



# **DIRECT NUMERICAL SIMULATION OF TRANSITION IN SUPERSONIC BOUNDARY LAYERS**

**Alexey Kudryavtsev and Dmitry Khotyanovsky**



Khristianovich Institute of Theoretical and Applied Mechanics  
Russian Academy of Sciences, Siberian Branch  
Novosibirsk

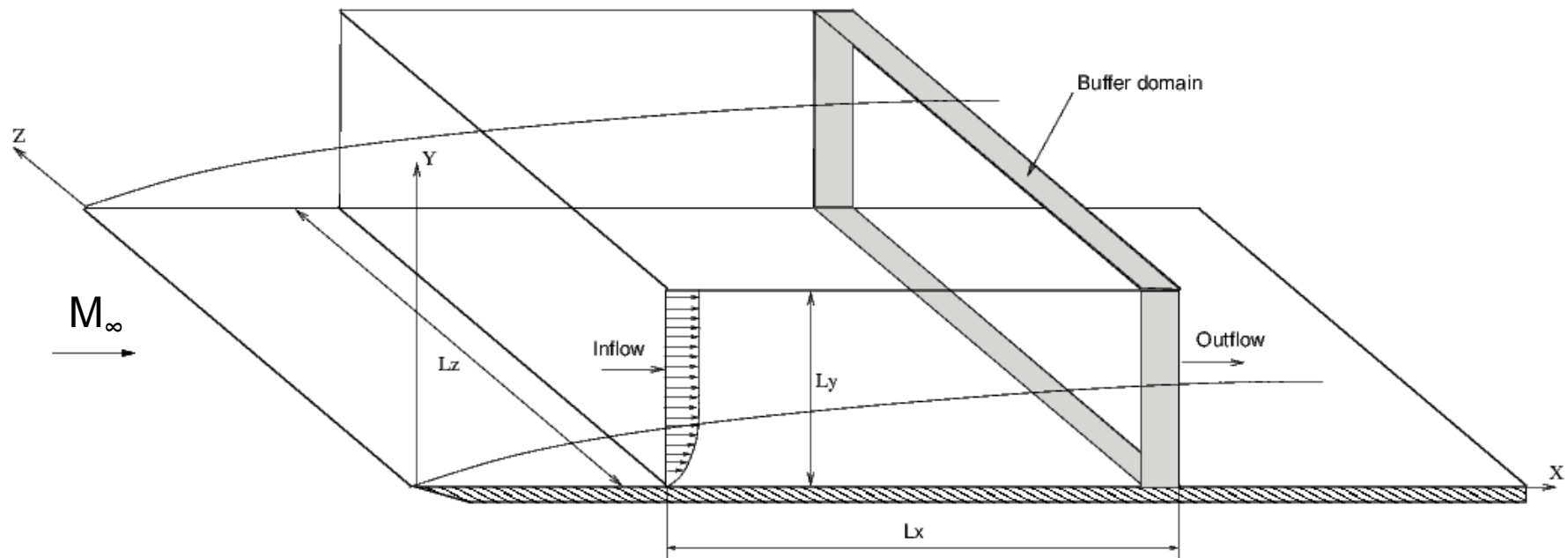
# Outline

- Motivation
- Problem formulation and numerical approach
- Transition to turbulence in supersonic boundary layer,  $M = 2$
- Transition to turbulence in hypersonic boundary layer,  $M = 6$
- Conclusions

# Motivation

- The transition to turbulence in supersonic and hypersonic boundary layers leads to a great increase in drag and thermal load on surface of flying vehicles.
- A better understanding of mechanisms of the transition can help to develop technologies for controlling and delaying of laminar-turbulent transitions.
- Such technologies can be a key to design of perspective aircraft of new generation, in particular civilian supersonic transport vehicles.
- Direct Numerical Simulation (DNS) of the transition in compressible flows in many aspects is interconnected with computational aeroacoustics. External acoustic waves can excite the instability via transforming to the internal flow instability waves (*receptivity process*), which themselves are of mixed vortical/acoustical nature. The turbulent boundary layer is a powerful source of aerodynamic noise. Both in DNS of transition and computational aeroacoustics high resolution numerical methods are required.

# Supersonic boundary layer on a flat plate



Schematic of the computational domain

## Numerical approach

- **3D compressible Navier – Stokes equations are solved with CFS3D or HyCFS codes:**
  - Finite difference WENO-5 scheme for convective fluxes
  - 4<sup>th</sup> order central differences on a compact stencil for diffusive fluxes
  - 4<sup>th</sup> order explicit Runge–Kutta-Gill algorithm for time integration
  - Grid is stretched in the normal direction, up to 40 millions cells are used
  - MPI parallelization for CFS3D solver, three-level (CUDA/OpenMP/MPI) parallelization for HyCFS solver
- **Boundary conditions:**
  - Self-similar B.L. solution (+ LST eigenfunctions) at the inflow boundary
  - Non-reflecting boundary conditions at the upper and outflow boundaries
  - Periodic conditions at the spanwise boundaries
  - Plate surface: no-slip velocity, zero-pressure gradient, adiabatic wall temperature
  - Sponge buffer domain near the outflow boundary, in which the flow is relaminarized to avoid numerical reflections and false upstream influence

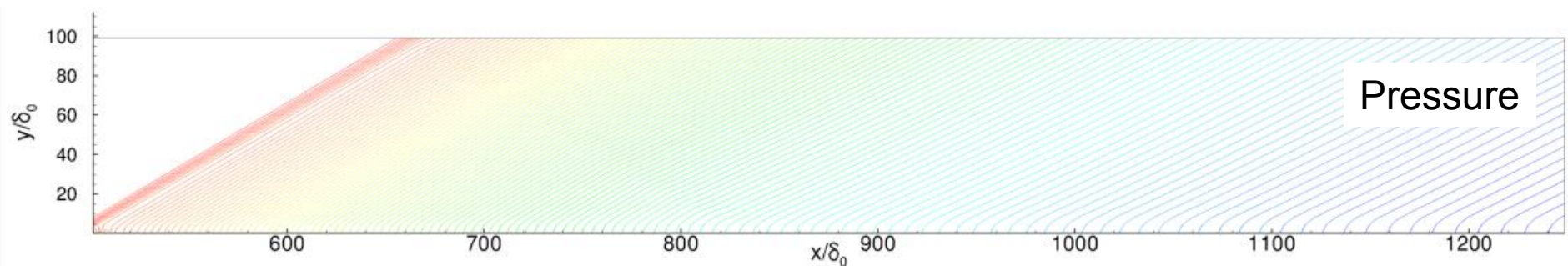
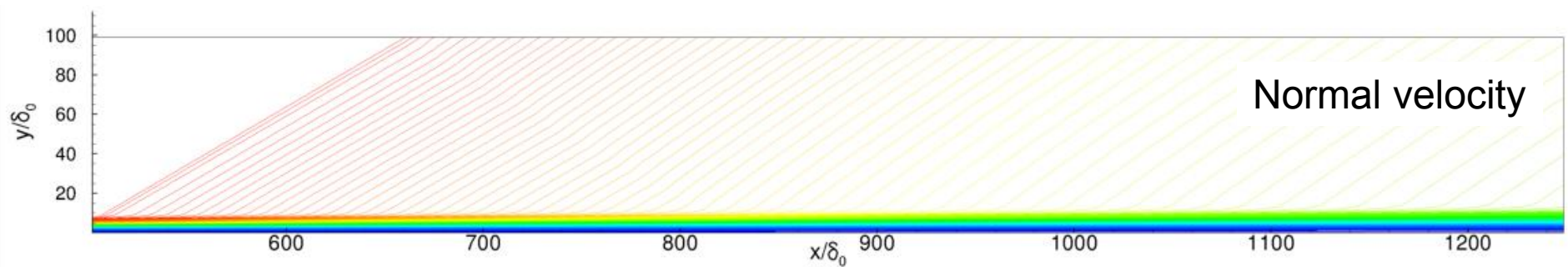
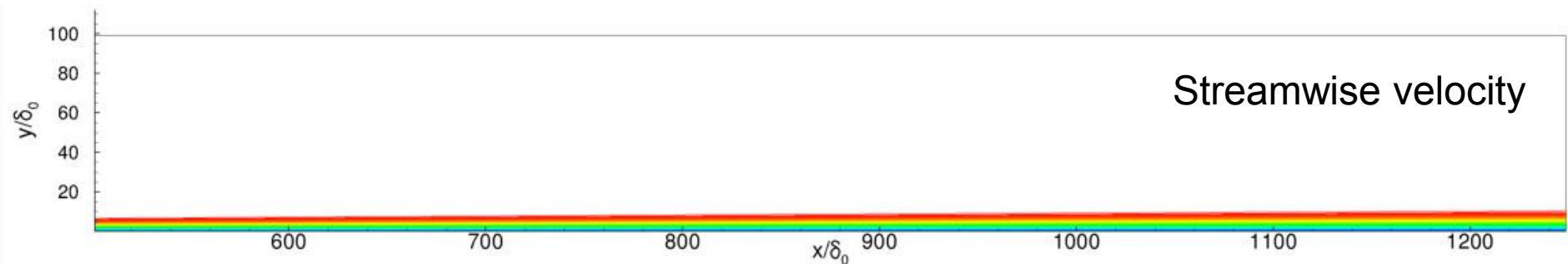
## Strategy of computations

