Goldstein Generalised Acoustic Analogy: Jet Noise Source Modelling

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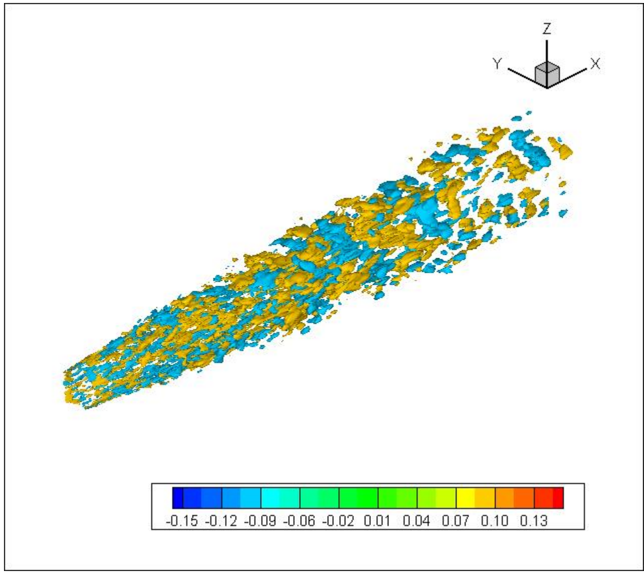
Outline

- Motivation
- Goldstein generalised acoustic analogy
- Correlation analysis of fluctuating turbulent stress
  - Similarities of effective jet noise sources and analytical source approximations
  - Eddy convection velocity vs the local meanflow velocity
  - Normality and quasi-normality hypnoses for noise source amplitudes modelling
  - Anisotropic effects
- Conclusions

Sources of the acoustic analogy

Low-order models

?



Detailed investigation of the source statistics to establish common trends and thus simplify the modelling

Noise sources

$$S(\boldsymbol{x},\omega) = \int_{V_y}\int_{V_\Delta} \hat{R}_{ijkl}(\boldsymbol{y},\boldsymbol{\Delta},\omega)\hat{\gamma}_{ij}(\boldsymbol{y},\omega|\boldsymbol{x})\hat{\gamma}^*_{kl}(\boldsymbol{y}+\boldsymbol{\Delta},\omega|\boldsymbol{x})\mathrm{d}\boldsymbol{\Delta}\mathrm{d}\boldsymbol{y}$$

where

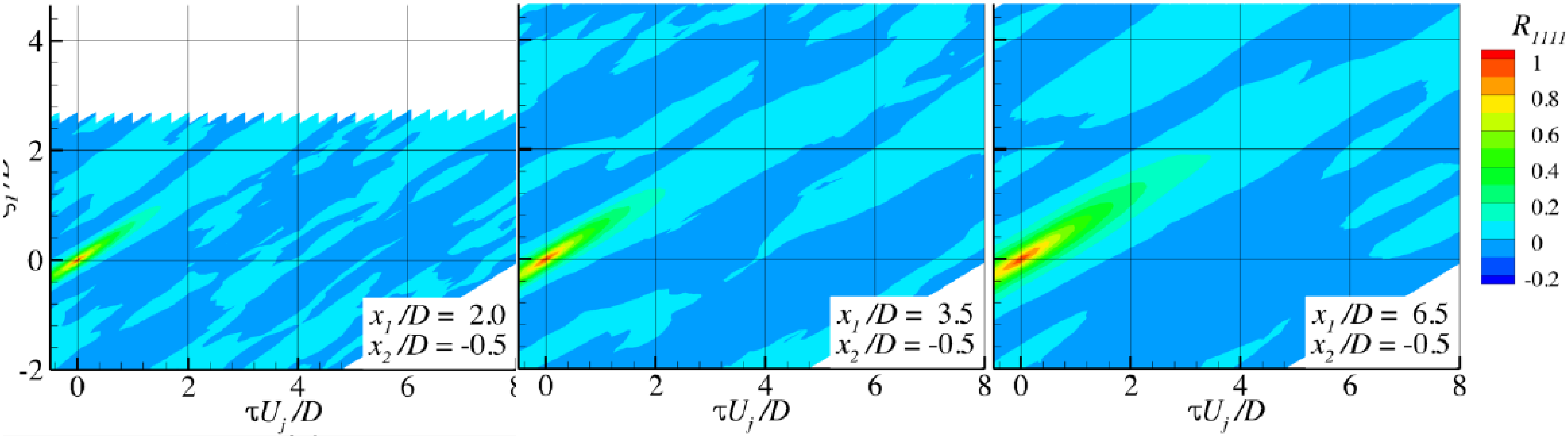
$$\hat{R}_{ijkl}(\boldsymbol{y},\boldsymbol{\Delta},\omega) = \int R_{ijkl}(\boldsymbol{y},\boldsymbol{\Delta},\tau)e^{-i\omega\tau}\mathrm{d}\tau = \overline{\hat{e}''_{ij}(\boldsymbol{y},\omega)\hat{e}''^*_{ij}(\boldsymbol{y}+\boldsymbol{\Delta},\omega)},$$

$$e''_{ij}(y,t) = -\left(\rho v'_i v'_j - \overline{\rho v'_i v'_j}\right)$$

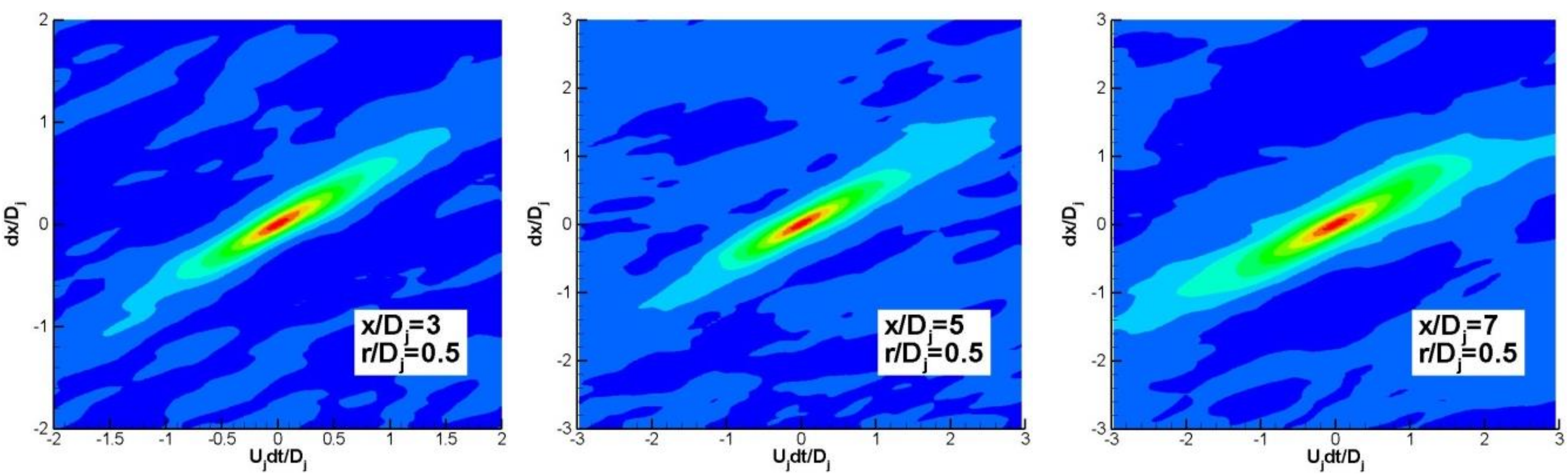
Needs modelling ...

The 4th Order Correlation Functions (R_{1111})

NASA SHJAR, SP07 (James Bridges, 2014)



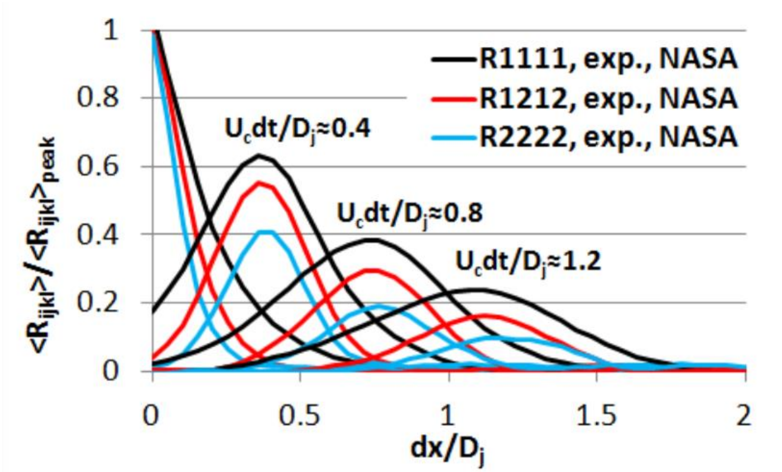
LES, SILOET cold static jet, M_a=0.9



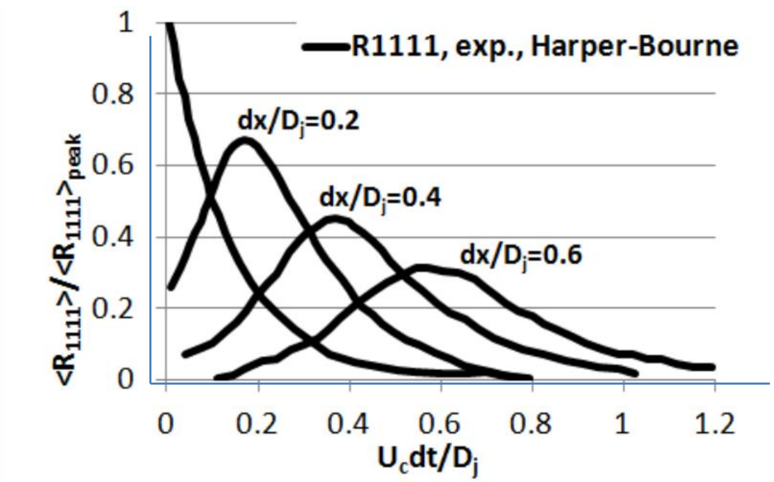
Comparison of Reynolds stress covariance functions in the time domain with the NASA SHJAR data

$$R_{ijkl}(\boldsymbol{y},\boldsymbol{\Delta},\tau)=\overline{e''_{ij}(\boldsymbol{y},t)e''_{ij}(\boldsymbol{y}+\boldsymbol{\Delta},t+\tau)},\quad e''_{ij}(y,t)=-\left(\rho v'_i v'_j-\overline{\rho v'_i v'_j}\right)$$

