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ABNORMAL AMPLIFICATION OF SOUND REFRACTED BY AN OBLIQUE SHOCK WAVE

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Motivation

Sound/Shock interaction results in the generation of 3 types of waves downstream the shock:

(1) refracted sound wave; (2) entropy wave; (3) vorticity wave.

Investigations of this problem:

- D. Blokhintsev, 1945, normal incidence of sound on a normal shock (ideal gas);
- J. Brillouin, 1955, oblique incidence on normal shock (ideal gas), erroneous results;
- V. Kantorovich, 1958, corrections and extension to an arbitrary EOS;
- J. McKenzie, K Westphal, 1968, oblique incidence-oblique shock general solution; linear theory has the solution at a certain incidence angles; no solution at critical and over-critical angles;
- A. Lubchich, M. Pudovkin, 2004, an alternative linear theory to obtain the solution at supercritical incidence;
- A. Kudryavtsev, A. Ovsyannikov, 2010; non-linear problem with DNS; incorrectness of Lubchich-Pudovkin and confirmation results by McKenzie-Westphal.

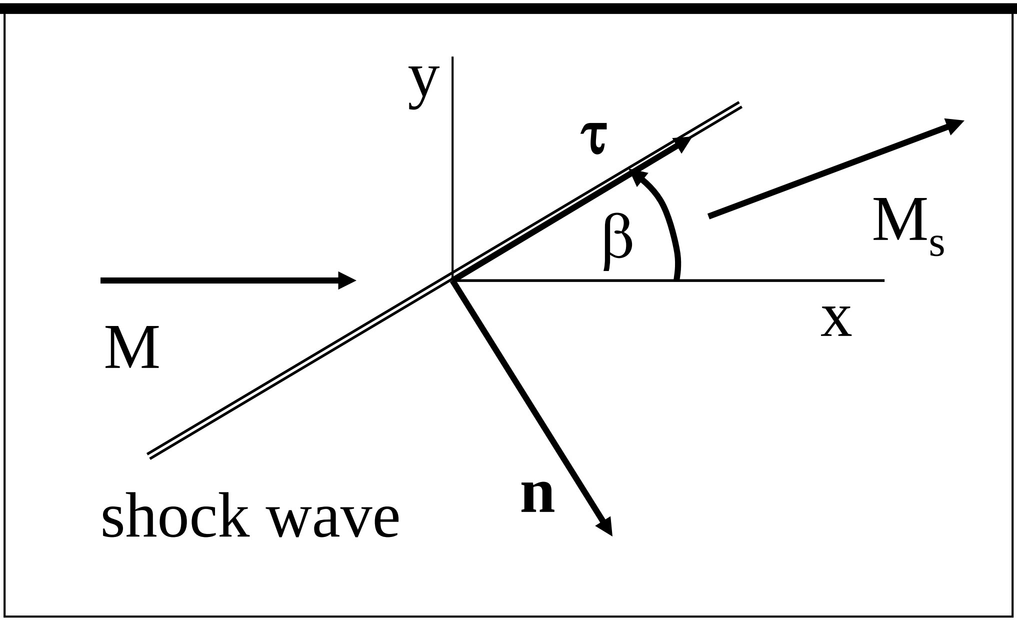
The present paper considers the linear theory of sound /shock interaction with focusing at the region of **critical and supercritical incidence**

Outline

Linear analysis of the interaction between plane monochromatic linear waves and an oblique shock wave:

i) basic notations;
ii) reflection and refraction laws, critical incidence angles for regular interaction;
iii) transmission, reflection, and generation coefficients at different incidence angles;

Nomenclature: base flow

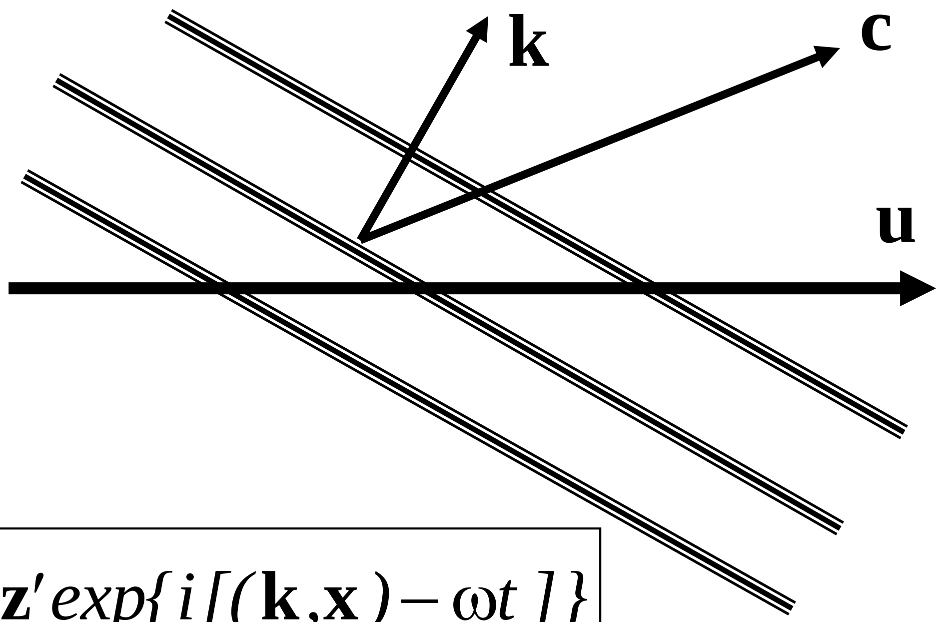


β= shock wave angle
n= unit normal vector
τ= unit tangent vector

Upstream parameters:
u = velocity vector;
a = speed of sound;
M = Mach number;

Downstream parameters:
$\mathbf{u}_s$ = velocity vector;
a_s = speed of sound;
M_s = Mach number;

Nomenclature: weak wave



$$\delta\mathbf{z} = \mathbf{z}' exp\{ i [(\mathbf{k},\mathbf{x}) - \omega t]\}$$

$\mathbf{z}'$ = amplitude

ω = circular frequency, $\omega>0$

k = wave vector

c = phase velocity, (**k**, **c**)= ω

4 kinds of weak waves:

Fast acoustic waves:

$$\omega = (\mathbf{u},\mathbf{k}) + ak; \ \mathbf{c} = \mathbf{u} + a\mathbf{k}/k$$

$$p' = \varepsilon_p; \ \rho' = \varepsilon_p / a^2; \ \mathbf{u}' = \frac{\varepsilon_p}{\rho a}\frac{\mathbf{k}}{k}$$

Entropy waves:

$$\omega = (\mathbf{u},\mathbf{k}); \quad \mathbf{c} = \mathbf{u}$$

$$p' = 0; \ \rho' = \varepsilon_\rho; \ \mathbf{u}' = 0$$

Slow acoustic waves:

$$\omega = (\mathbf{u},\mathbf{k}) - ak; \ \mathbf{c} = \mathbf{u} - a\mathbf{k}/k$$

$$p' = \varepsilon_p; \ \rho' = \varepsilon_p / a^2; \ \mathbf{u}' = -\frac{\varepsilon_p}{\rho a}\frac{\mathbf{k}}{k}$$

Vorticity waves:

$$\omega = (\mathbf{u},\mathbf{k}); \quad \mathbf{c} = \mathbf{u}$$

$$p' = 0; \ \rho' = 0; \ \mathbf{u}' = \varepsilon_v \mathbf{m}$$

Wave vector restriction:

The condition $\omega>0$ imposes a restriction on the wave vector as follows:

$$\left(\frac{\mathbf{u}}{u},\frac{\mathbf{k}}{k}\right)\begin{cases} >1/M, & \textit{slow acoustic wave} \\ >-1/M, & \textit{fast acoustic wave} \\ >0, & \textit{entropy or vorticity wave} \end{cases}$$

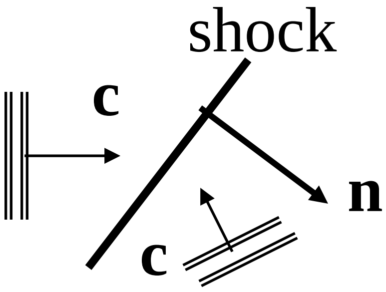
In supersonic flows: all 4 kinds of weak waves can exist
In subsonic flows: only fast acoustic, entropy, and and vorticity waves exist; slow acoustic waves can not be realized.

