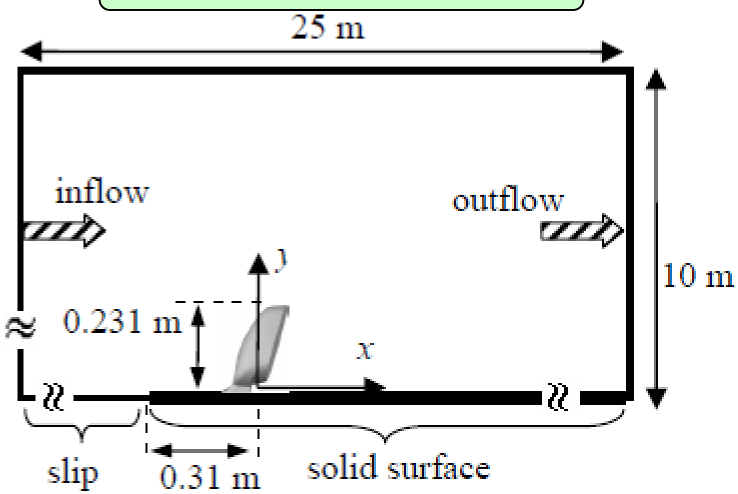
How does small change in shape effects acoustics?



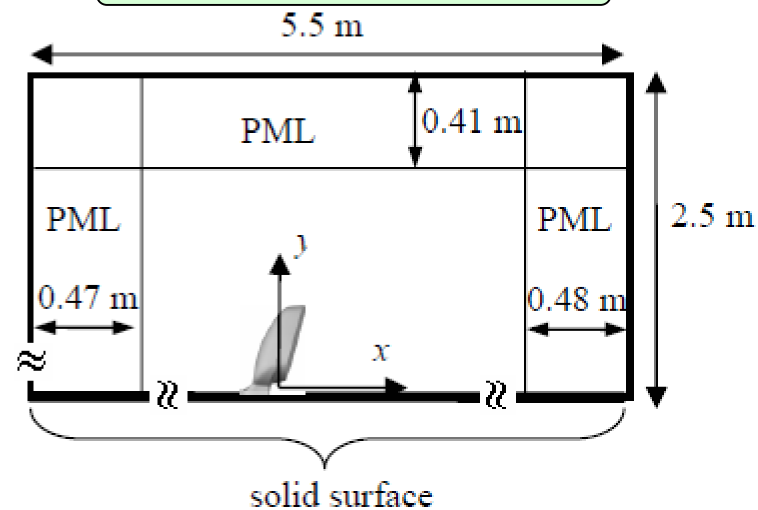
Simplified model: mirror with a short stay

Two cases: smooth edge (case 1) and sharp edge (case 2)

Flow calculation domain



Flow calculation domain

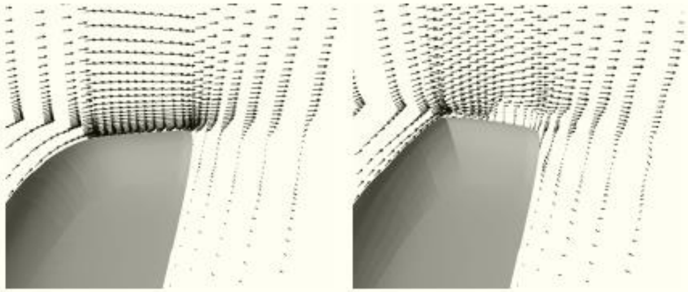


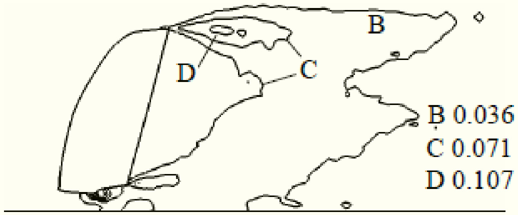
Single-block structured grid 309x143x159 in 25x10x20 m domain, wall grid spacing 0.4 mm, solid surface at x>-0.31 m, initial data:

$$p_* = 0,\ u_* = 38.9 m/s,$$

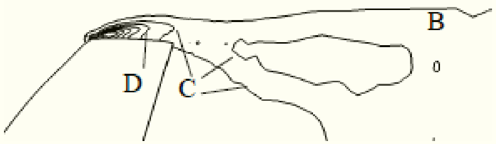
$$\mathrm{Re} = 253000/0.1m,\ M = 0.113$$