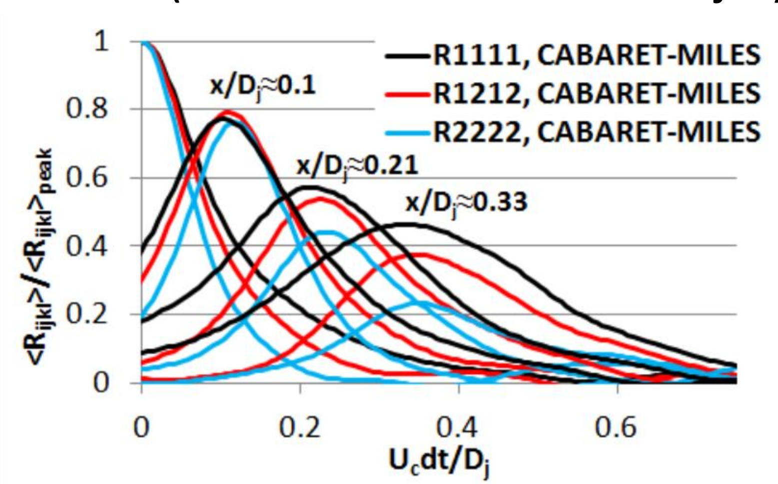
NASA SHJAR data (Bridges, 2014)



Harper-Bourne experiment (2003)



LES data (static isothermal SILOET jet)



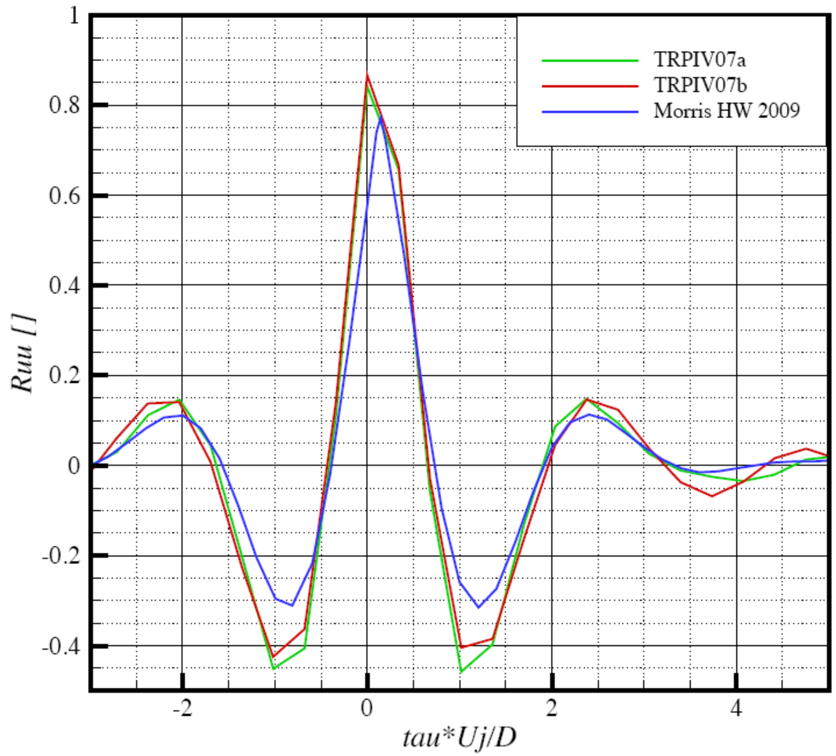
- The same qualitative behavior
- Temporal and spatial scales

for R_{1111}, R_{1212} =R_{2121} and R_{2222} are different

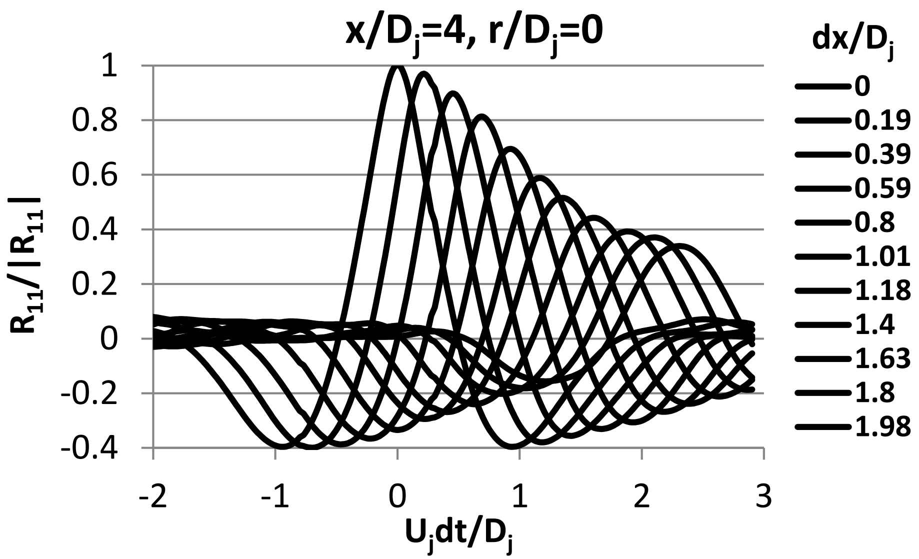
Is it possible to collapse the source correlation data for different jets to a useful dimensionless form?

Qualitative Comparison of the Second order Correlations

Experiment data

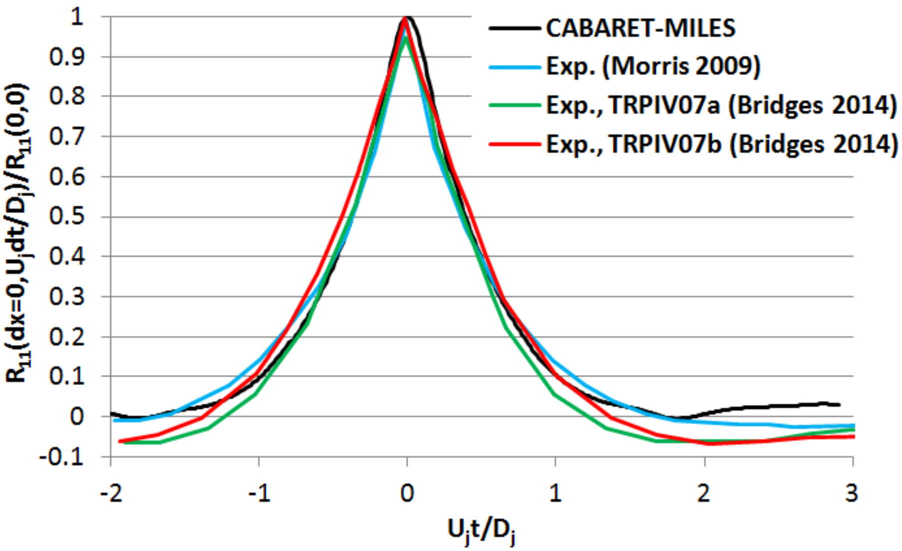


LES data

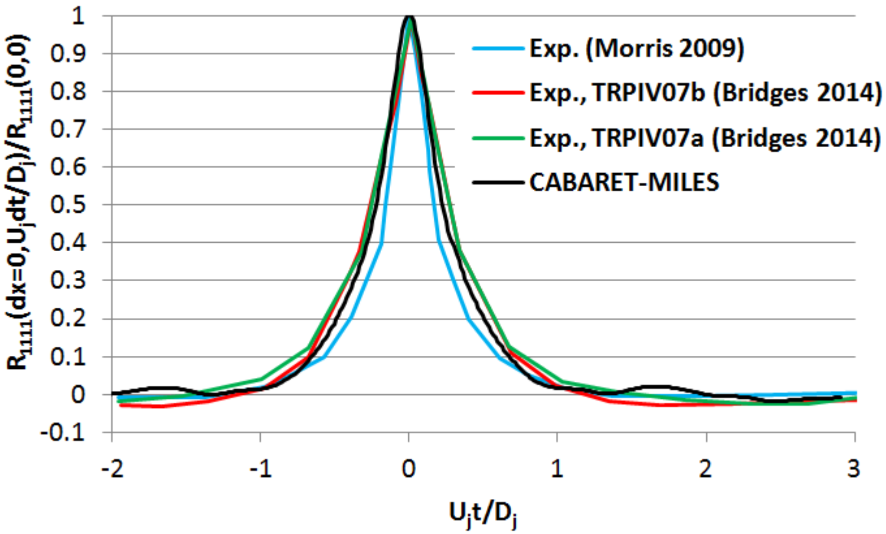


Quantitative Comparison of the Second and Forth Order Correlation Functions

$R_{11}(\tau)=<u'_1(t)u'_1(t+\tau)>$



$R_{1111}(\tau)$



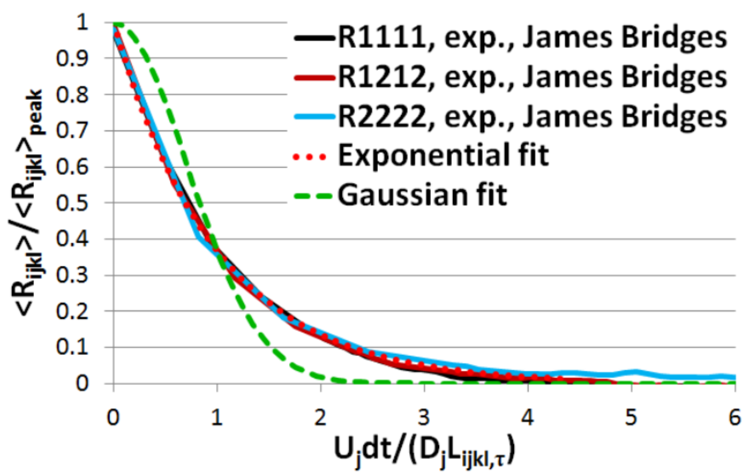
LES data vs Experiment

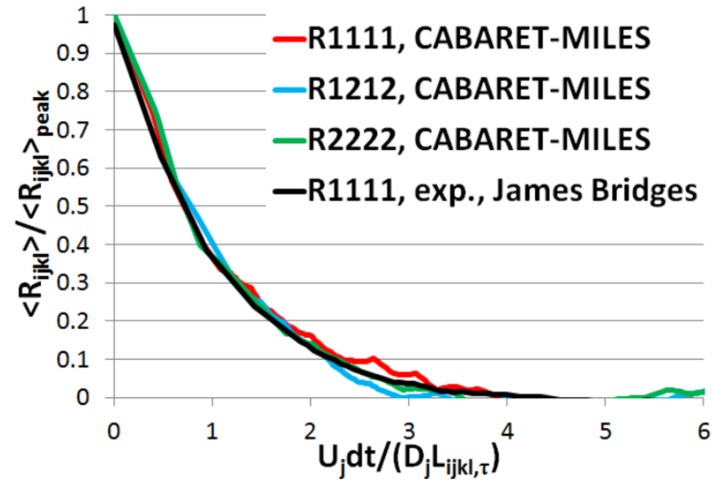
Similarity of jet noise sources: Universal shapes of Reynolds stress covariance functions