- Compute steady 2D laminar basic flow using self-similar inflow profiles. Set adiabatic wall.
- Expand the flow in 3D. Set isothermal wall.
- Perform LST analysis for the inflow profiles.  
Determine unstable disturbances.
- Impose LST eigenfunctions as the inflow forcing at unstable frequencies.
- Run 3D computations and see what happens.

## Boundary layer transition at Mach 2

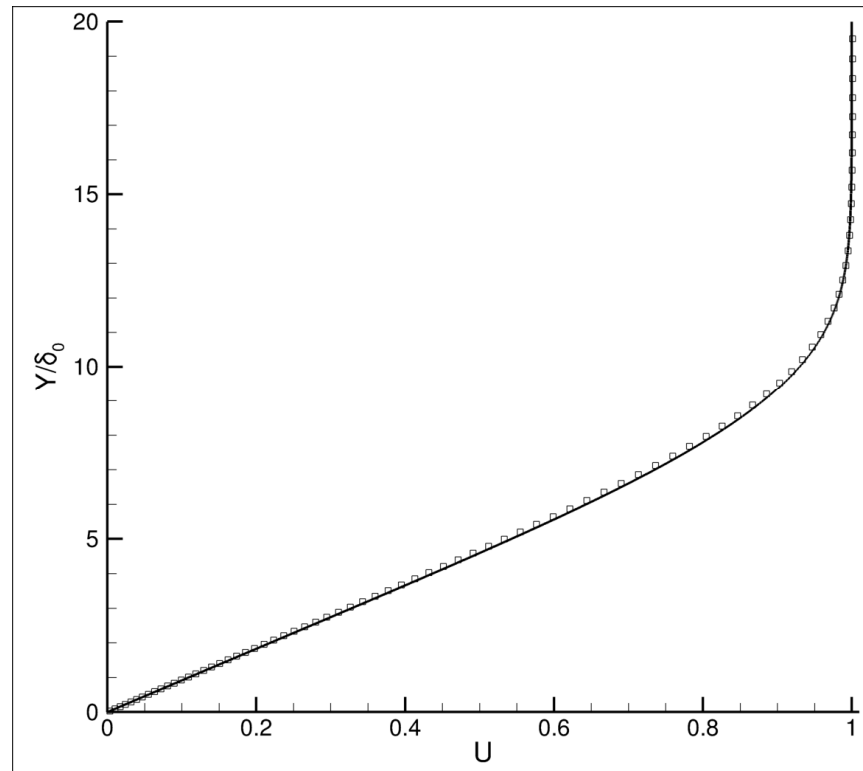
- Two different mechanisms of nonlinear wave interaction leading to transition are discussed for boundary layers at moderate supersonic speeds: subharmonic resonance and oblique breakdown.
- *Kosinov, Semenov et al.* 1990-present. Numerous experimental studies of the B.L. transition at  $M=2$ . Observation of subharmonic resonance was claimed. Other authors found evidence of oblique breakdown in the same experiments.
- *Fasel, Thumm, Bestek.* (1993) Direct numerical simulation of transition in supersonic boundary layers. Oblique breakdown.
- *Sandham, Adams, Kleiser.* (1995) Direct simulation of breakdown to turbulence following oblique instability waves in a supersonic boundary layer.
- *Terzi, Mayer, Fasel.* (2008) The late nonlinear stage of oblique breakdown to turbulence in a supersonic boundary layer.
- In the current work we simulate transition to turbulence at  $M = 2$  via oblique breakdown.