Direction of wave propagation:

Weak waves are classified into 2 groups by the direction of propagation of disturbances with respect to the shock front. These 2 groups are:

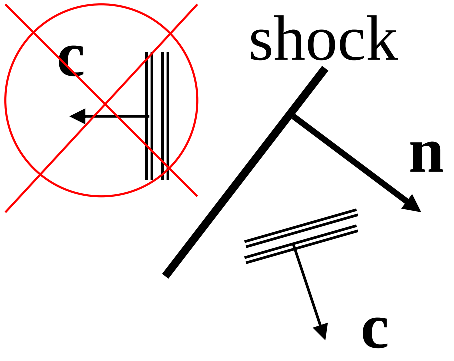
i-wave (incident)

$$\begin{cases} in\ upstream & (\mathbf{c},\mathbf{n}) > 0 \\ in\ \ downstream & (\mathbf{c},\mathbf{n}) < 0 \end{cases}$$



r-wave (running away)

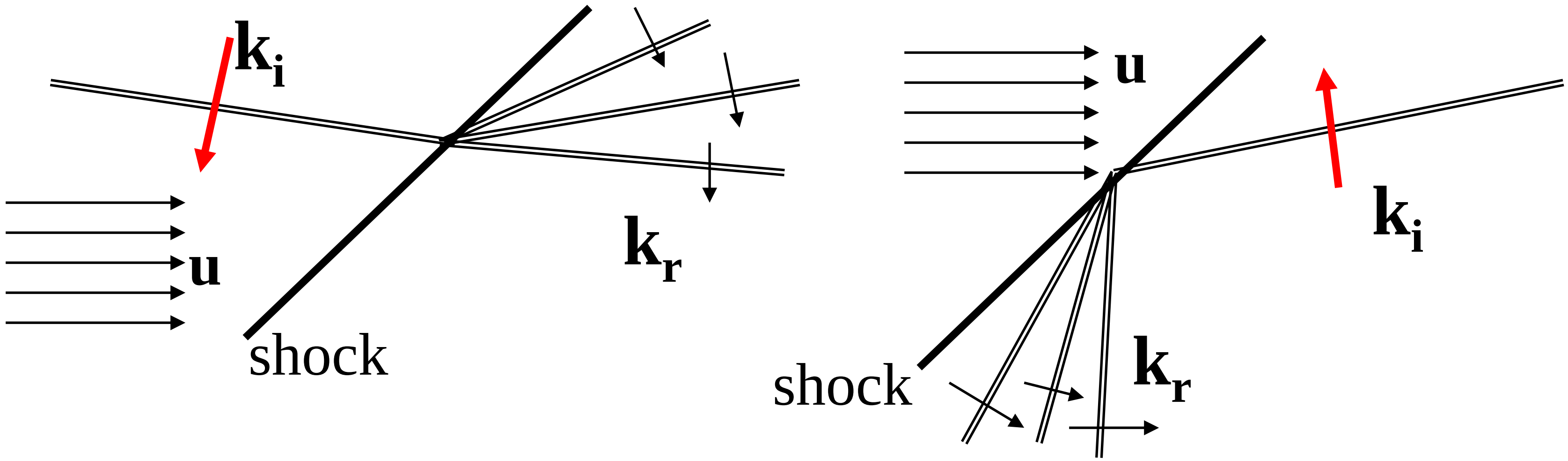
$$\begin{cases} in\ upstream & (\mathbf{c},\mathbf{n}) < 0 \\ in\ \ downstream & (\mathbf{c},\mathbf{n}) > 0 \end{cases}$$



Refraction & Reflection:

<u>**2 types of sound/shock interaction**</u>:

Wave refraction ***Wave reflection***



The problem is to define what r-waves are generated and determine their wave vectors $\mathbf{k_r}$ and amplitudes.

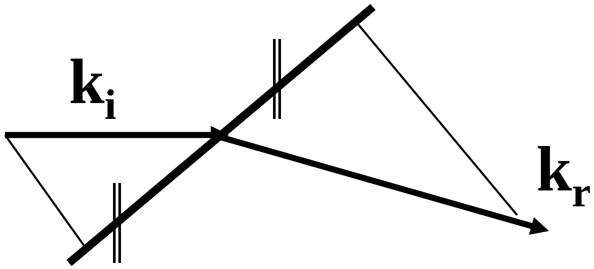
Note: length scale is taken so that $|\mathbf{k_i}|=1$ ($\lambda_i=2\pi$)

Defining wave vectors:

Wave vector $\mathbf{k_r}$ must satisfy the following conditions.

i) continuity of wave vectors tangent to the shock:

$$[(\mathbf{k},\tau)]=0$$



ii) continuity of frequencies:

$$[\omega]=0; \quad \omega=(\mathbf{u}_s,\mathbf{k}_r)-\varepsilon_r a_r k_r$$

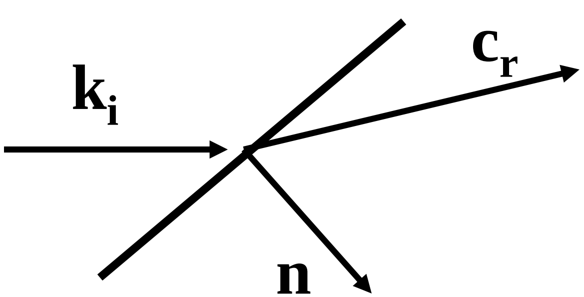
$\varepsilon_r = -1$ for fast acoustic wave

$\varepsilon_r = +1$ for slow acoustic wave

$\varepsilon_r = 0$ for entropy/vorticity wave

iii) run away:

$$(\mathbf{c}_r,\mathbf{n})>0$$



Solving r-wave vectors K_r:

Analysis of the above relations:

i) For ***entropy/vorticity wave*** the solution $\mathbf{K}_r$ is found under any wavevector $\mathbf{K}_i$ of the *i*-wave =>

$$\mathbf{k}_r = \mathbf{k}_i + \frac{(\mathbf{u}-\mathbf{u}_s, \mathbf{k}_i)}{(\mathbf{u}, \mathbf{n})}\mathbf{n}$$

ii) For ***acoustic wave*** the solution $\mathbf{K}_r$ is found only for *i*-waves with wavevectors satisfied the following condition:

$$\pm\left[z + \frac{(\mathbf{u}-\mathbf{u}_s, \mathbf{k}_i) \pm a}{u_{sn}}\right] \geq \frac{\sqrt{(1-M_{sn})(1-z^2)}}{M_{sn}}$$

$$z = (\mathbf{k}_i, \mathbf{n}) = \cos(\psi_i); \quad u_{sn} = (\mathbf{u}, \mathbf{n}); \quad M_{sn} = u_{sn} / a_s$$

upper sign "+" = fast acoustic i-wave,
upper sign "-" = slow acoustic i-wave

Analyzing sound refraction case:

i) Only one wave – fast or slow acoustic wave can be generated in the flow behind the shock;

ii) There are 2 angles ψ_*^- and ψ_*^+, $0 < \psi_*^- < \psi_*^+$ (**critical angles**) that determines what kind of acoustic wave is generated in the shocked area:

$\psi_*^- < |\psi_i| < \pi$ => entropy + vorticity + **slow acoustic** waves

$|\psi_i| < \psi_*^+$ => entropy + vorticity + **fast acoustic** waves

$\psi_*^+ < |\psi_i| < \psi_*^-$ => **no solution** for the acoustic wave

iii) Incidence is restricted by the **Mach cone angle**:

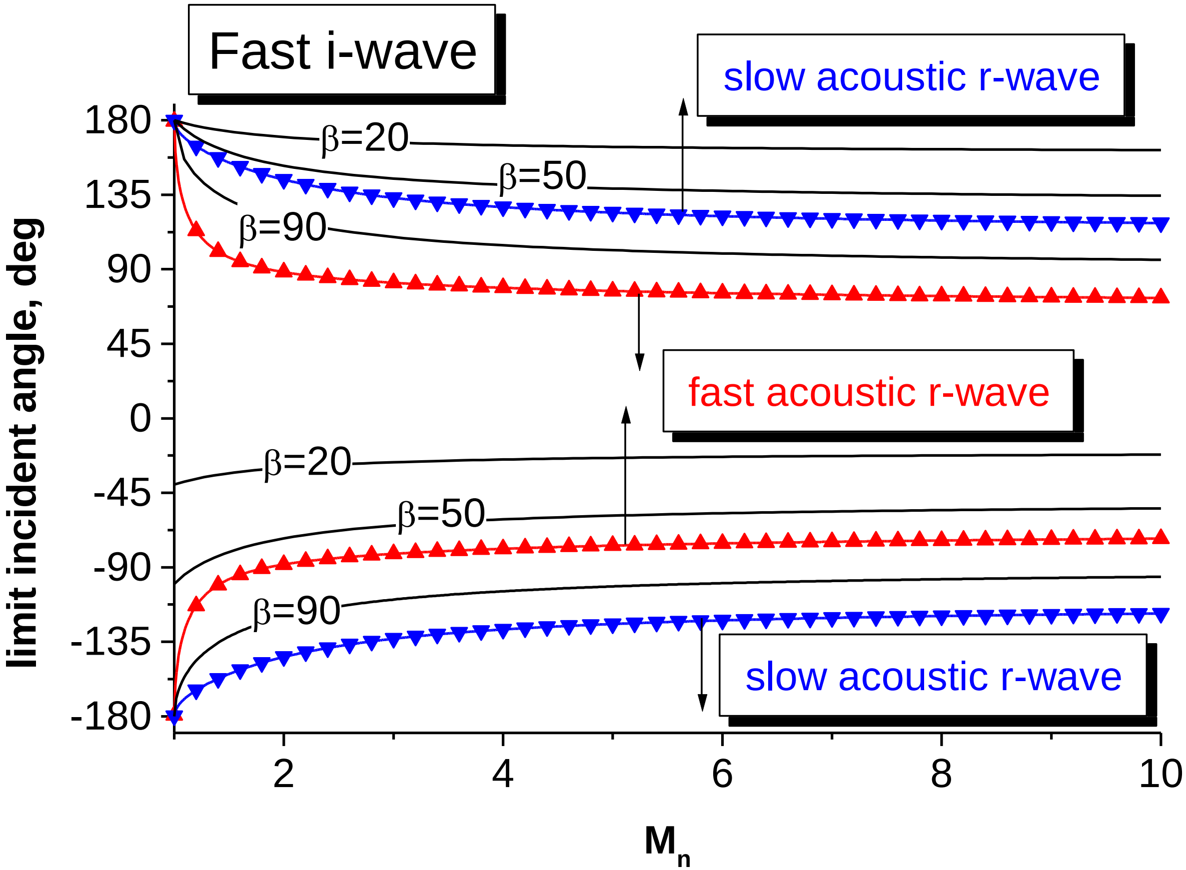
$$\arcsin(\varepsilon / M) - \beta < \psi_i < \pi - \beta - \arcsin(\varepsilon / M)$$

$\varepsilon = -1,\ +1,\ 0$ for fast acoustic, slow acoustic, and entropy/vorticity wave

Critical angles: fast acoustic incidence

Critical angles depend on upstream Mach number M_n, only:

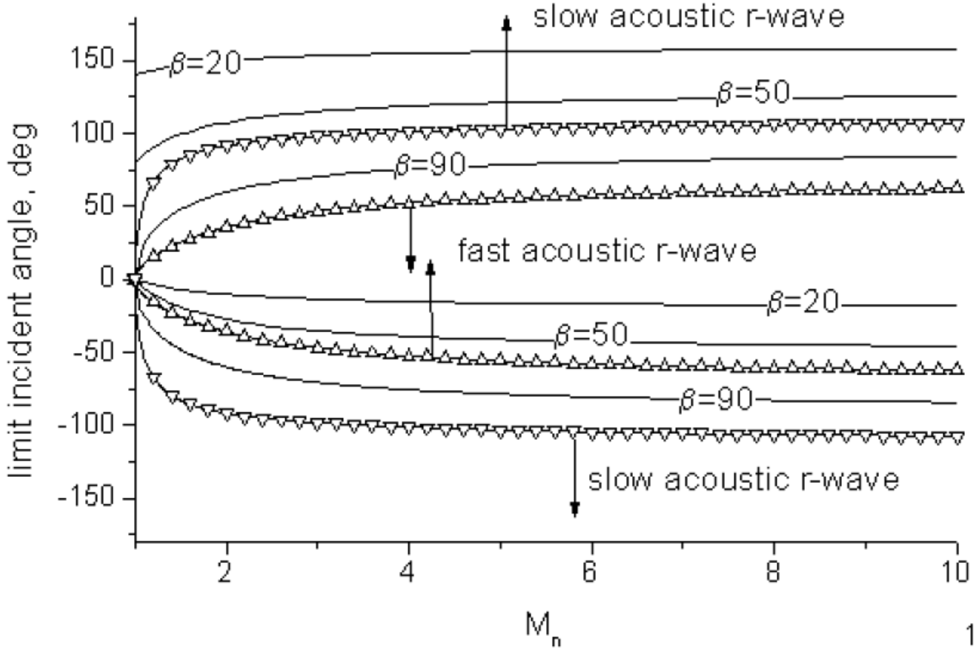
$$z_*^{\pm} = \cos(\psi_*^{\pm}) = \frac{M_n \varepsilon \pm f\sqrt{M_n^2 + f^2 - \varepsilon}}{M_n^2 + f^2} \qquad f = f(M_n) = a_s\sqrt{(1 - M_{sn}^2)} / a$$