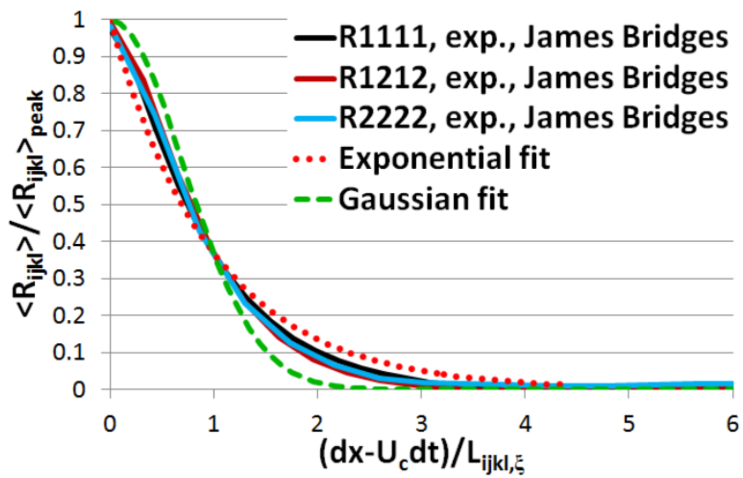
$$R_{ijkl}(\boldsymbol{y},\boldsymbol{\Delta},\tau) = \overline{e''_{ij}(\boldsymbol{y},t)e''_{ij}(\boldsymbol{y}+\boldsymbol{\Delta},t+\tau)},\quad e''_{ij}(y,t) = -\left(\rho v'_i v'_j - \overline{\rho v'_i v'_j}\right)$$

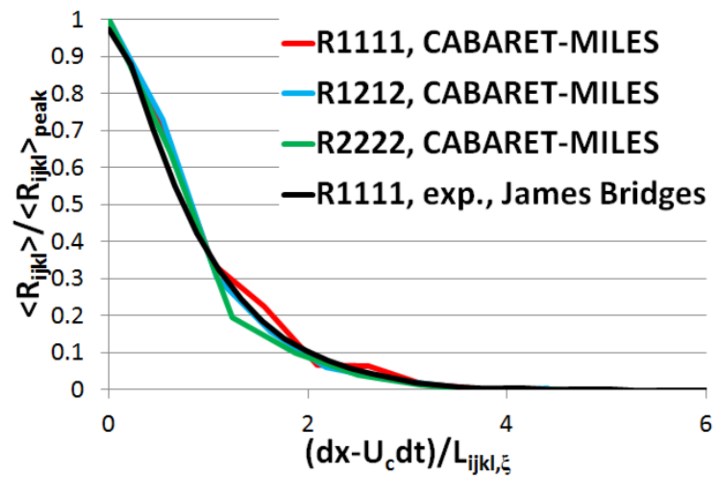
Experiment data

LES data vs Experiment







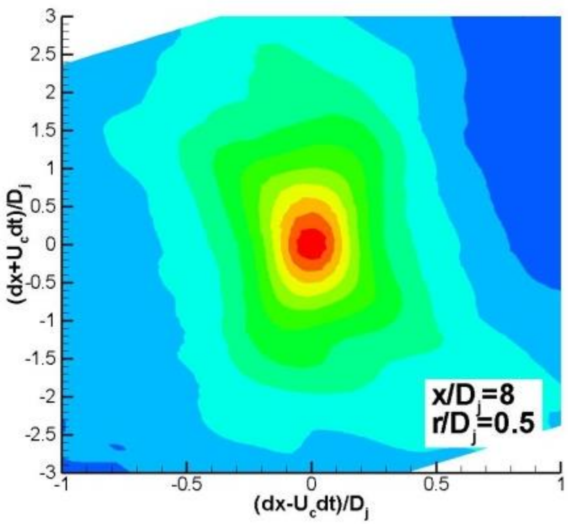


Time-domain model of the Reynolds stress covariance

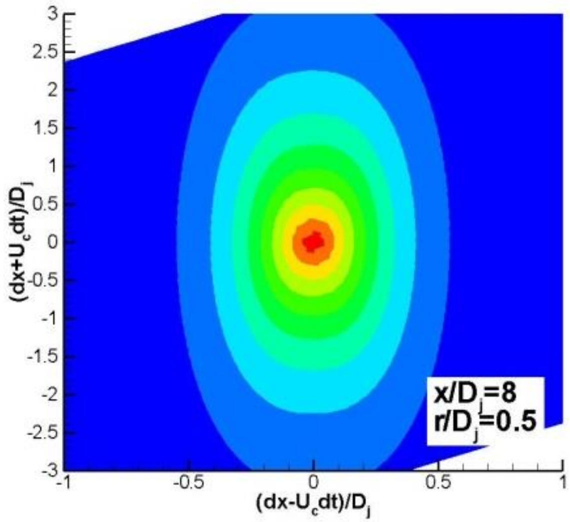
$$R_{ijkl}(\mathbf{y},\boldsymbol{\eta},\tau) = |R_{ijkl}| \exp\left(-\sqrt{\left(\frac{|\tau|}{L_\tau}\right)^a + \left(\frac{|\eta_1 - U_c\tau|}{L_1}\right)^{b_1} + \left(\frac{|\eta_2|}{L_2}\right)^{b_2} + \left(\frac{|\eta_3|}{L_3}\right)^{b_3}}\right)$$

$$a = 2,\ b_1 = b_2 = b_3 = 2.5 \qquad U_c = \mathrm{const}$$

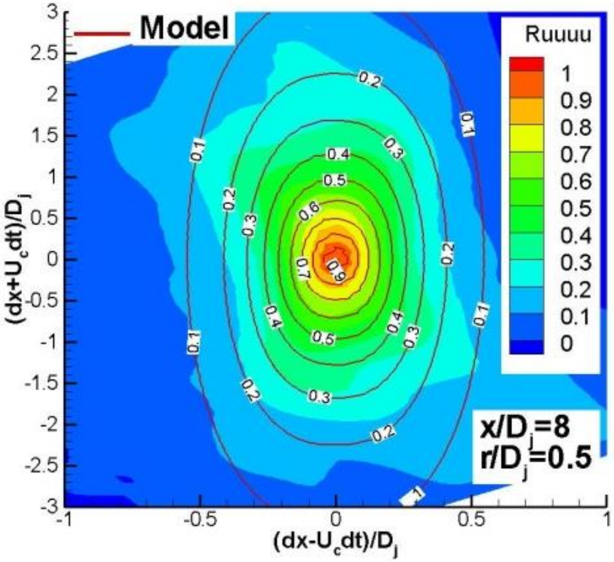
[Goldstein&Leib 2008]
[Bassetti et al 2007]



LES data



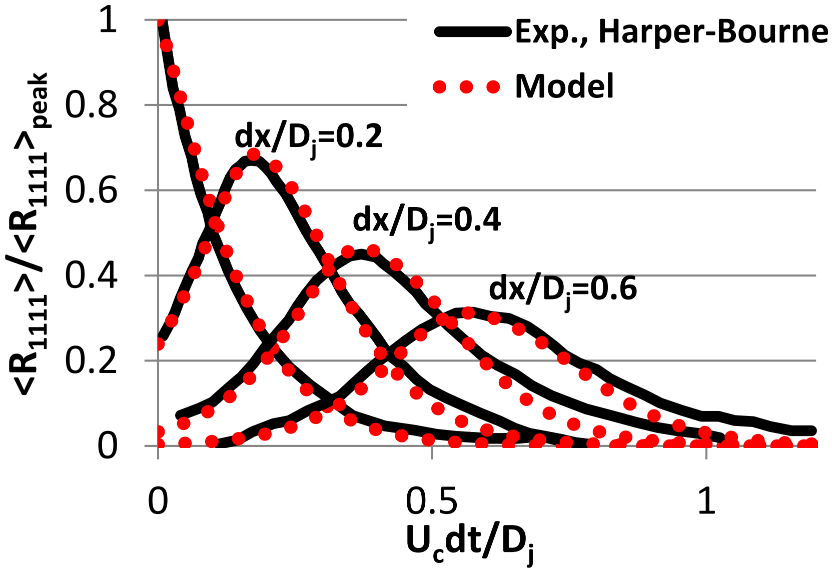
Model



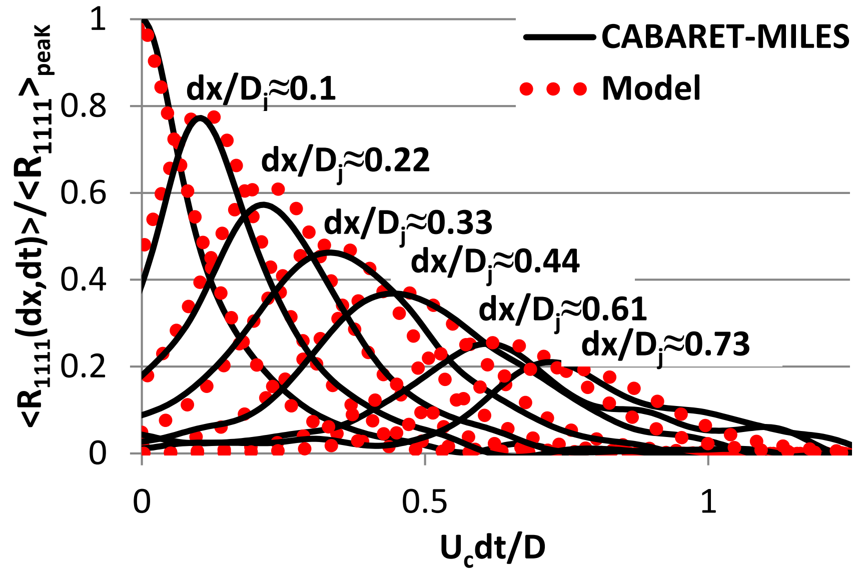
Note that this model of the Reynolds stress covariance is non-separable, e.g. analytically not integrable even with most simple Green's functions