## Laminar basic flow. $M=2$ , $Re_{\delta_0}=500$

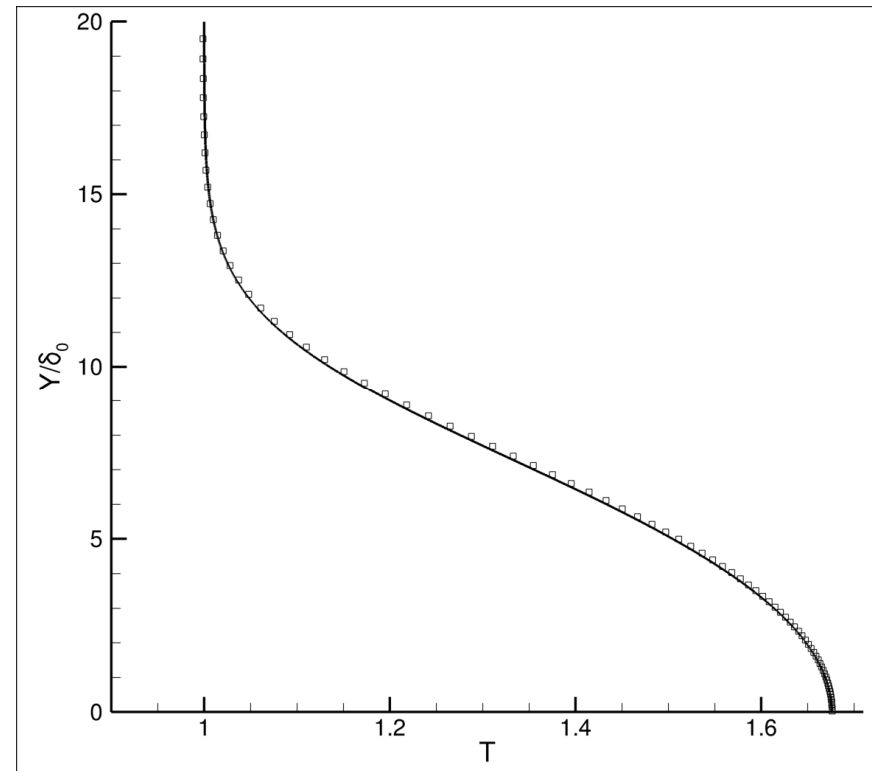




## Laminar basic flow profiles at $x/\delta_0=2000$ . $M = 2$



Streamwise velocity



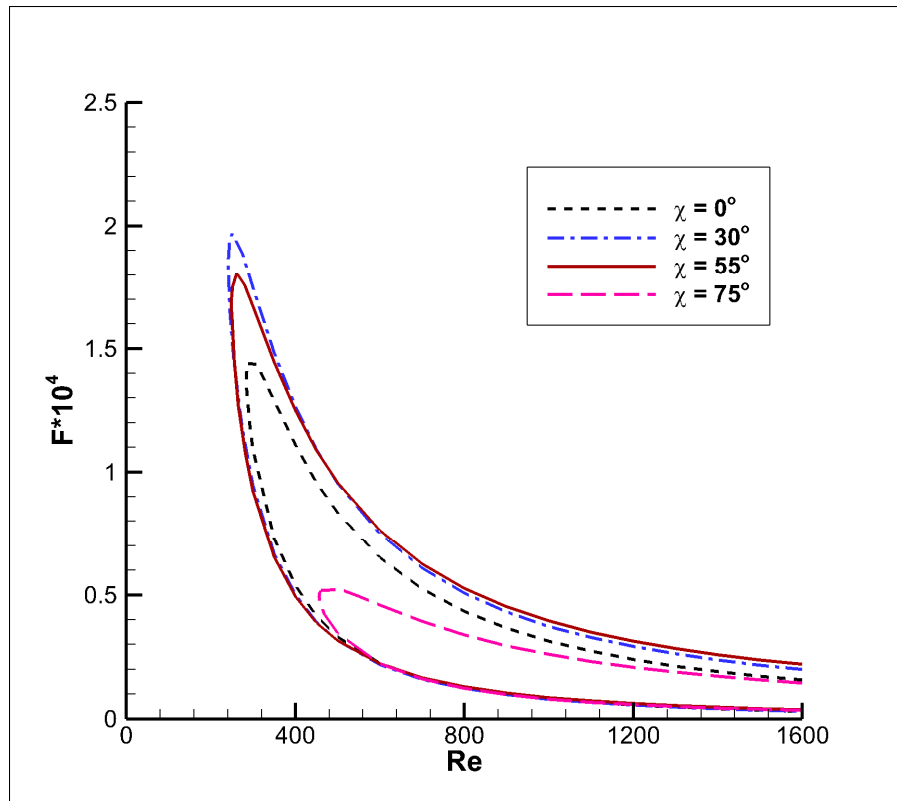
Temperature

Solid curves correspond to self-similar solution

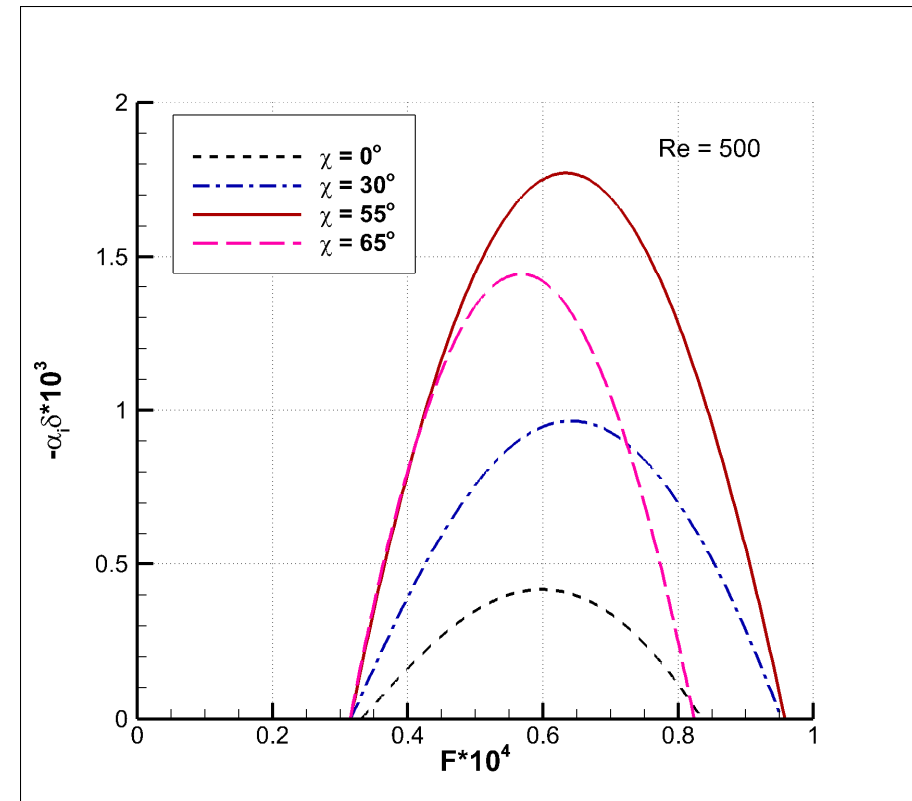
## Linear stability analysis

- Viscous linear stability eigenvalue problem is solved in quasi-parallel assumption with a LST code using iterative shooting method. Range of unstable frequencies is determined, other stability characteristics such as linear growth rates are calculated.
- In supersonic boundary layer at  $M = 2$  only one mode of disturbances is unstable (***first mode***). It represents vortical disturbances very similar to Tollmien-Schlichting waves observed in incompressible boundary layer flows.
- However, in contrast with the incompressible case, the most amplified disturbances are three-dimensional waves propagating approximately at the angle  $\chi = 55^\circ$  to the mean flow direction.
- Linear stability characteristics are used to choose the flow parameters for DNS, the LST eigenfunctions are employed for inflow forcing.

## Results of LST analysis at $M = 2$



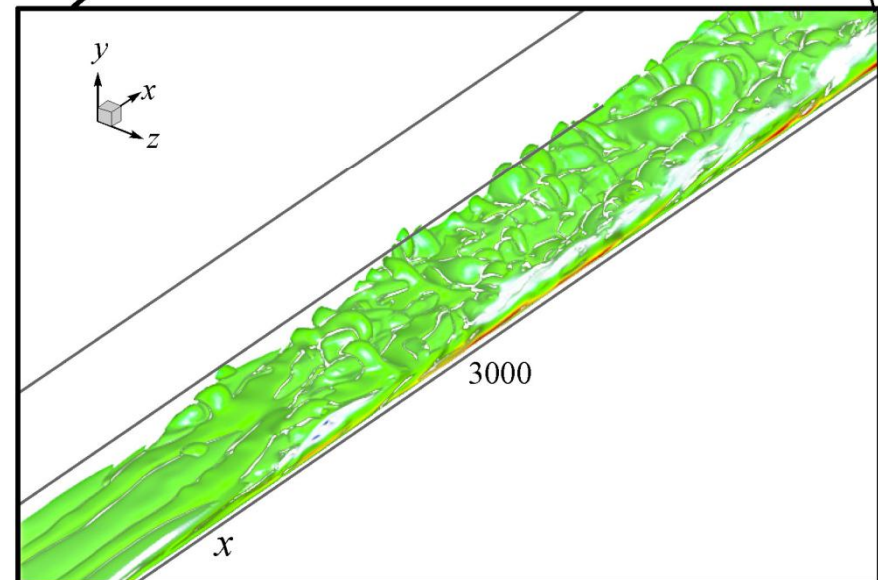
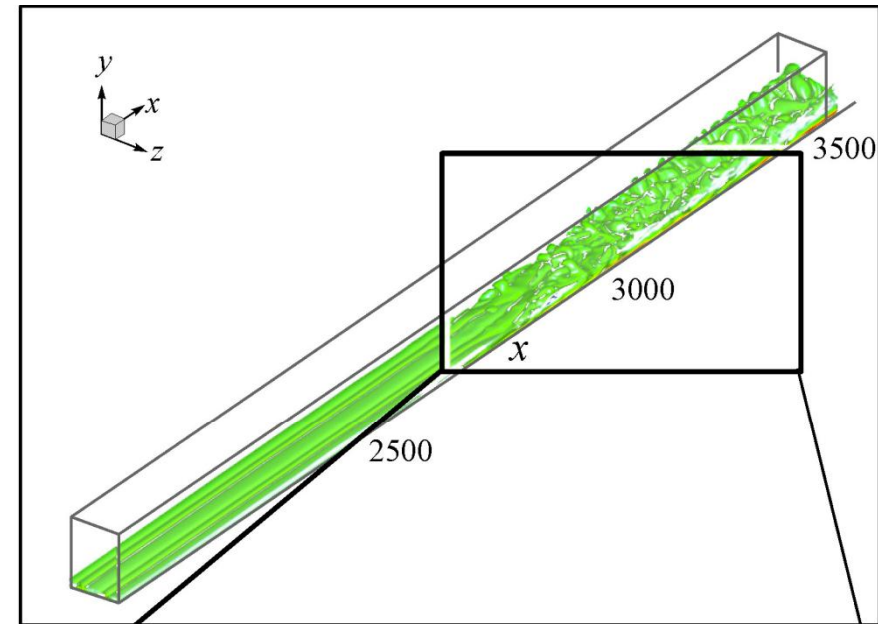
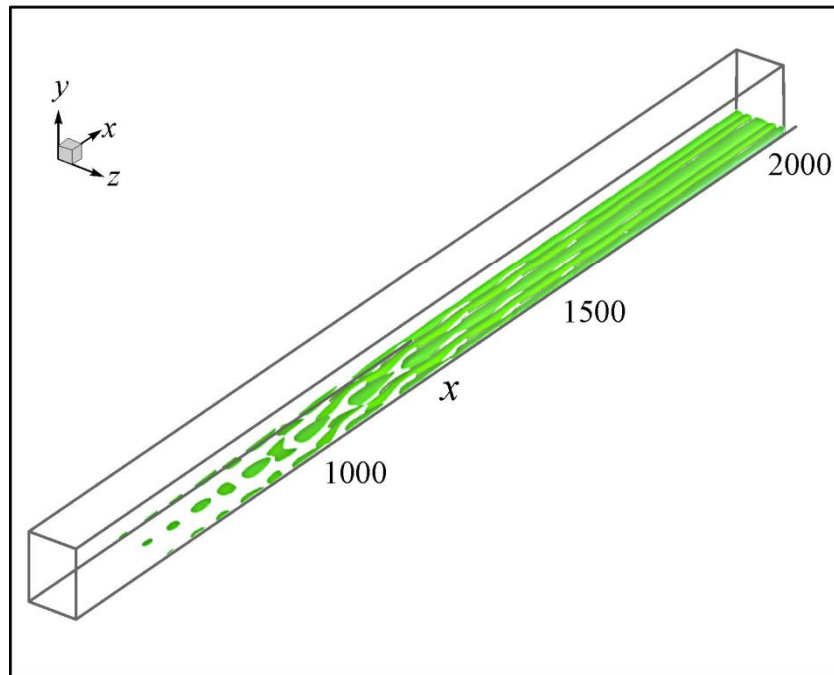
Neutral curves