Single-block structured grid 283x150x160, grid spacing - 1mm (wall), 26 mm (domain), 15 points/wave 1.3KHz, PML b.c. (20 cells), time step – 2.56d-6 s (flow), 0.64d-6 s (acoustics), region of resolution – 1-2 kHz computed velocity field: separation

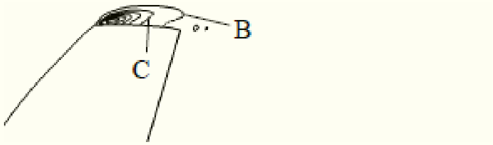




(a) Case 1.



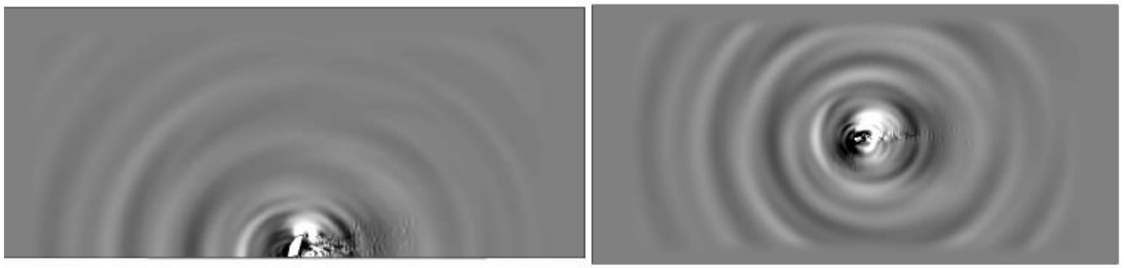
(b) Case 2 : in the tip region.



(c) Case 2 : in the tip region with a filter between 1 kHz and 2 kHz.

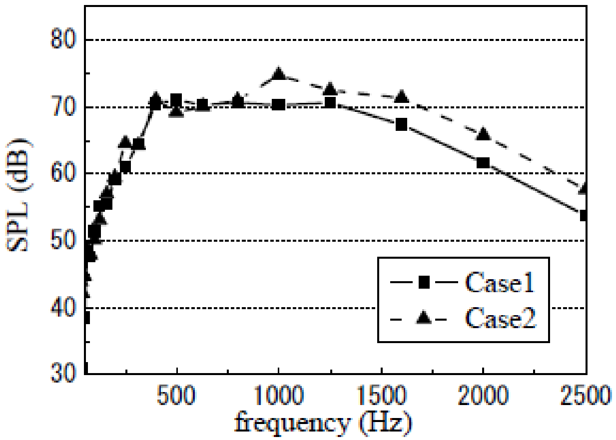
root-mean-square pressure fluctuation, $(p_*)_{rms}$

sound pressure in far field



x-y plane

ground plane

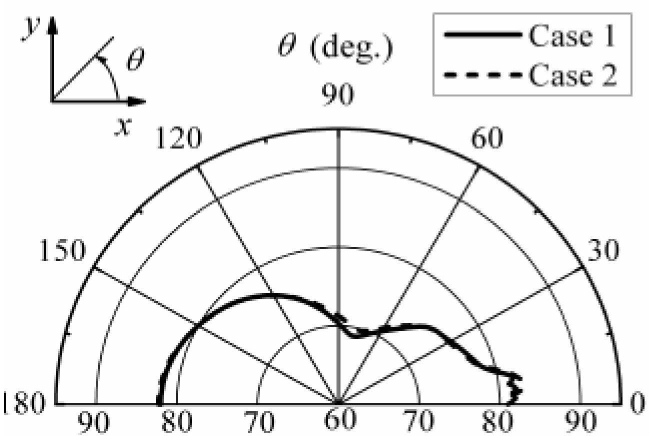


sound pressure spectrum at (x,y,z)=0, 0.8, 0)