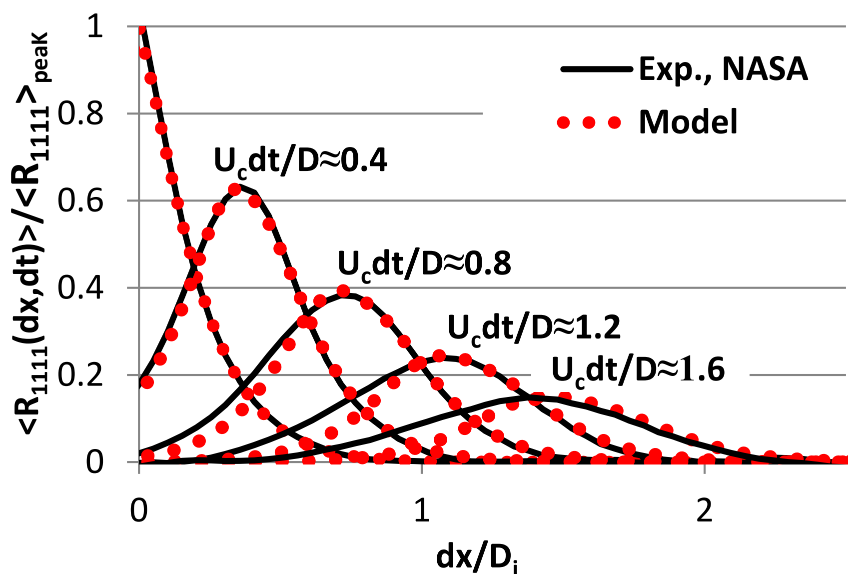
Comparison of the model with the LES data and with the NASA and Harper-Bourne experimental data

Model vs experimental data for other jets



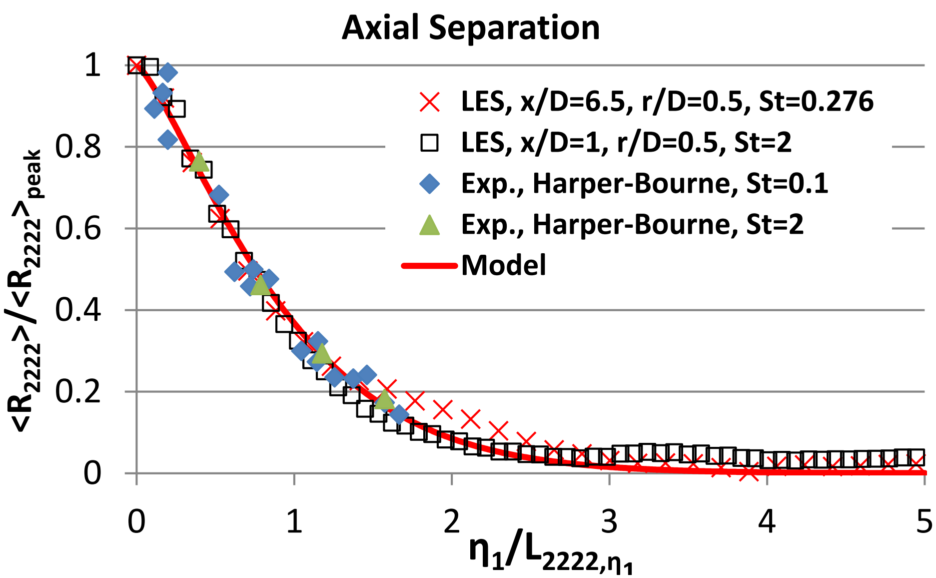
Model vs the LES data

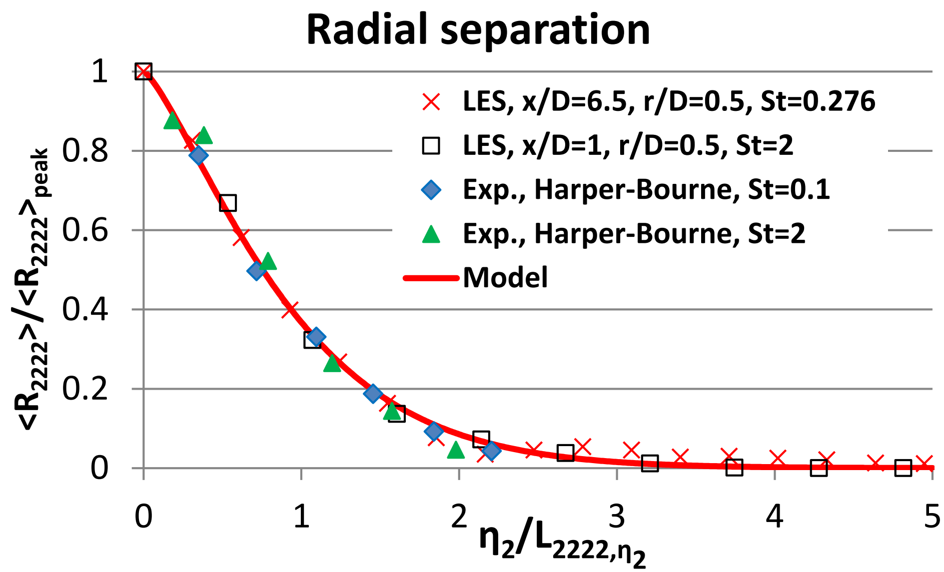


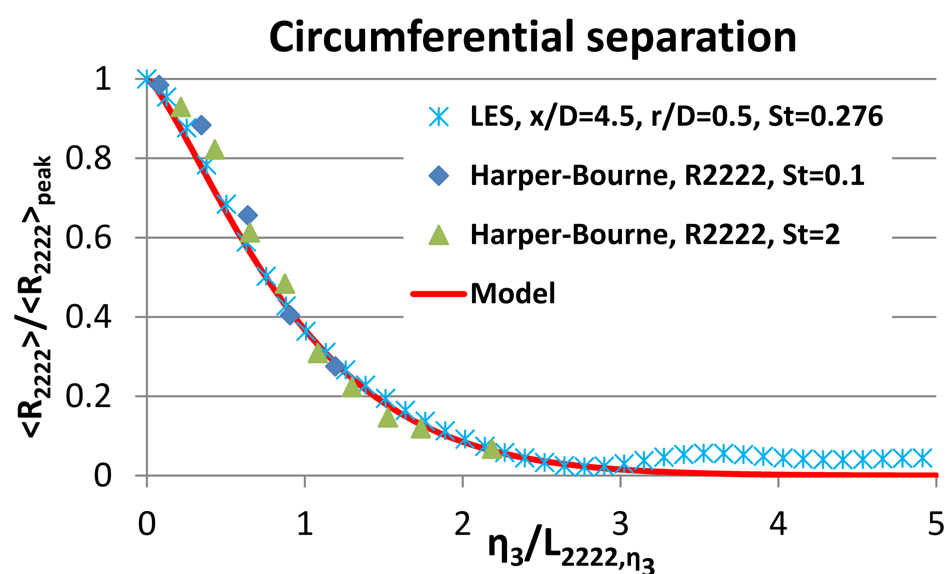


Despite some deviations at large separations, the model agrees well with the reference LES data and the experiments for the other jets

Dimensionless Reynolds stress covariance function (R_{2222}) at different locations and frequencies: the model vs the LES data for the isothermal jet and the Harper-Bourne experiment