Growth rates vs. frequency

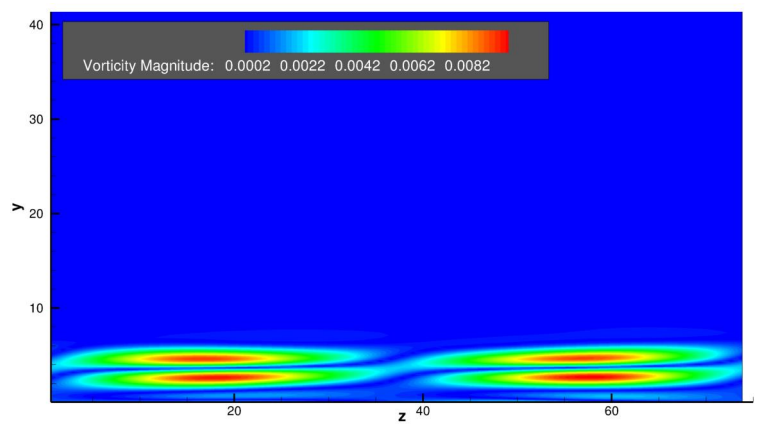
**DNS results at  $M = 2$ ,  
 $Re_{\delta_0} = 500$ ,  $\chi = \pm 55^\circ$ ,  $A = 5 \cdot 10^{-3}$**

Isosurfaces of velocity fluctuations.