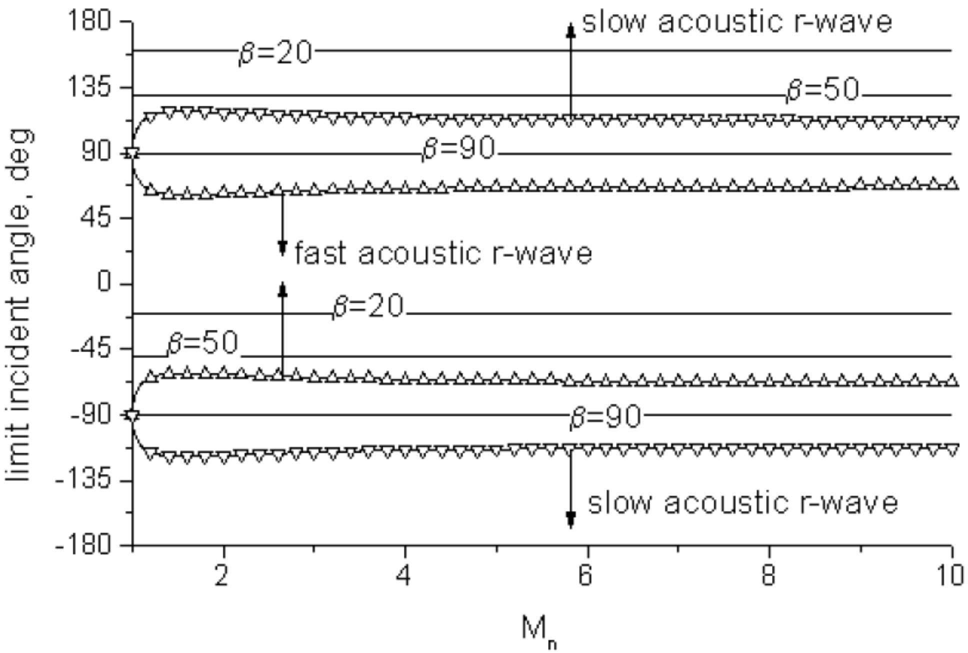
red = $\pm\psi_*^{+}$

blue = $\pm\psi_*^{-}$

Critical angles: slow acoustic and entropy/vorticity incidence

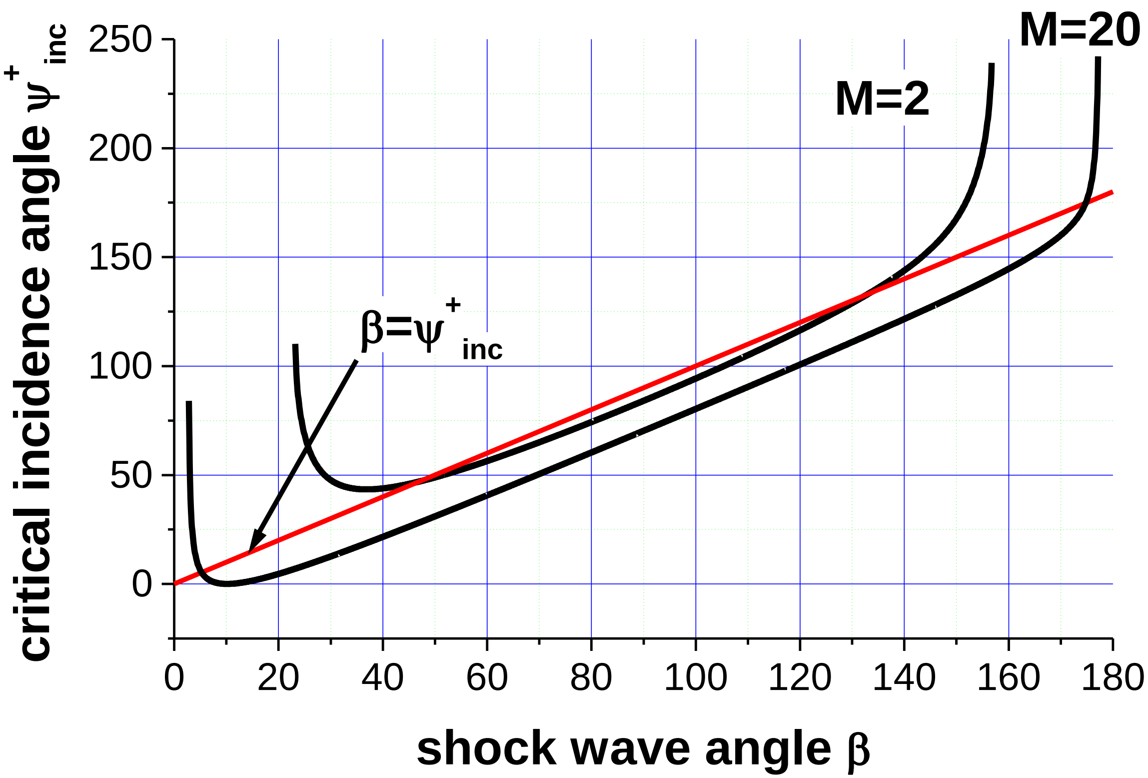


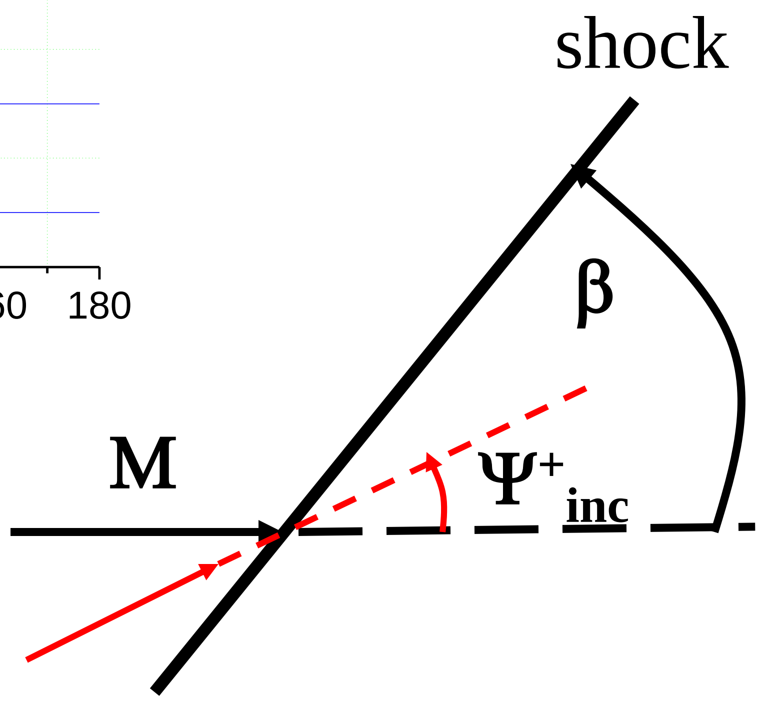
Entropy/vorticity wave incidence



Slow acoustic wave incidence

Critical angles: fast acoustic incidence





Analyzing sound reflection case:

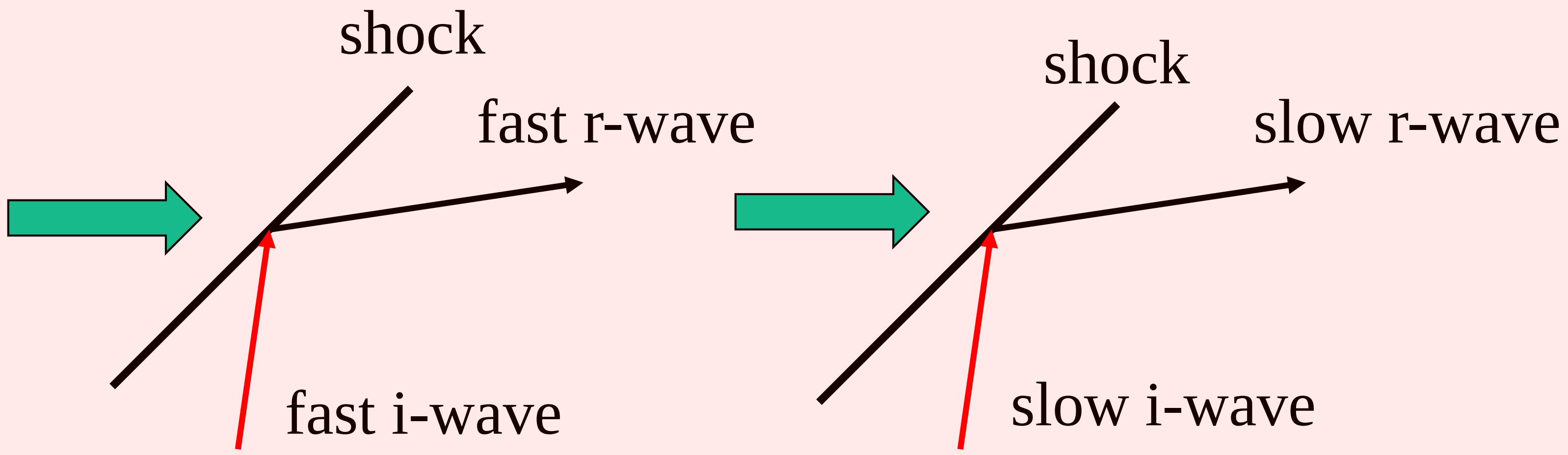
i) No entropy/vorticity wave incidence;

ii) Acoustic wave incidence is limited by the following conditions:

$$\varepsilon(\mathbf{k_i},\mathbf{n}) > M_{sn}; \quad (\mathbf{M}_s,\mathbf{k_i}) > \varepsilon; \qquad \pm[zM_{sn}-\varepsilon] \geq \sqrt{(1-M_{sn})(1-z^2)}$$