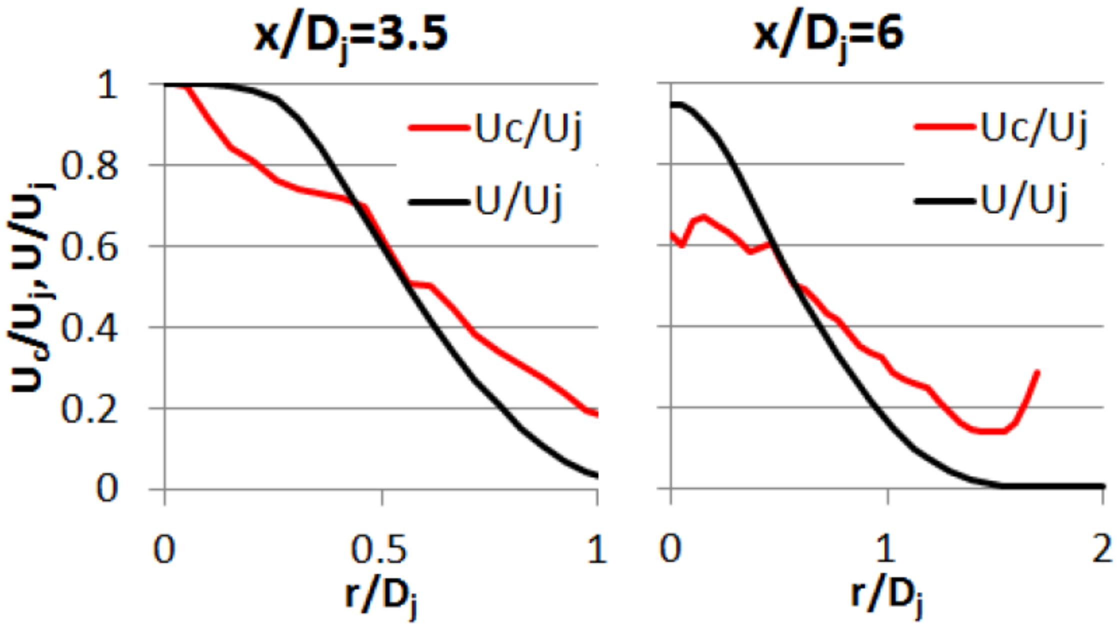


The ‘universal’ shape doesn’t depend on location or frequency

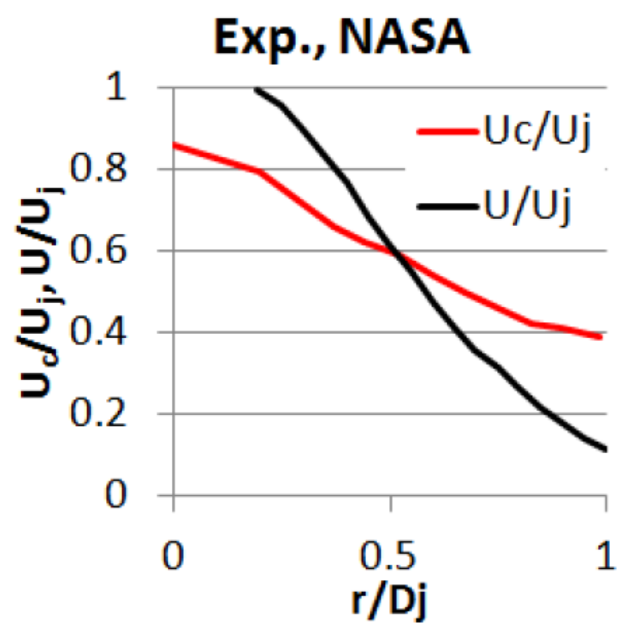
LES results and the experimental data

Eddy convection velocity vs the local meanflow velocity

LES data



Experiment



x/D_j=5, U_j=87m/s, D_j=0.0508m

[Morris and Zaman, 2010]

The LES data and the experiment show a very similar behaviour

Quasi-normality hypothesis (single point and two-point)

Known from RANS

Covariance matrix $\Sigma = \begin{pmatrix} \overline{u_1'^2} & \overline{u_1' u_2'} \\ \overline{u_1' u_2'} & \overline{u_2'^2} \end{pmatrix}$

Needs to be modelled

$$R_{ijij} = \left(\overline{u_i'^2}\right)\left(\overline{u_j'^2}\right) + \left(\overline{u_i' u_j'}\right)^2$$

Normality

$$R_{iiii} = 2R_{ii}^2$$

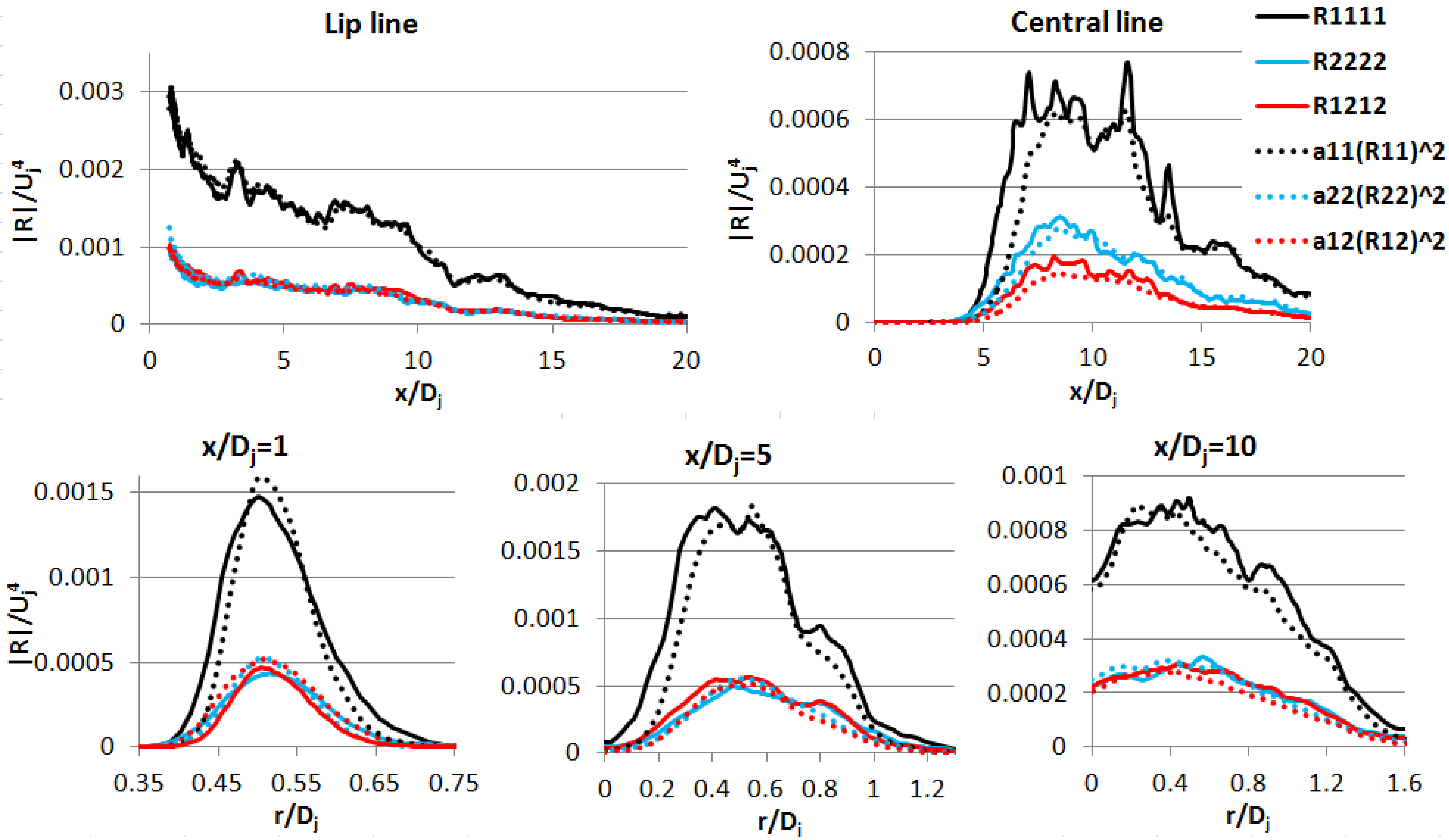
$$R_{ijij} = R_{ii}R_{jj} + R_{ij}^2$$

Quasi-normality

$$R_{iiii} = \alpha_{ii}R_{ii}^2$$

$$R_{ijij} = \alpha_{ij}\left(R_{ii}R_{jj} + R_{ij}^2\right)$$

Normality/quasi-normality

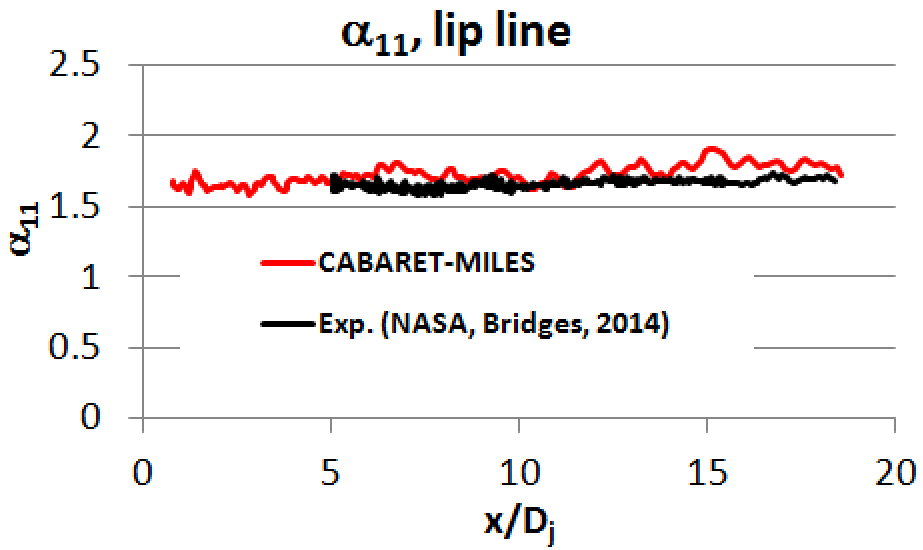


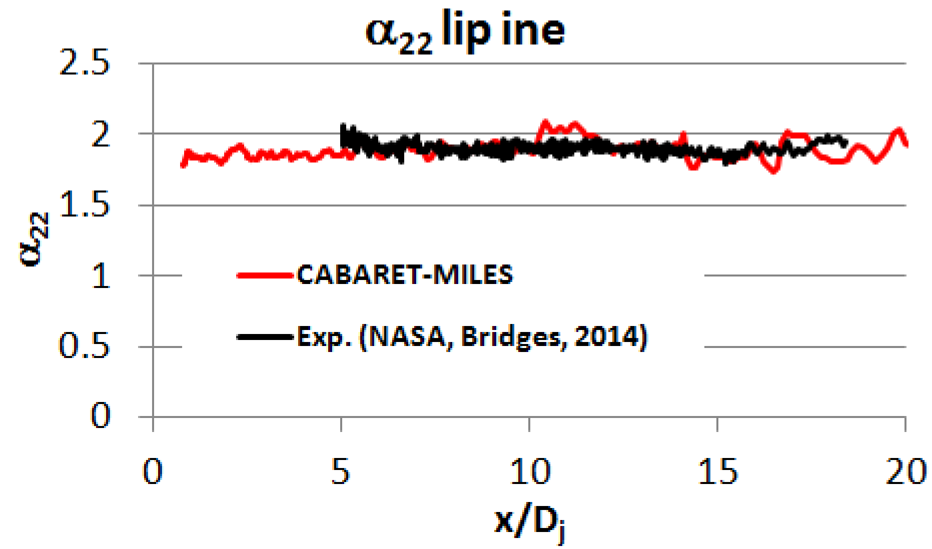
	Normal	Quasi-normal
α_{11}	2	1.7
α_{12}	2	1.92
α_{13}	1	0.92