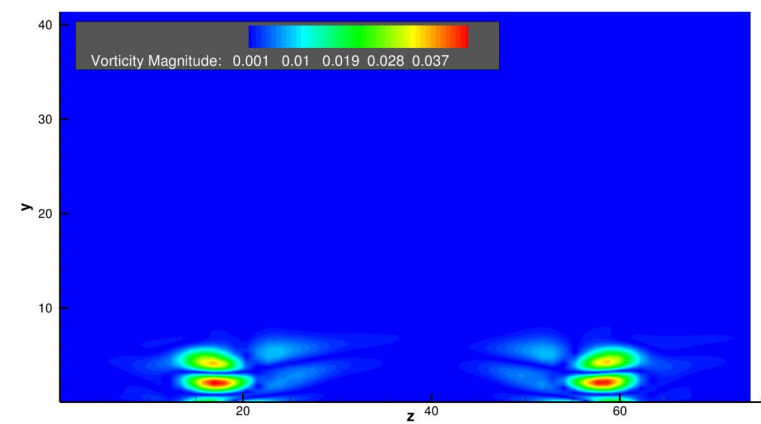


## Flow in Y-Z cross-sections

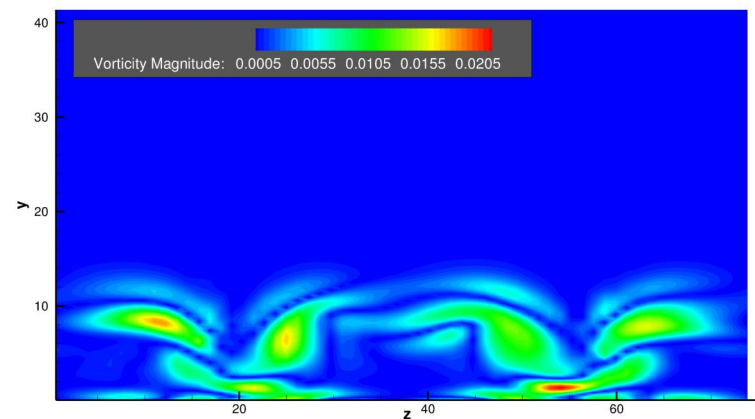
Vorticity magnitude contours



$X/\delta_0=1000$

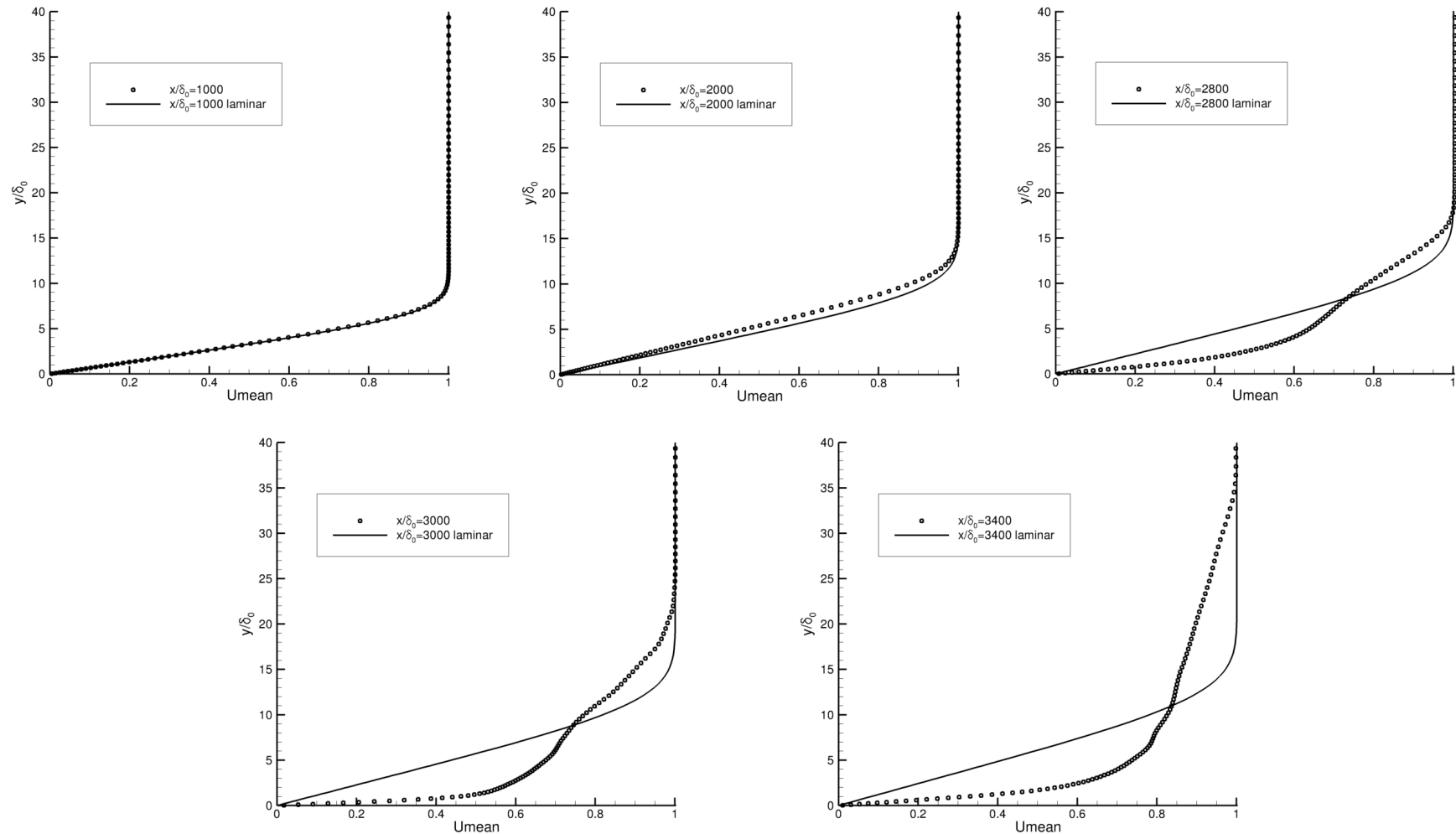


$X/\delta_0=2000$



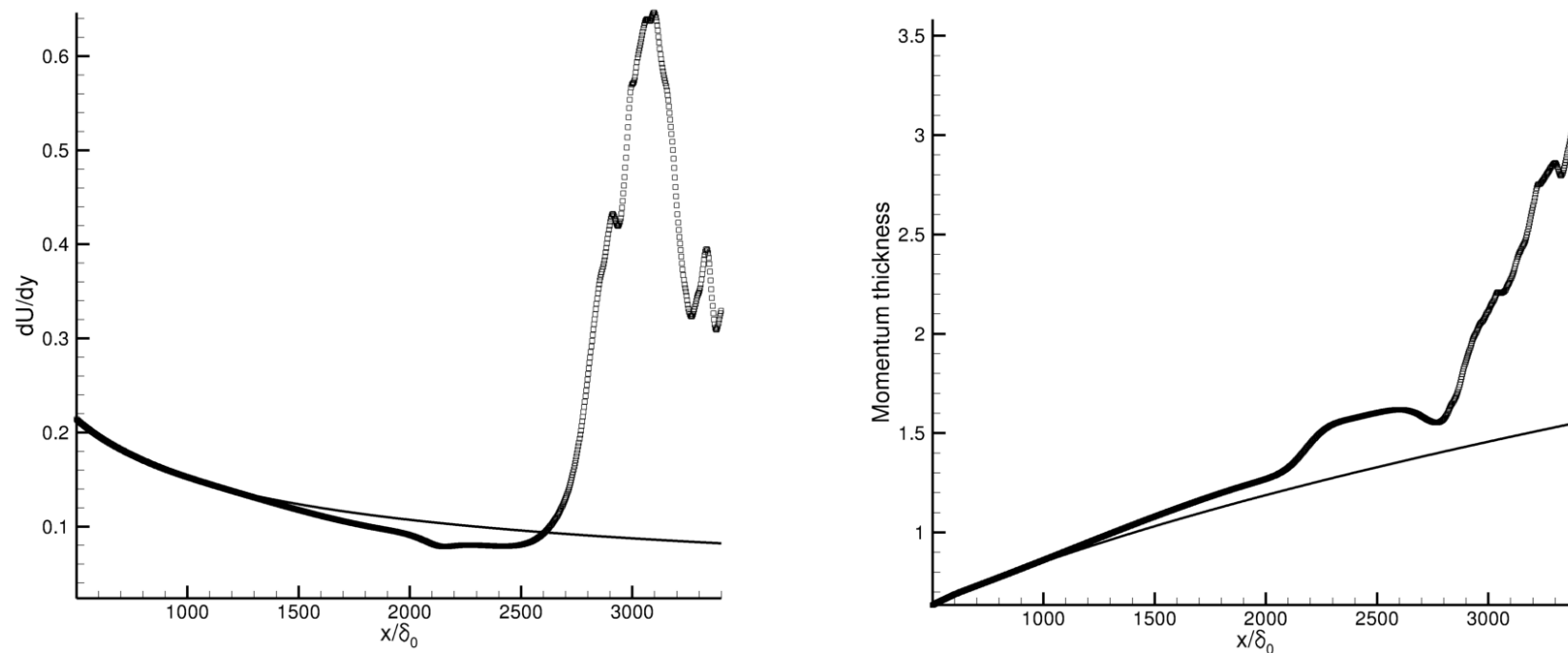
$X/\delta_0=2600$

# Mean velocity profiles in several cross-sections



Solid curves correspond to laminar basic flow

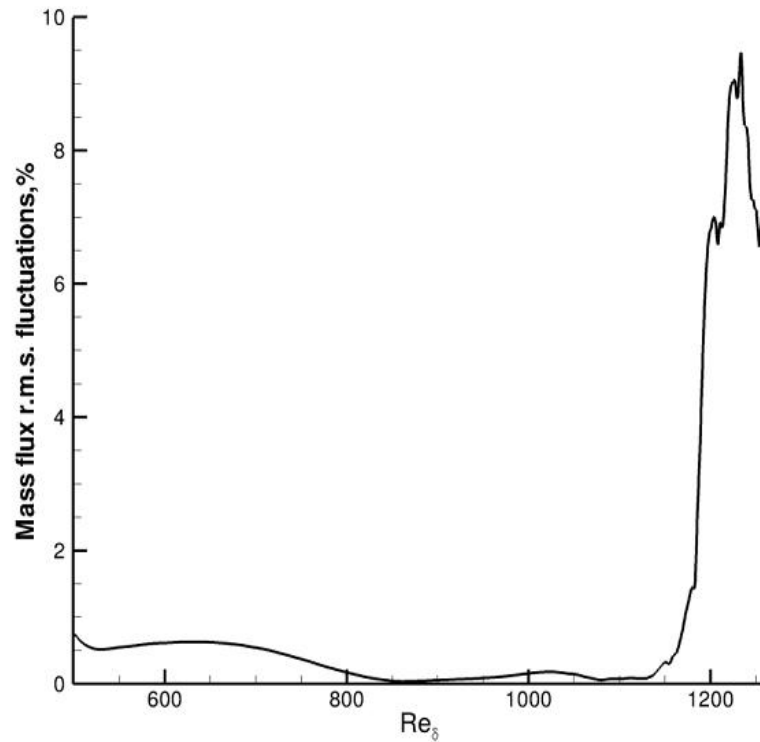
## Wall shear stress and b.l. thickness variation



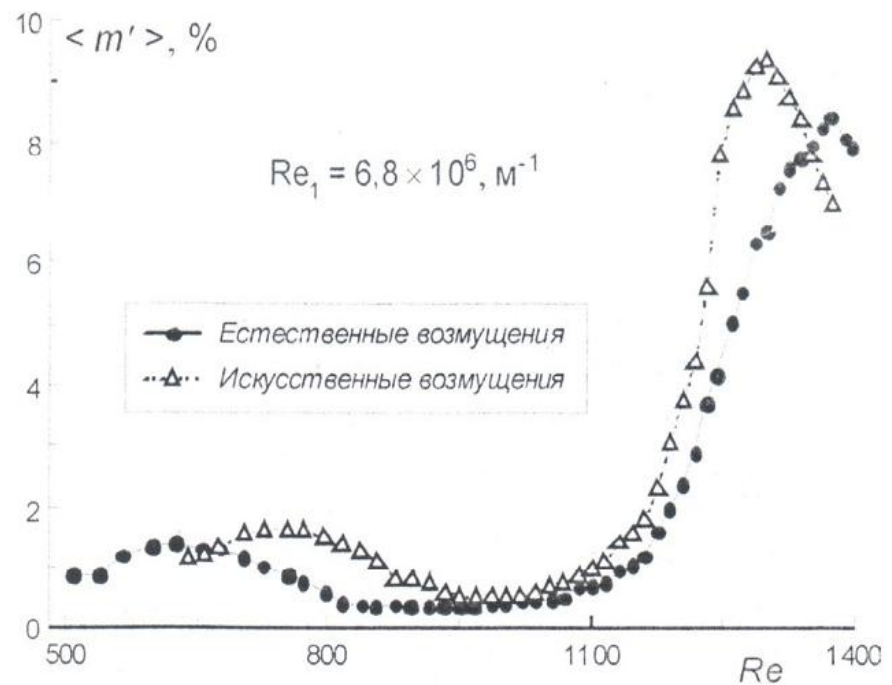
Solid curves correspond to self-similar laminar boundary layer solution

## Comparison with experimental data

Mass flow rate r.m.s. fluctuations



Current DNS



Experiments of Yermolaev, Kosinov and Semyonov (2008)