$\varepsilon = -1, +1$ for fast acoustic, and slow acoustic wave

Only 2 situations admissible:



Determining wave amplitudes

The amplitudes of 3 waves: entropy, vorticity and fast (or slow) acoustic must be defined along with ***the velocity of the perturbed shock front***.

The linearization of the Rankine-Hugoniot relations yields:

$$A_s \mathbf{q}'_s - A\mathbf{q}' = C_{sh}(\mathbf{b}_s - \mathbf{b})$$

$\mathbf{q}'$ =amplitude vector, C_{sh}=velocity of shock, A=A($\mathbf{q}$), $\mathbf{b}$=$\mathbf{b}$($\mathbf{q}$)

Reflection problem $\Rightarrow$ $\mathbf{q}' = 0,\ \mathbf{q}'_s = \mathbf{q}'_{s,i} + \mathbf{q}'_{s,r}$

Refraction problem $\Rightarrow$ $\mathbf{q}' = \mathbf{q}'_i,\ \mathbf{q}'_s = \mathbf{q}'_{s,r}$

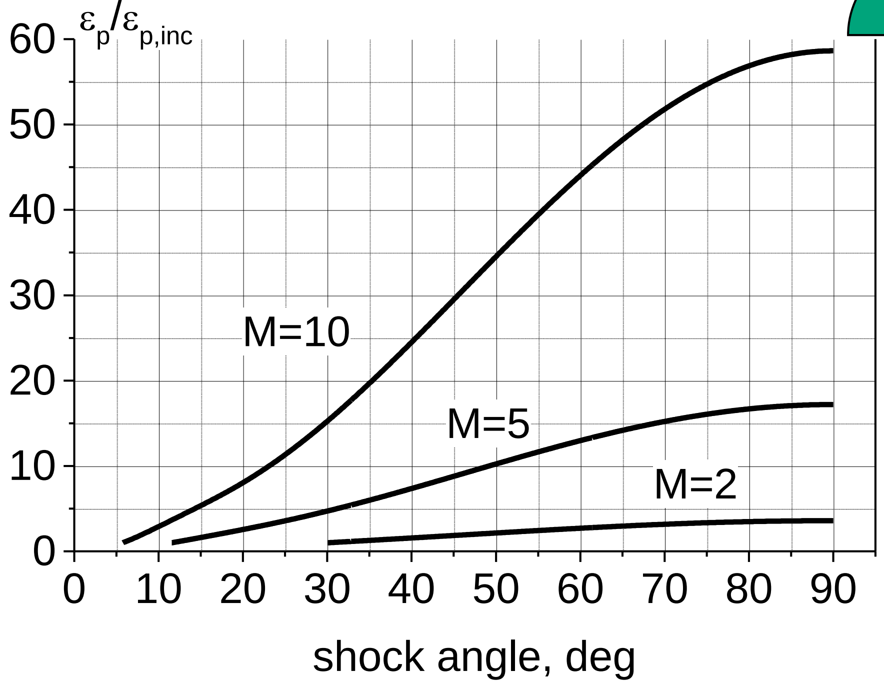
The r-field amplitude is decomposed as $\mathbf{q}'_{s,r} = q_\rho \mathbf{e}_{en} + q_p \mathbf{e}_{ac} + q_u \mathbf{e}_{vrt}$

$q_{\rho,p,u}$ = wave characteristic amplitudes to be defined

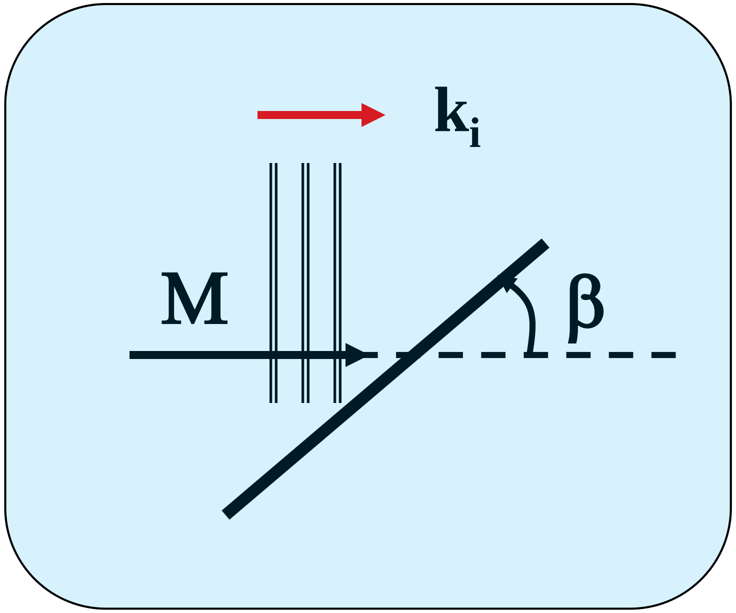
$\mathbf{e}_{en,ac,vrt} = \mathbf{e}_{en,ac,vrt}(\mathbf{q}) =$ normalized amplitude vectors