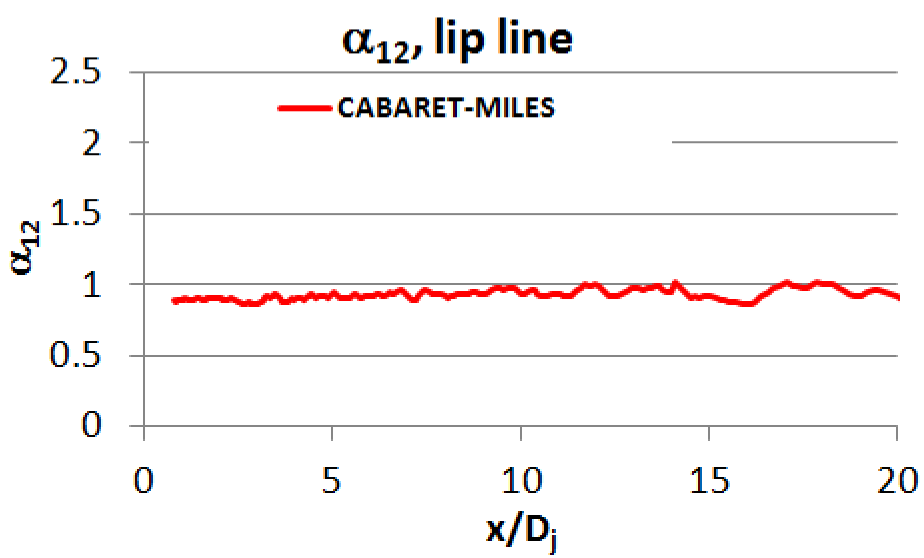
Minor difference
(1.7/2 ~ 0.7dB error)!

[Kreitzman, Nichols, AIAA, 2015]
rectangular heated supersonic jet with chevrons

Quasi-normality







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Minor difference

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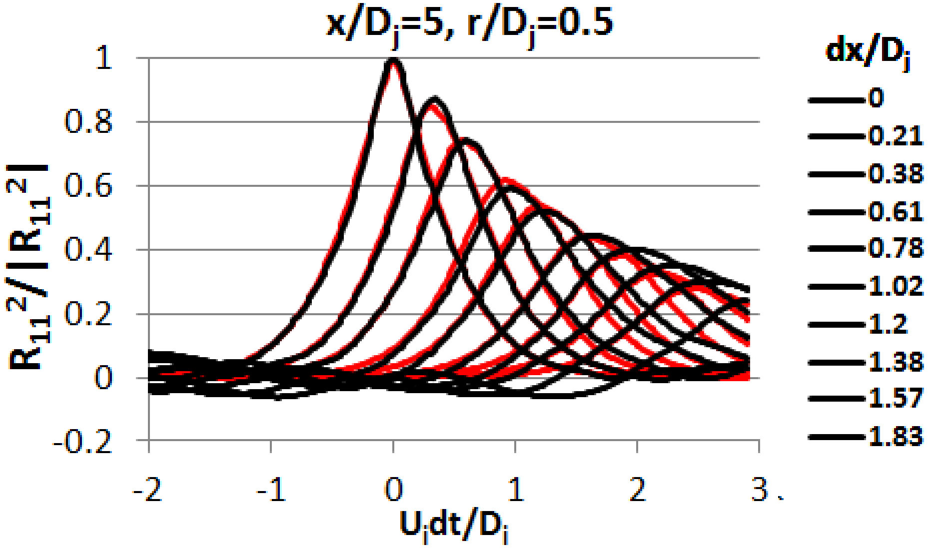
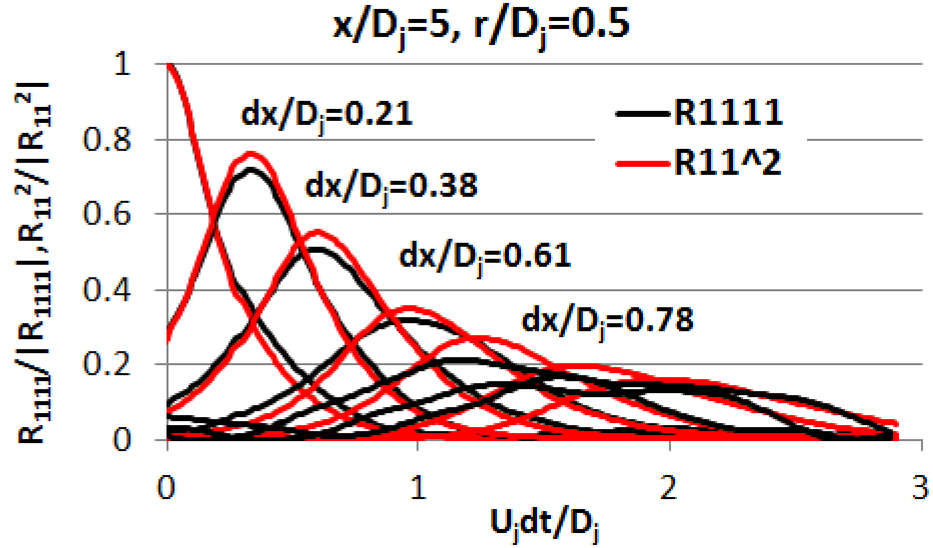
[Kreitzman, Nichols, AIAA, 2015]
rectangular heated supersonic jet with chevrons

Quasi-normality of Correlation functions

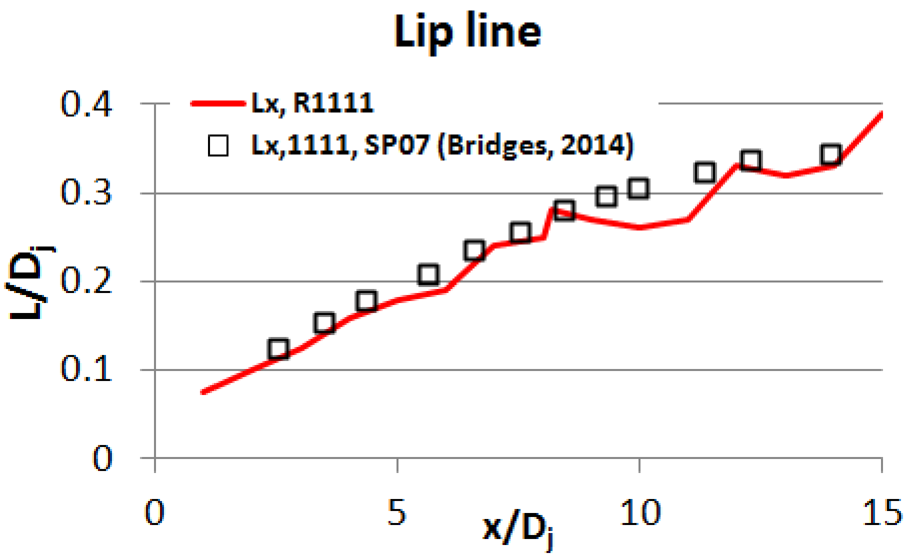
The same model is still valid for $R_{11}^2(\tau)$