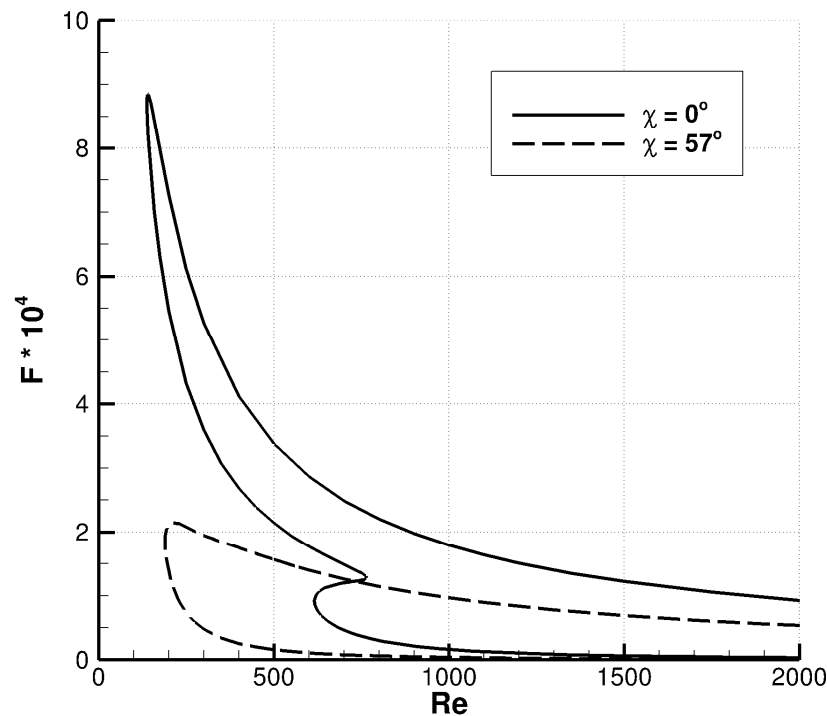
## Boundary layer transition at Mach 6

- Transition to turbulence at hypersonic speeds is a less studied and understood subject than transition at moderate supersonic Mach numbers.
- Experimental studies by *Demetriades*(1974), *Kendall* (1975), *Stetson et al.* (1983) and, more recently, by *Maslov, Shiplyuk et al.* (ITAM SB RAS, 1997-present) showed the presence of instability waves which were identified with the so-called second mode disturbances predicted by LST.
- Extensive numerical investigations of early 2D stages of the instability were performed in the 2000s by *Egorov, Fedorov & Sudakov* (TsAGI, MFTI), in particular they studied receptivity of the boundary layer to external disturbances. This problem was also studied by *Zhong* (UCLA).
- Some numerical simulations of transition (Pruett et al, 1995, Adams & Kleiser, 1996) were performed at slightly lower Mach number  $M = 4.5$ .

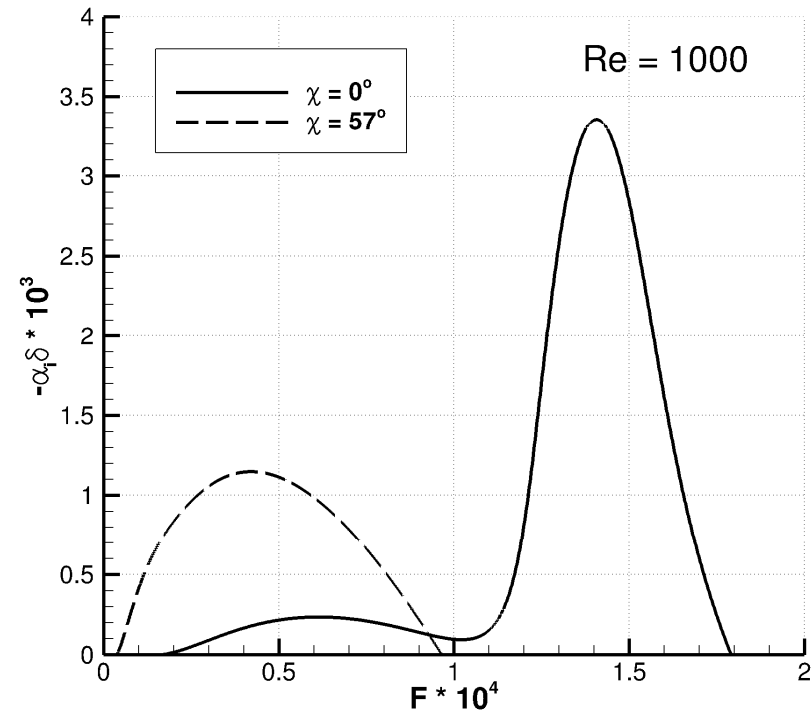
## Linear stability analysis

- At hypersonic speeds in addition to the 1<sup>st</sup> vortical mode there are multiple unstable acoustic modes (Mack's modes after *L. Mack*).
- In hypersonic boundary layer at  $M = 6$  the most unstable disturbances are those of the so called **second mode**. Disturbances of the 1<sup>st</sup> mode are also unstable, however grow much slower. The most unstable disturbances of 1<sup>st</sup> mode are waves propagating at angles 55-50° to the main flow direction.
- At the same time, 1<sup>st</sup> mode disturbances can stay unstable much longer downstream than 2<sup>nd</sup> mode waves.
- Different combinations of 1<sup>st</sup> and 2<sup>nd</sup> mode disturbances are used as inflow forcing in the current DNS to study various possible routes to turbulence at hypersonic speeds.