Transmission coefficients

Incidence of fast acoustic waves; k_i **collinear upstream flow**



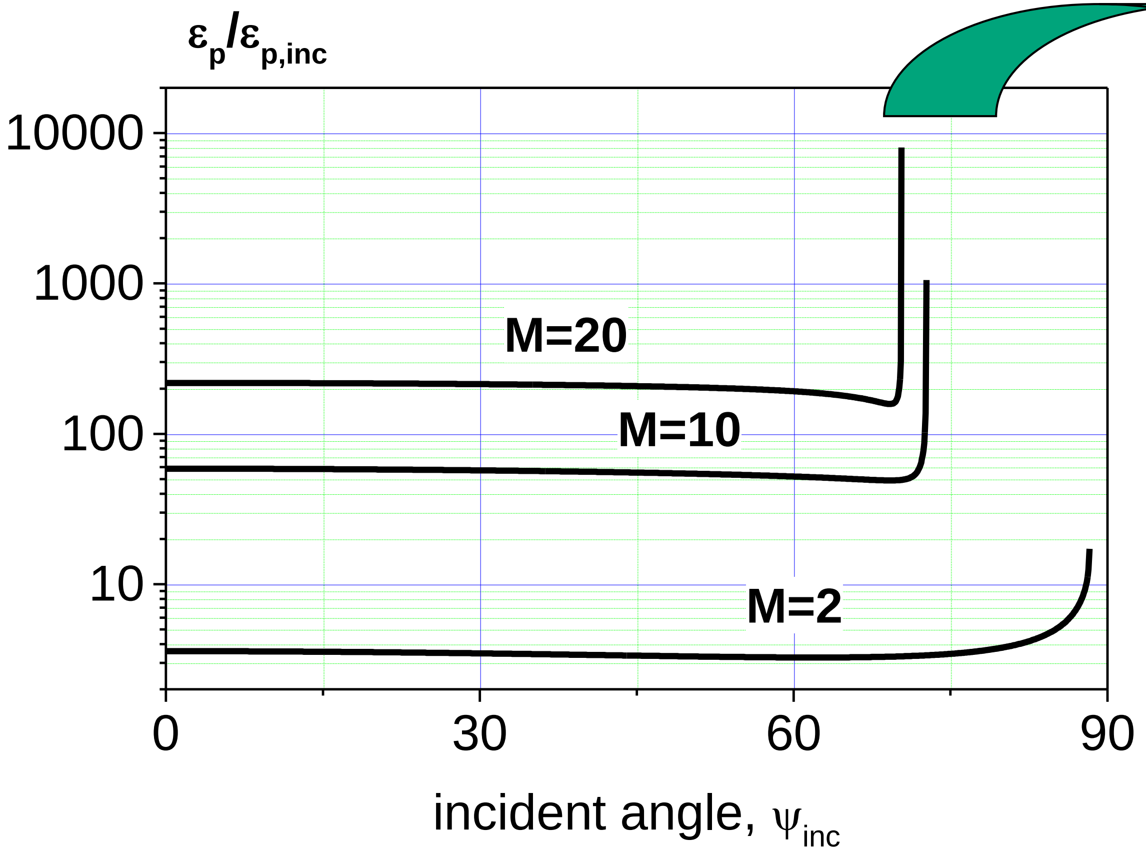
$$(\varepsilon_p/\varepsilon_{p,inc})_{max} \sim M^2$$



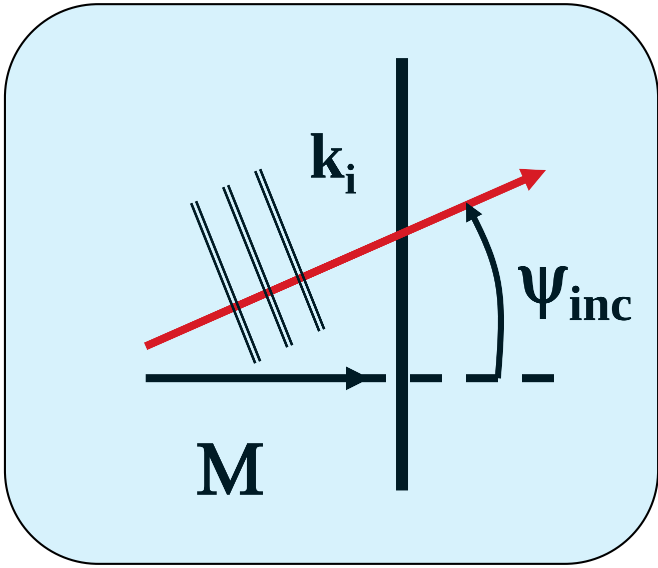
Amplification of pressure amplitude vs shock angle β

Transmission coefficients

Incidence of fast acoustic waves; normal shock



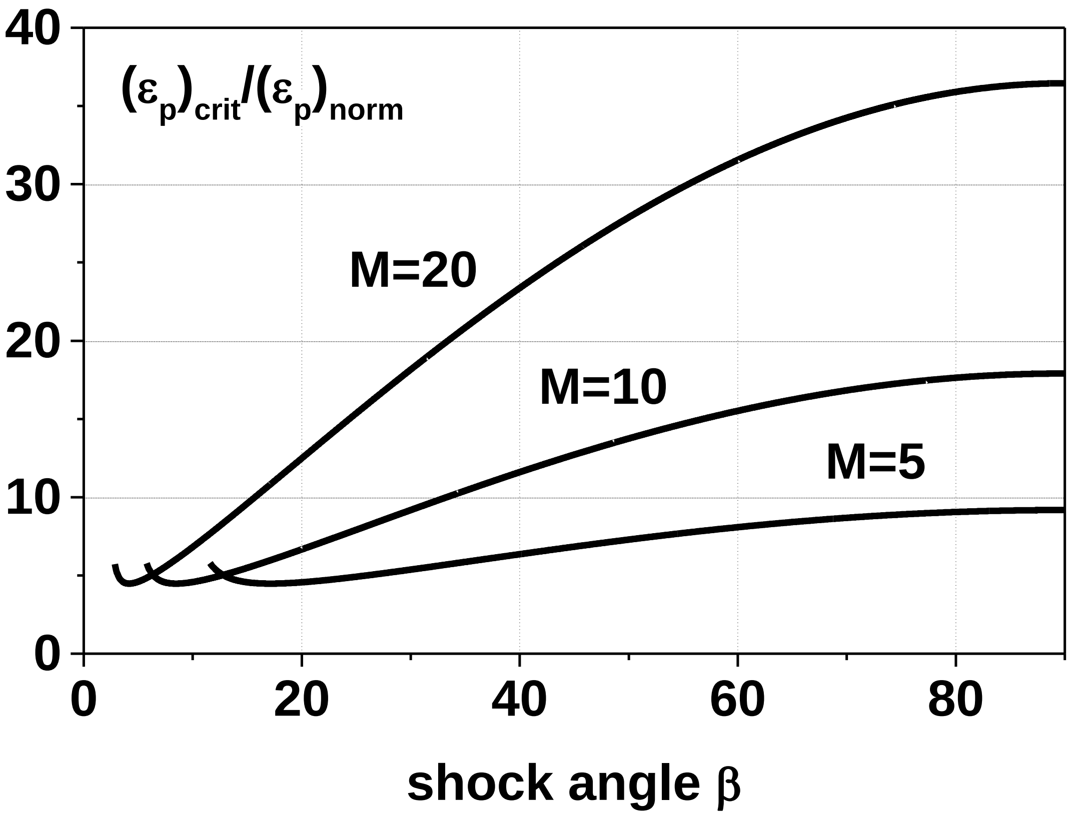
$(\varepsilon_p/\varepsilon_{p,inc})_{max} \sim M^3$