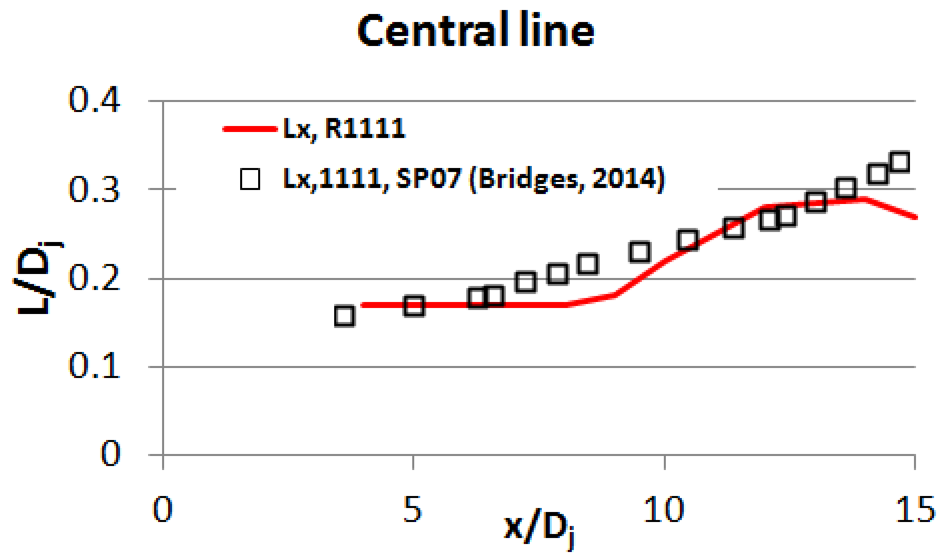


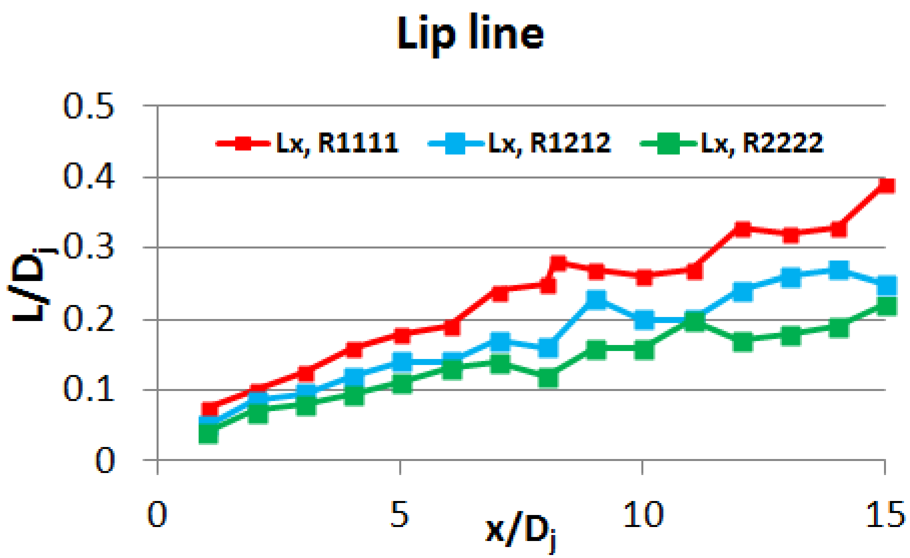
$R_{1111}(\tau)$ vs $R_{11}^2(\tau)$

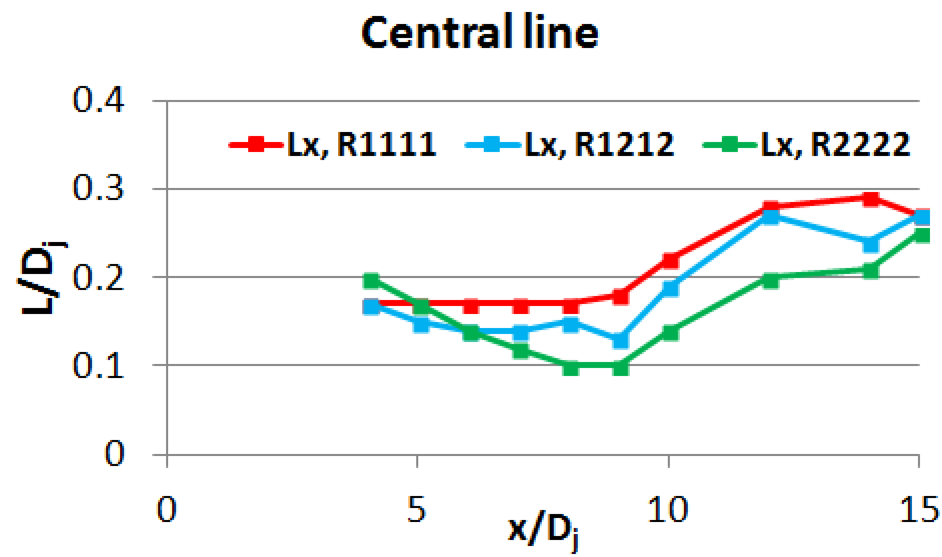


Axial Length Scales

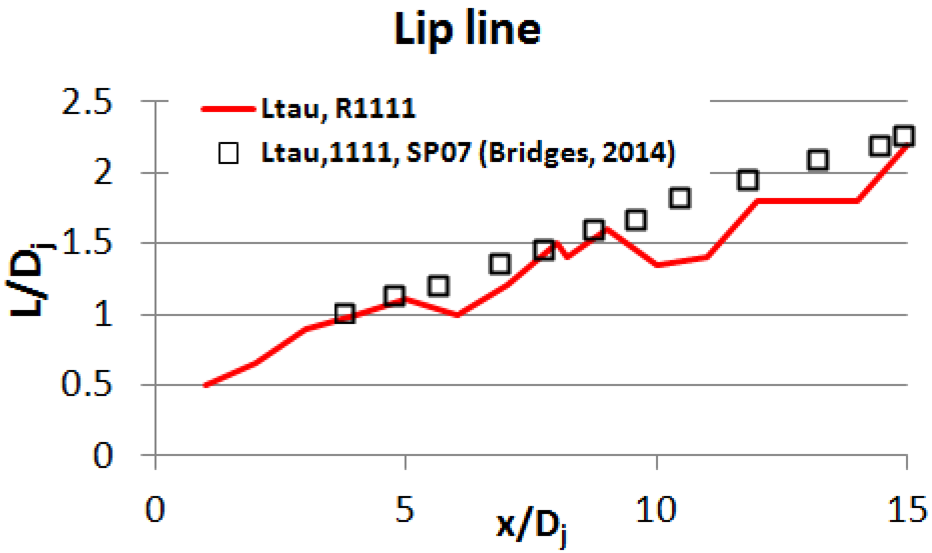


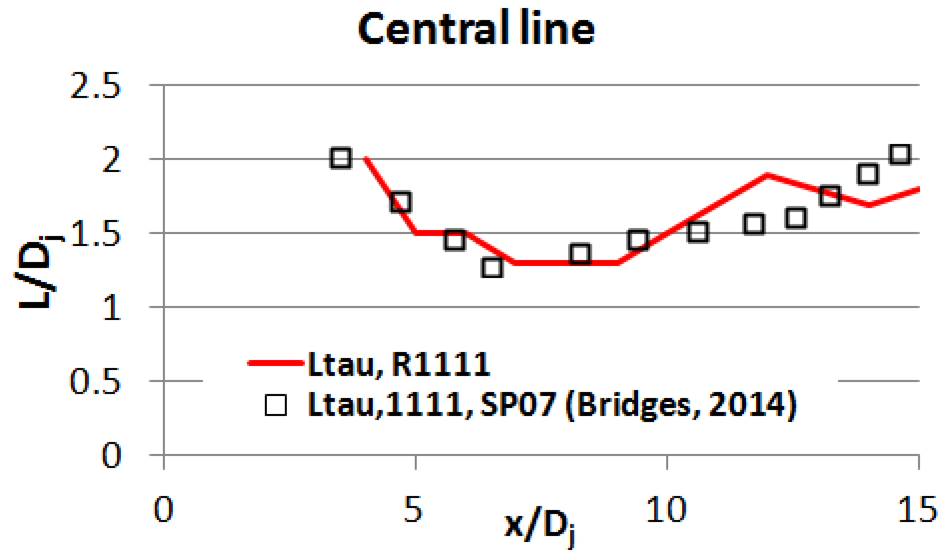


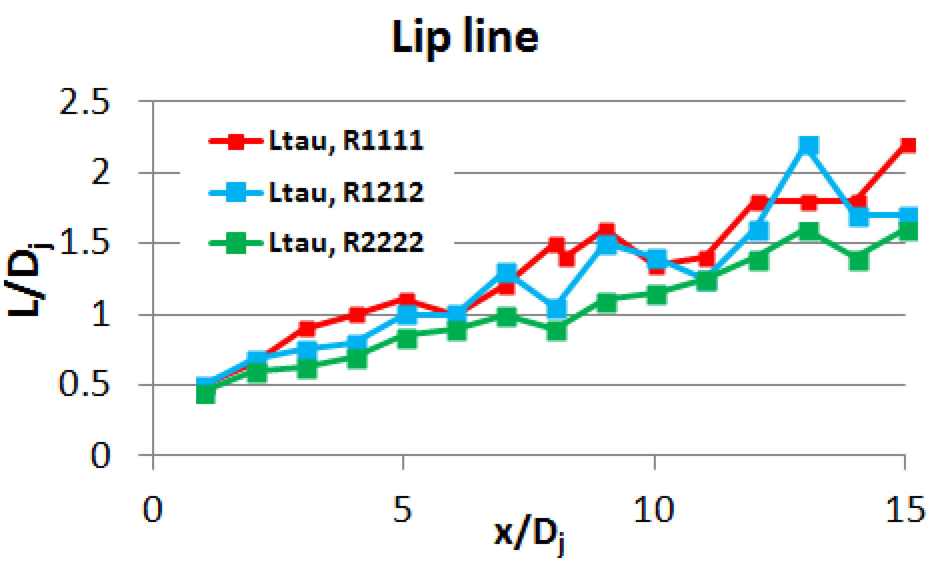


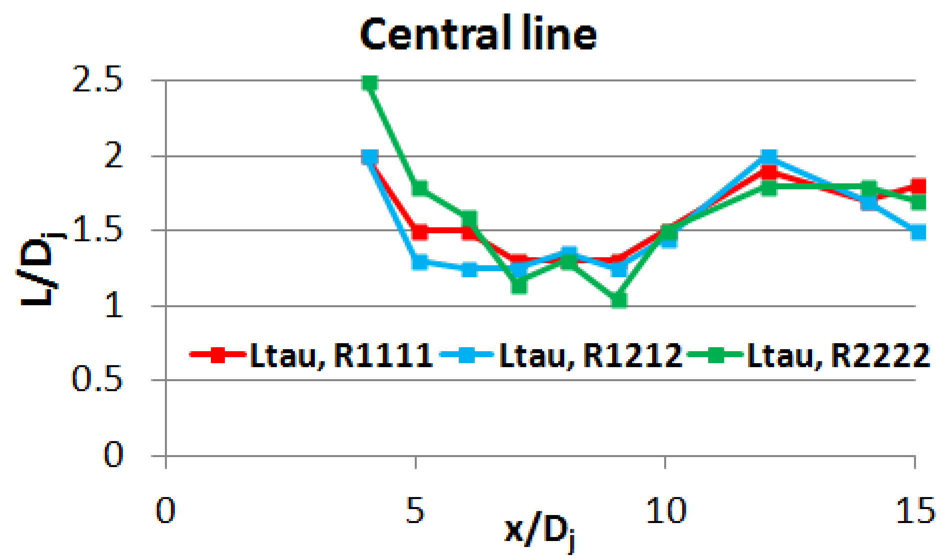


Temporal Scales









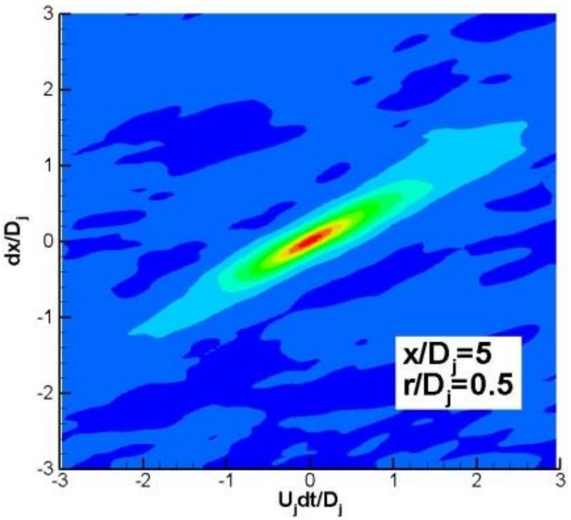
Temporal and Length Scales Modelling

$$R_{ijkl}(\mathbf{x},\eta,\tau) = |R_{ijkl}|\, exp\left[-\sqrt{\tau^2 + |\eta_1|^{2.5} + |\eta_2|^{2.5} + |\eta_3|^{2.5}}\right]$$