## Results of LST analysis at $M = 6$

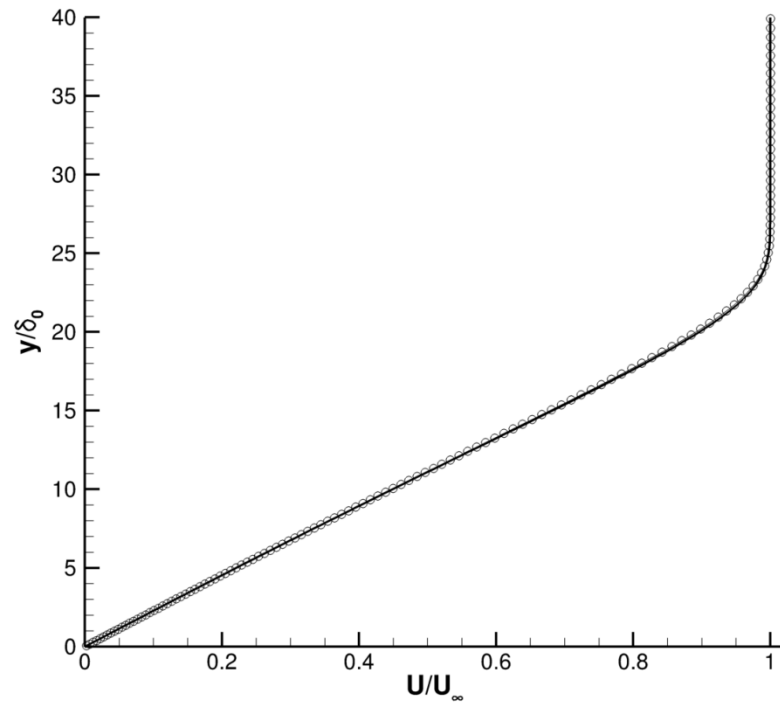


Neutral curves

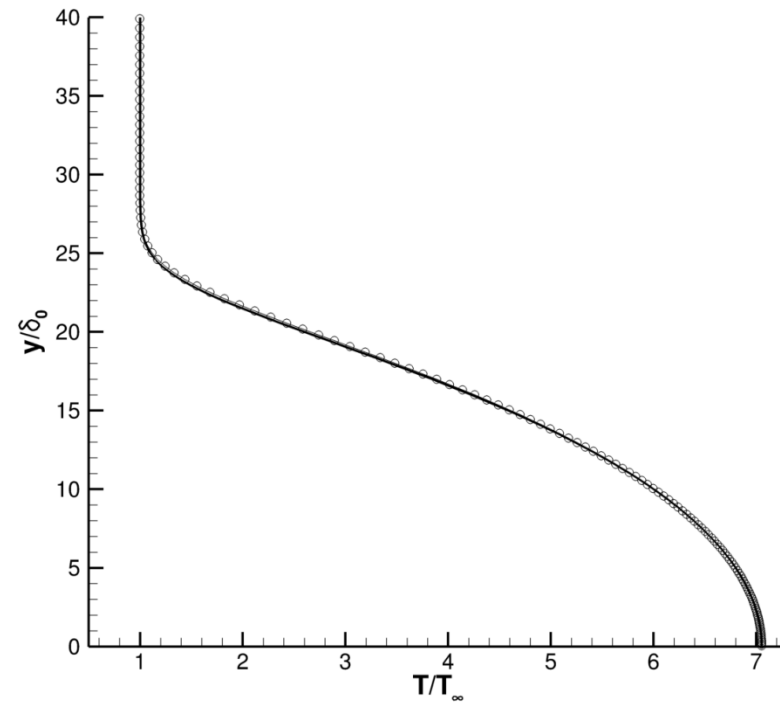


Growth rates vs. frequency

## Laminar basic flow profiles at $x/\delta_0=2000$ . $M = 6$



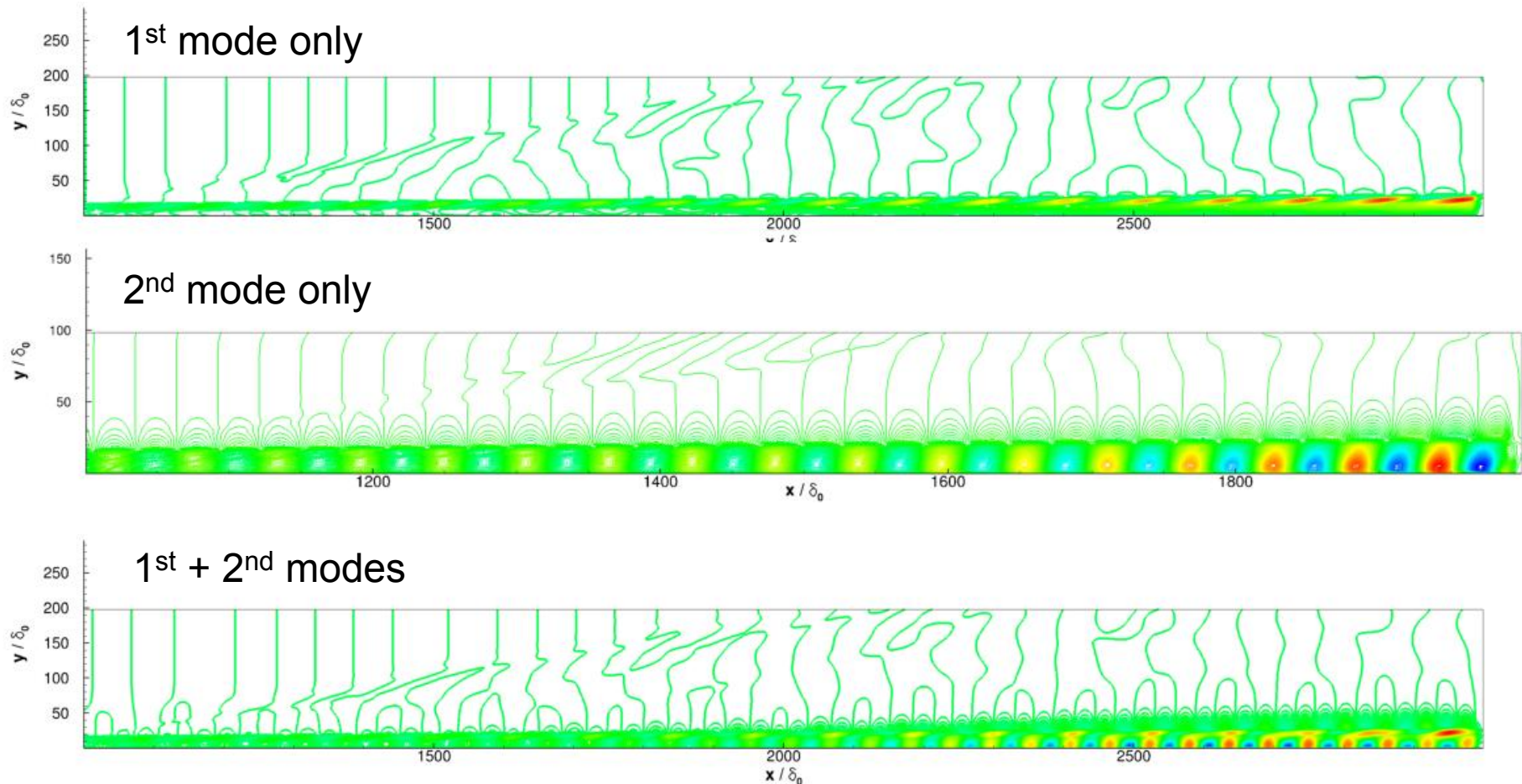
Streamwise velocity



Temperature

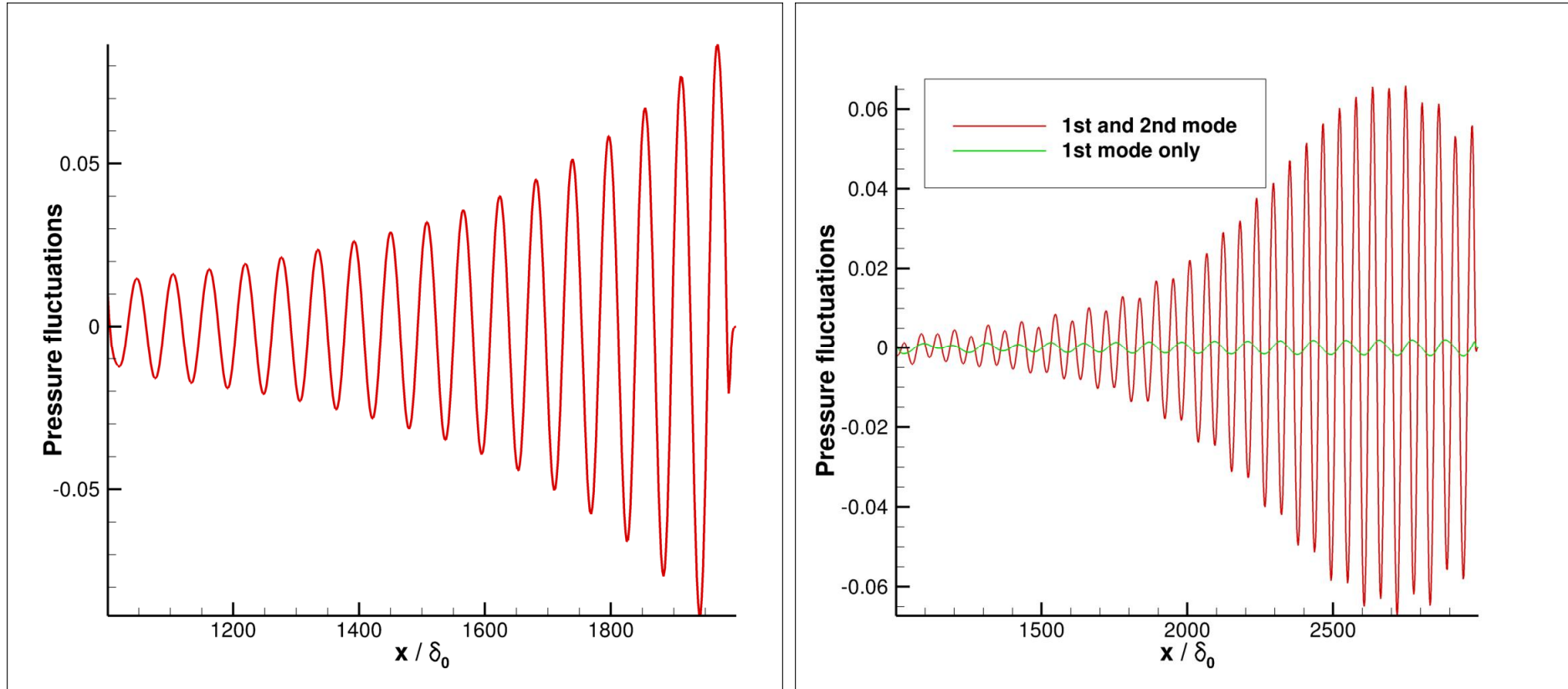
Solid curves correspond to self-similar solution