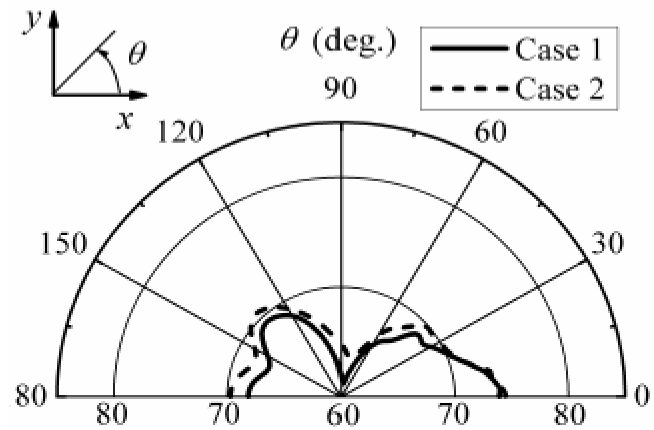
Generation of vortices around the tip is the reason of increasing sound

Application: automobile rear-view mirror

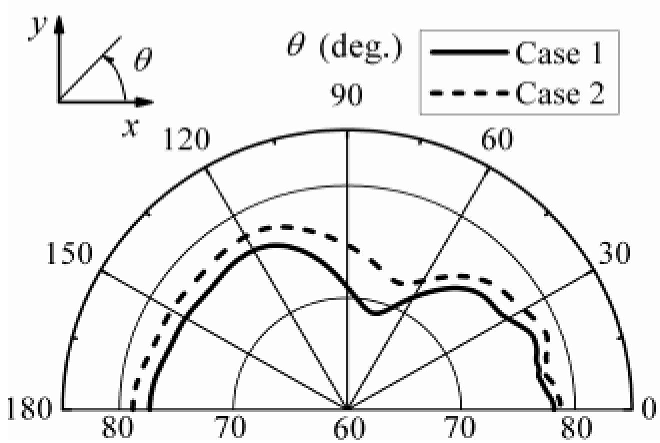
Directivity of sound: SPL at r=1.6 m from the origin in the x-y plane



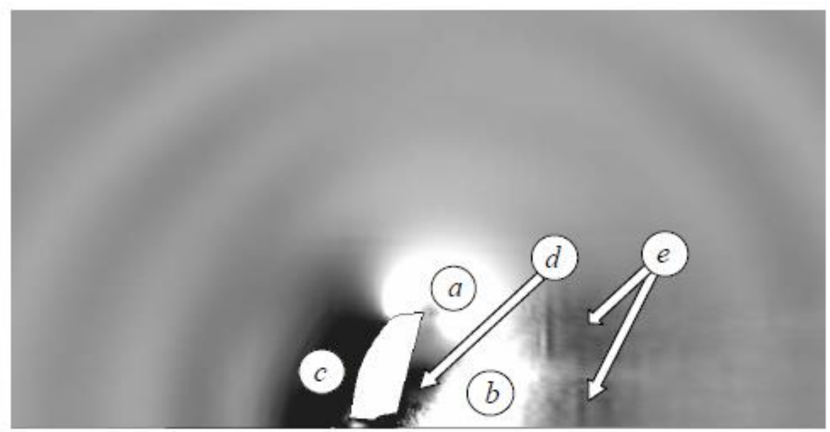
SPL of the overall frequency band



SPL for the frequency band from 1 to 2 kHz



SPL for the frequency band from 1 to 2 kHz, r=0.8 m



acoustic field with a filter 1-2 kHz, near field

A flow/acoustic splitting method based on asymptotic expansions has been developed to simulate near and far sound field generated by low Mach number flows:

- base flow equation given by incompressible N.-S. model + density fluctuation equation;
- acoustic equations – hyperbolic system obtained as the difference between compressible and incompressible equations;

The method has been verified and validated on the sound field generated from the flow around a cylinder (aelion tone) and the flow around a block on the plane solid surface; comparison with theoretical predictions and experimental data has shown good results.

The method has been applied to model near and far sound field around rear-view automobile mirror installed on a solid plate. Influence of small change in geometry on the radiated sound has been investigated. Flow separation at the sharp tip was shown to increase SPL at frequences above 1 kHz.