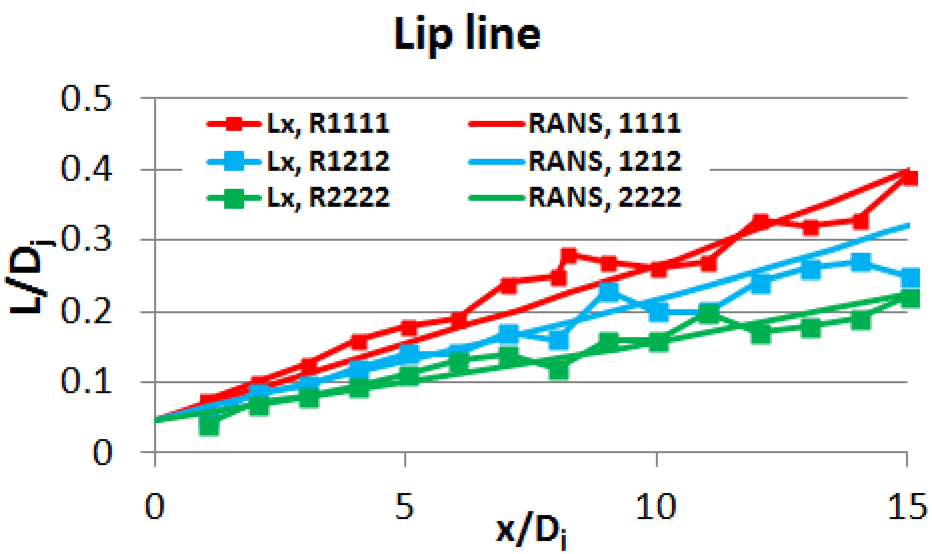
$$\tau = \frac{dx + U_c dt}{L_{ijkl,\tau}}, \quad \eta_1 = \frac{dx - U_c dt}{L_{ijkl,1}}, \quad \eta_2 = \frac{dy}{L_{ijkl,2}}, \quad \eta_3 = \frac{dz}{L_{ijkl,3}}$$

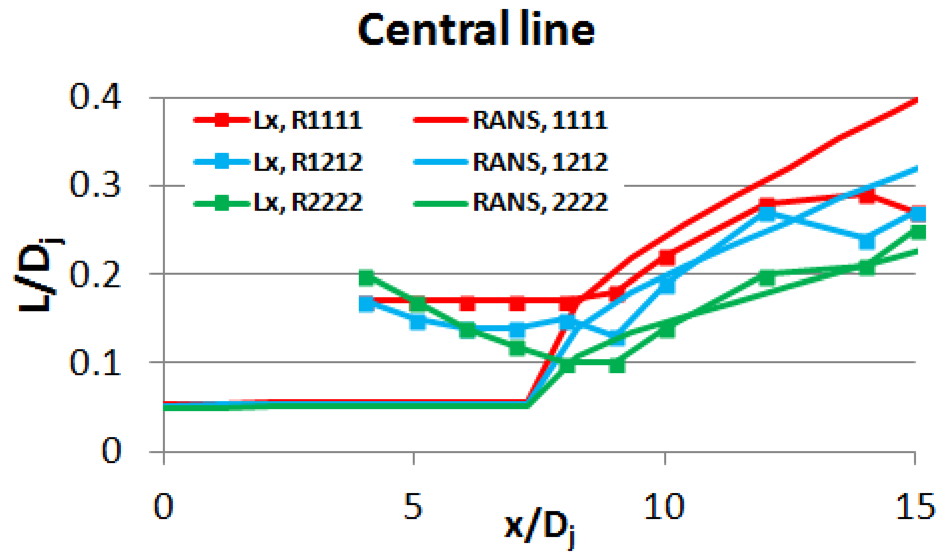
$$\frac{L_{ijkl,\mu}(\mathbf{x})}{D_j} = c_{ijkl,\mu} \frac{k^{3/2}(\mathbf{x})}{\varepsilon(\mathbf{x}) D_j}, \quad \mu = \tau, 1, 2, 3$$

Temporal scales obey to the same scaling law as spatial scales



Axial Length Scales vs RANS



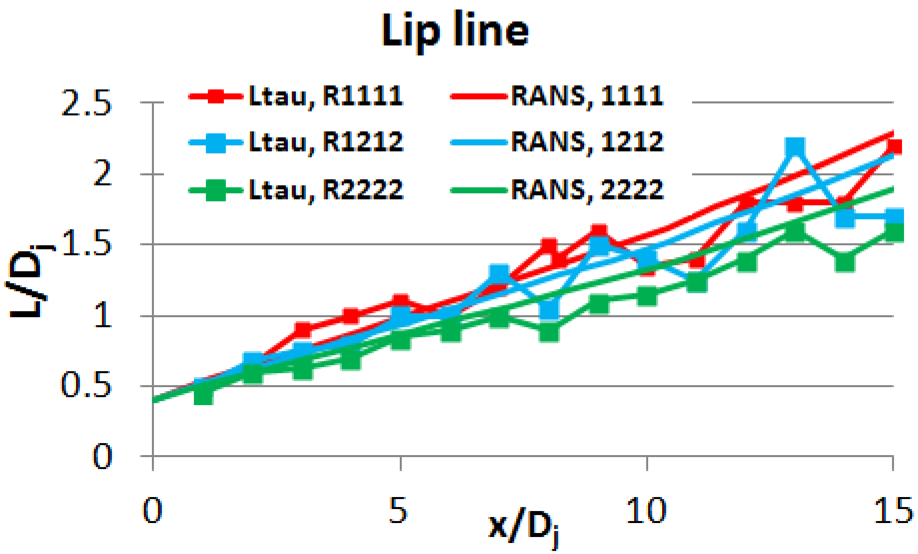


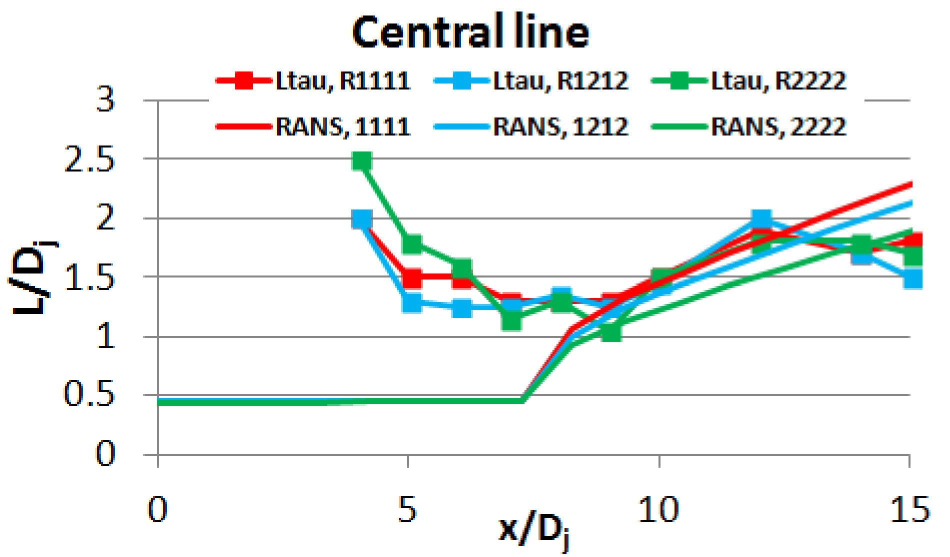
$c_{x,1111}$ **2.25**

$c_{x,1212}$ **1.75**

$c_{x,2222}$ **1.15**

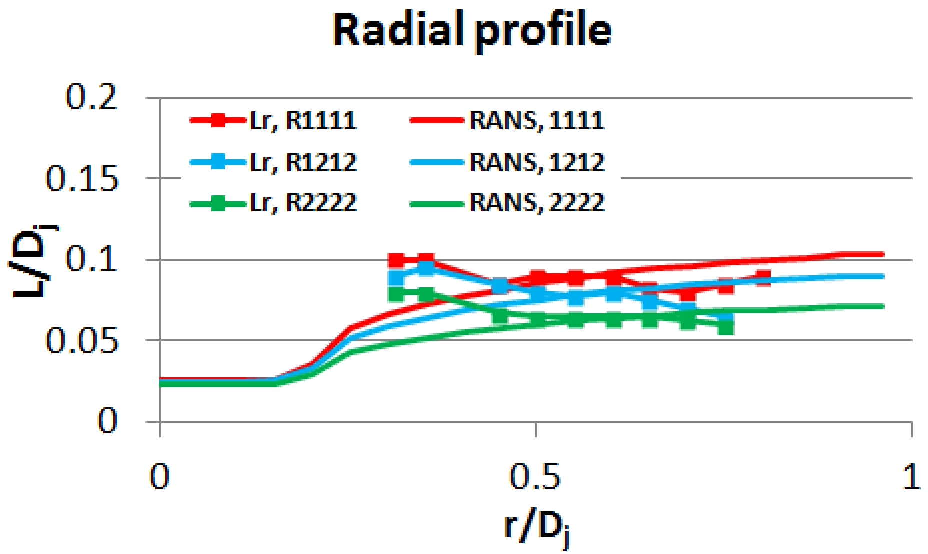
Temporal Scales vs RANS





$c_{\tau,1111}$	**12**
$c_{\tau,1212}$	**11**
$c_{\tau,2222}$	**9.5**

Radial Length Scales vs RANS



Anisotropy

$c_{r,1111}$	**1.3**
$c_{r,1212}$	**1.1**
$c_{r,2222}$	**0.8**

$c_{x,1111}$	**2.25**
$c_{x,1212}$	**1.75**
$c_{x,2222}$	**1.15**

Conclusions

- An analytical exponential function is suggested for approximating the auto-correlation fluctuating stress function in accordance with the LES data and the experiments
- The eddy convection speed is very similar to the local axial jet velocity in the shear layer location in accordance with Harper-Bourne (2003), Morris and Zaman (2010) and Bridges (2014)
- Scaling factors (1.7,1.92,0.92) for the single point quasi-normality study has been found which are similar to the theoretical values (2,2,1)
- Quasi-normal hypothesis also works for different separations, so the second order correlation functions can be also model in the universal same way
- Anisotropic length scales were obtain calibrating RANS solution