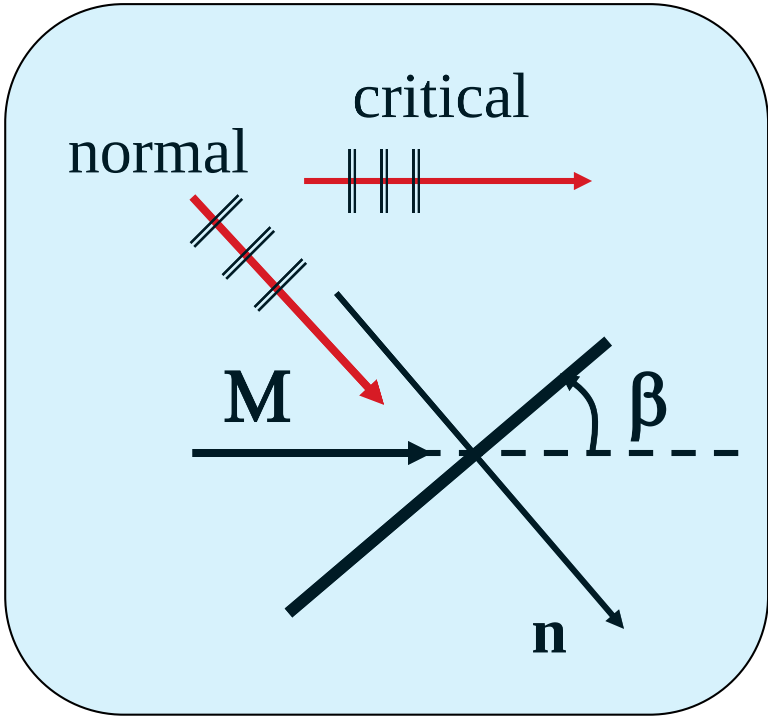


Amplification of pressure amplitude vs incident angle ψ_{inc}

Transmission coefficients

Incidence of fast acoustic waves at critical and normal angles

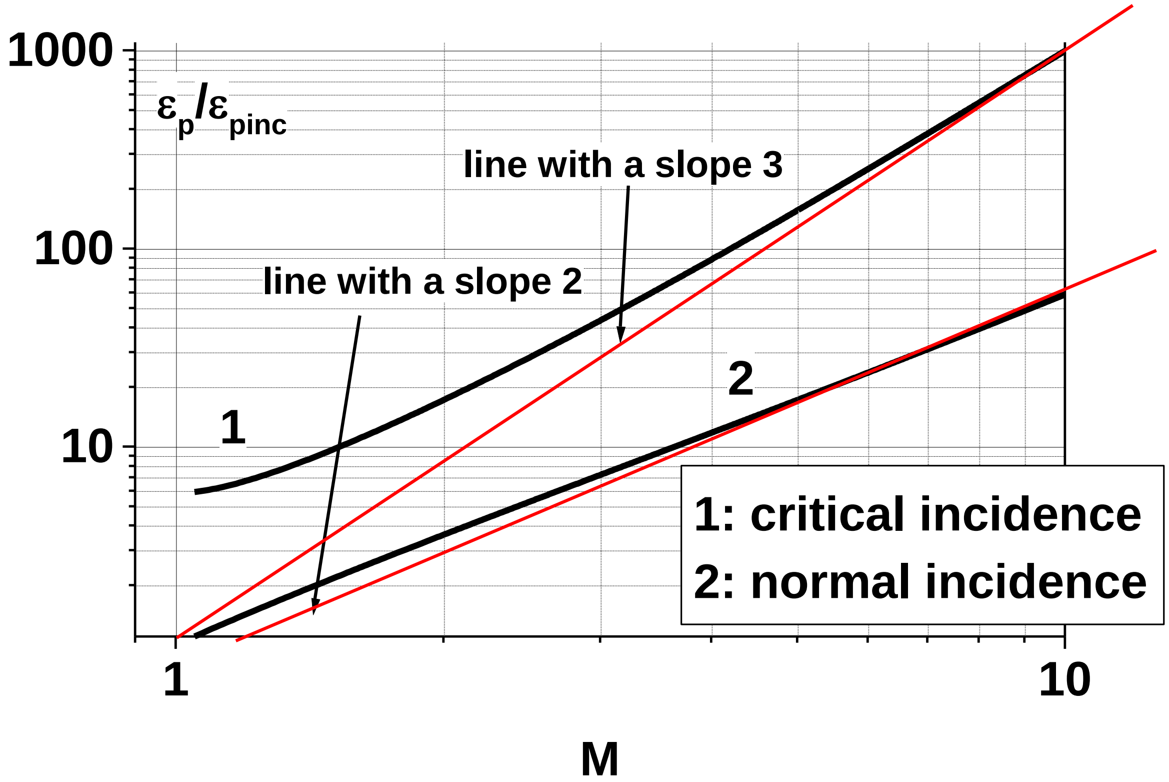




Ratio of wave amplitude transmitted at critical and normal incidence vs shock angle

Transmission coefficients

Normal shock: critical and normal incidence



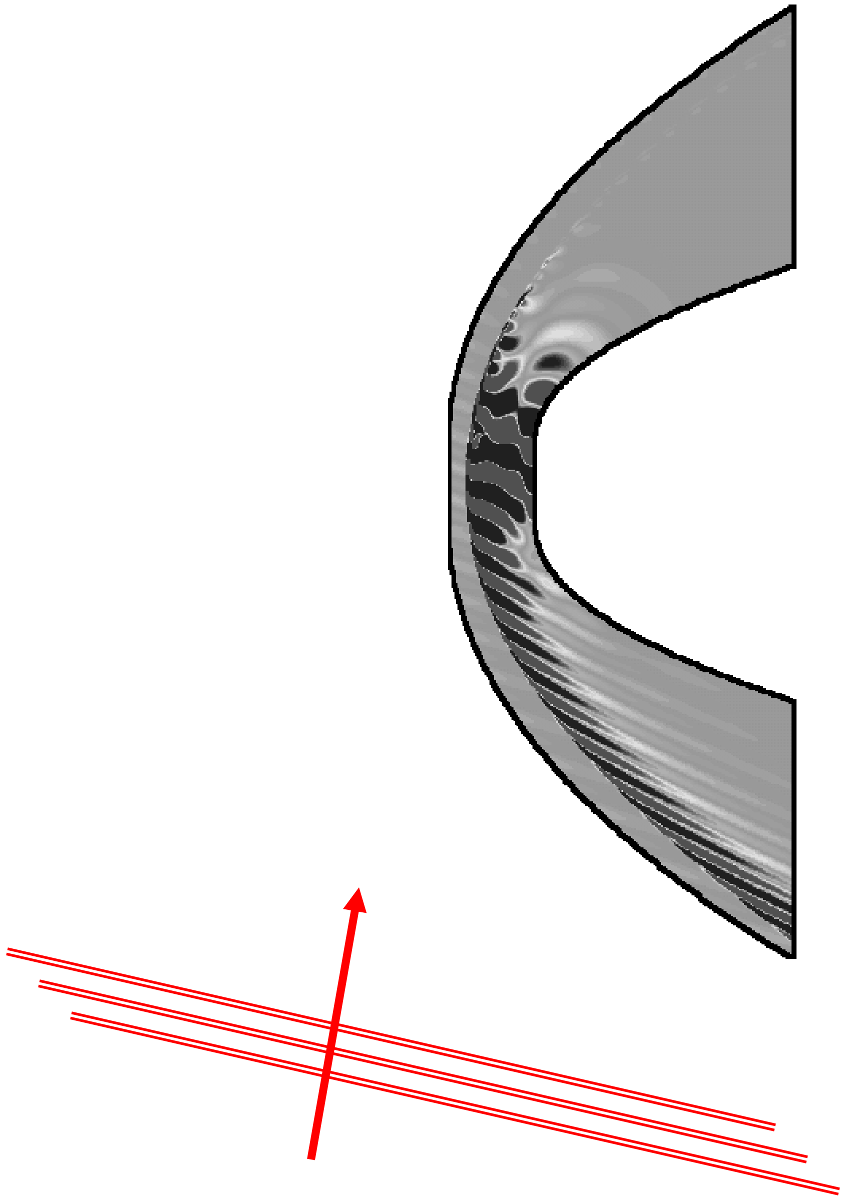
Amplification of pressure amplitude vs Mach number

Conclusions

i) Normally, the amplitude of the transmitted wave in the sound/shock interaction is of $O(M^2)$ greater than that in the incident wave.