## Development of disturbances of 1<sup>st</sup> and 2<sup>nd</sup> modes



Instantaneous pulsation velocity contours in plane  $z = 0$

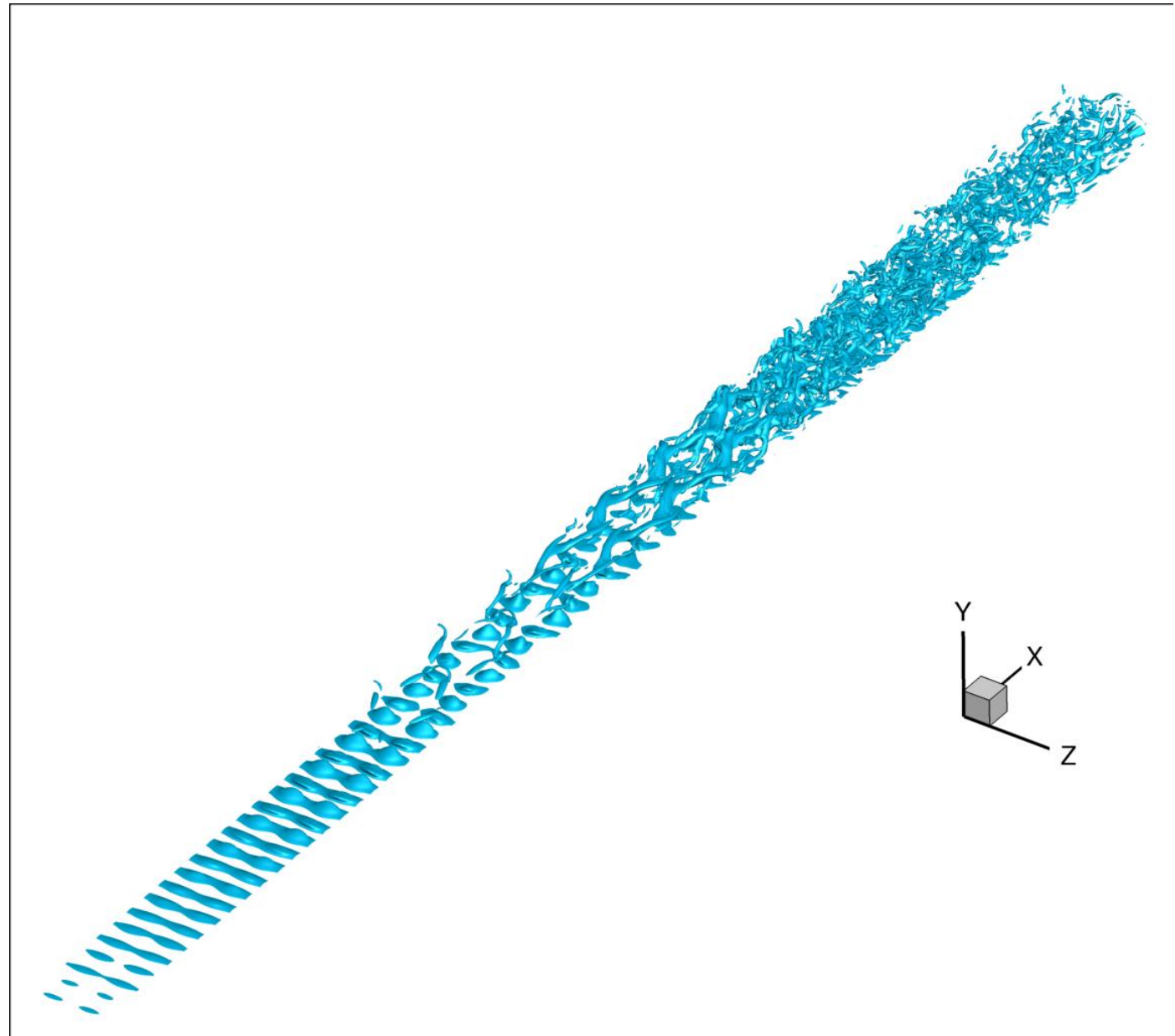
## Wall pressure pulsations



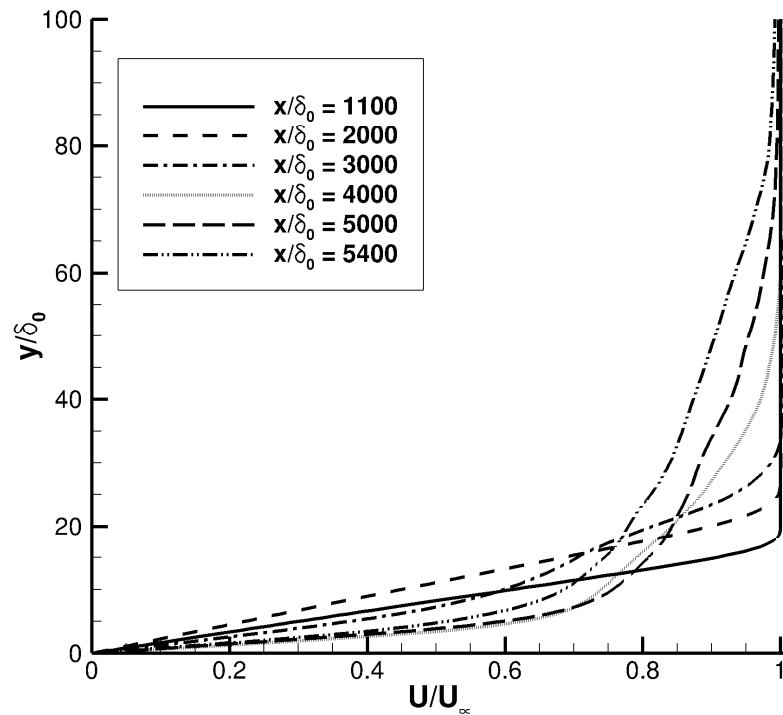
# Nonlinear development and transition

Superposition of disturbances of 1<sup>st</sup> and 2<sup>nd</sup> modes

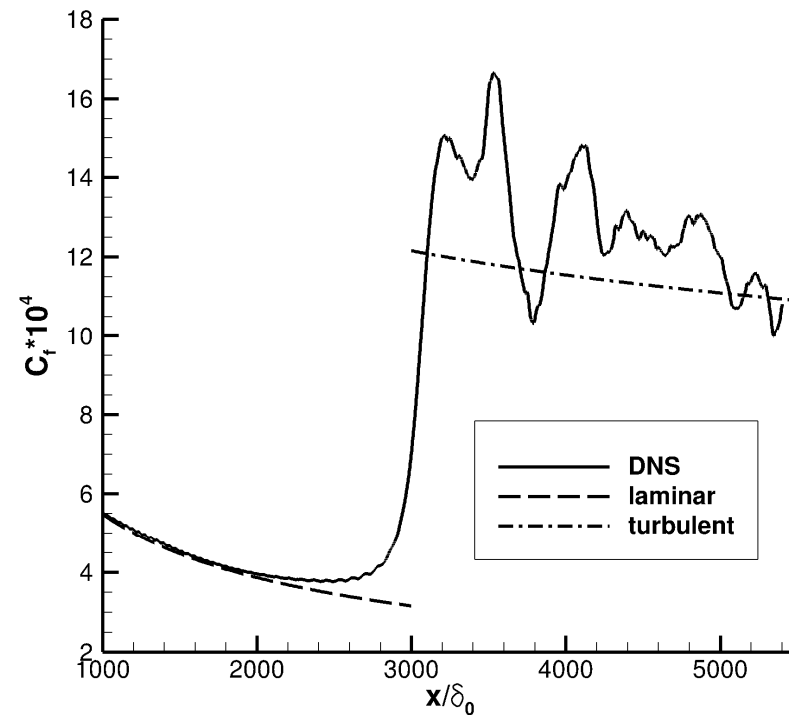
Q-criterion isosurface



## Mean flow profiles and skin friction



Mean velocity profiles at several cross-sections



Skin friction distribution along plate



## Conclusions

- Transition to turbulence in supersonic and hypersonic boundary layers (at  $M = 2$  and  $6$ ) including early linear stages, later nonlinear stages as well as stochastization, breakdown of laminar flow and transition itself was numerically simulated.
- At  $M = 2$  the oblique breakdown scenario of transition was reproduced: non-linear evolution of two symmetrical instability waves causes formation of elongated streamwise vortex structures and, at some downstream position leads to fast growth of 3D small-scale pulsations and transition to turbulence. Good agreement with experiments on the location of the transition onset is observed.
- At  $M = 6$  2D disturbances of the 2<sup>nd</sup> mode are dominant at early stages and rapidly grow, the 1<sup>st</sup> mode's main input is the transverse component of the fluctuations. Farther downstream 2<sup>nd</sup> mode stabilizes while 1<sup>st</sup> mode keeps steadily growing, 3D fluctuations become more visible. Onset of the laminar-turbulent transition is observed at significantly larger distance from the leading edge in comparison with the  $M = 2$  case.

**Thank you for your attention!**