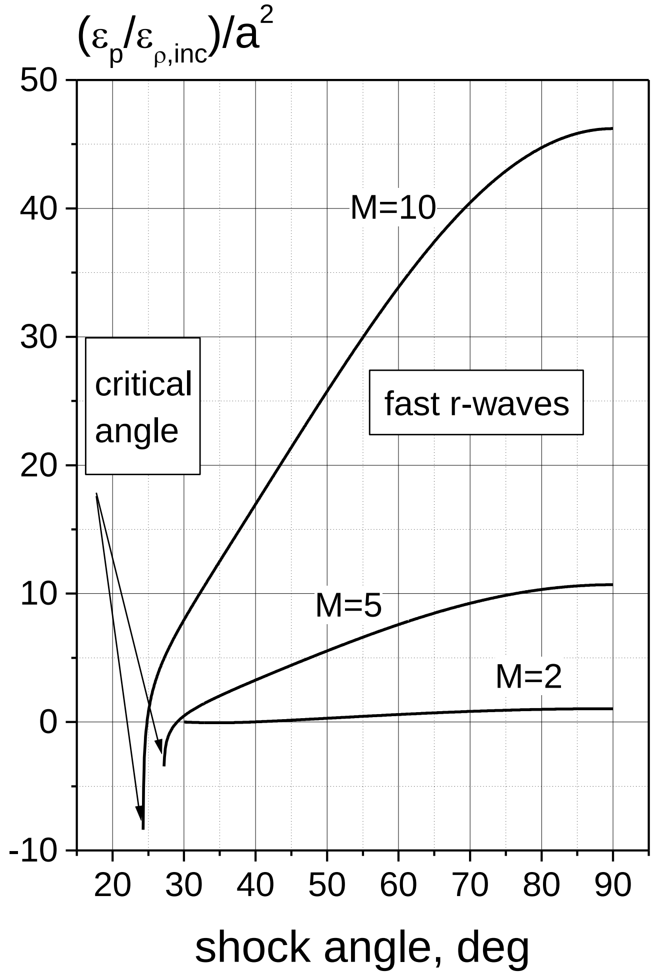
ii) As the incidence angle increases, the amplitude amplification slightly decreases.

iii) However, when the sound wave runs the shock at near the critical angle, the pressure amplification factor is abruptly grows up, taking asymptotically order $O(M^3)$.

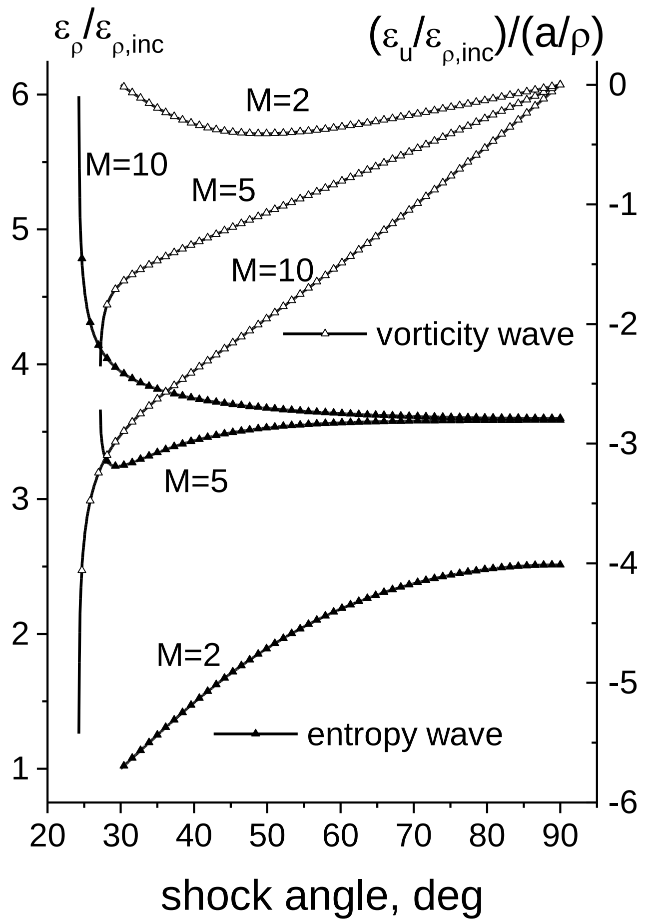


Transmission and generation coefficients

Incidence of entropy wave



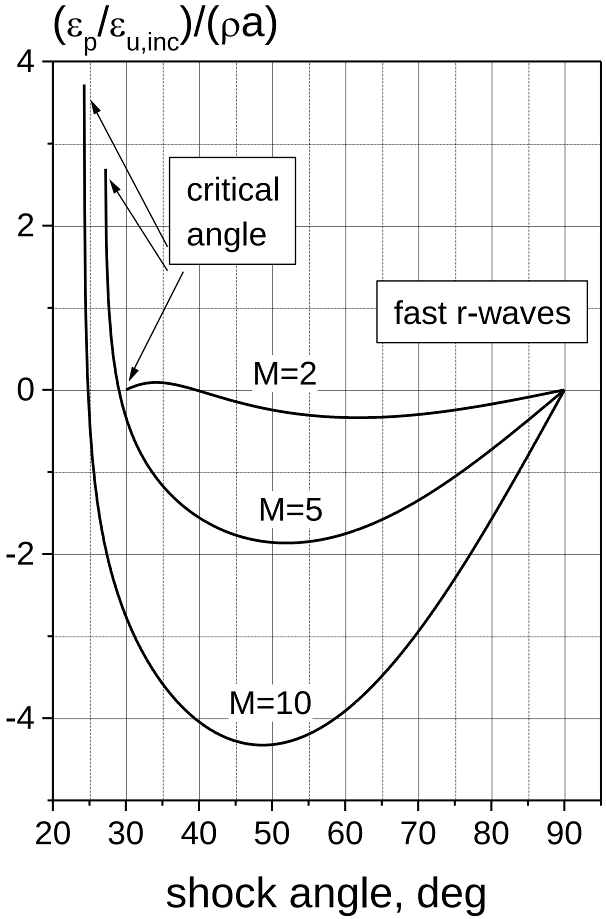
Generation coefficient
(fast wave generated)



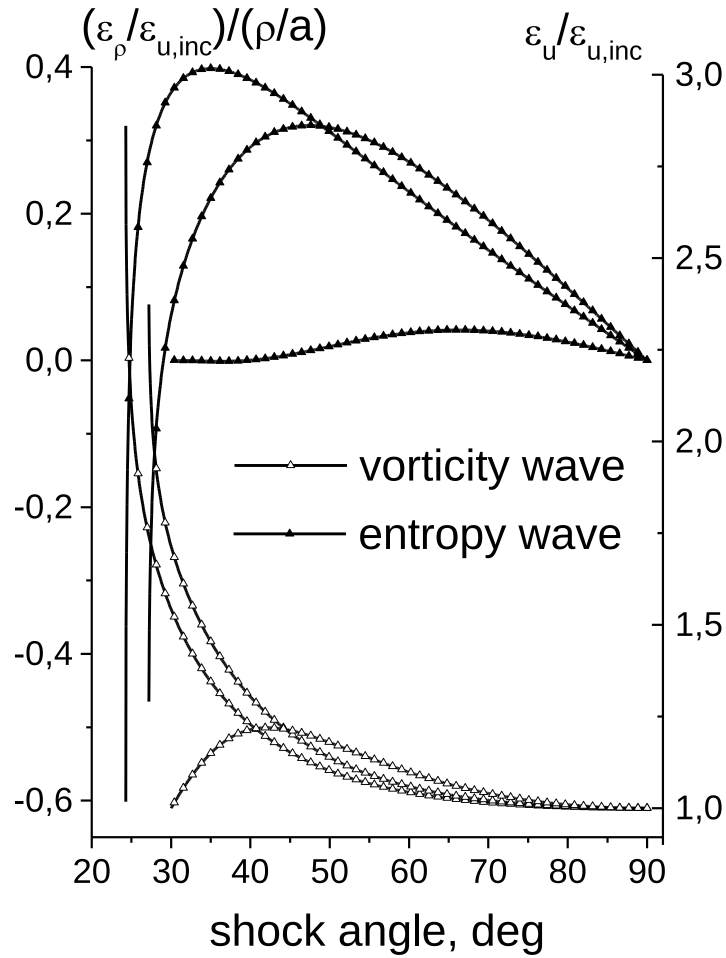
Generation and transmission coefficients

Transmission and generation coefficients

Incidence of vorticity wave



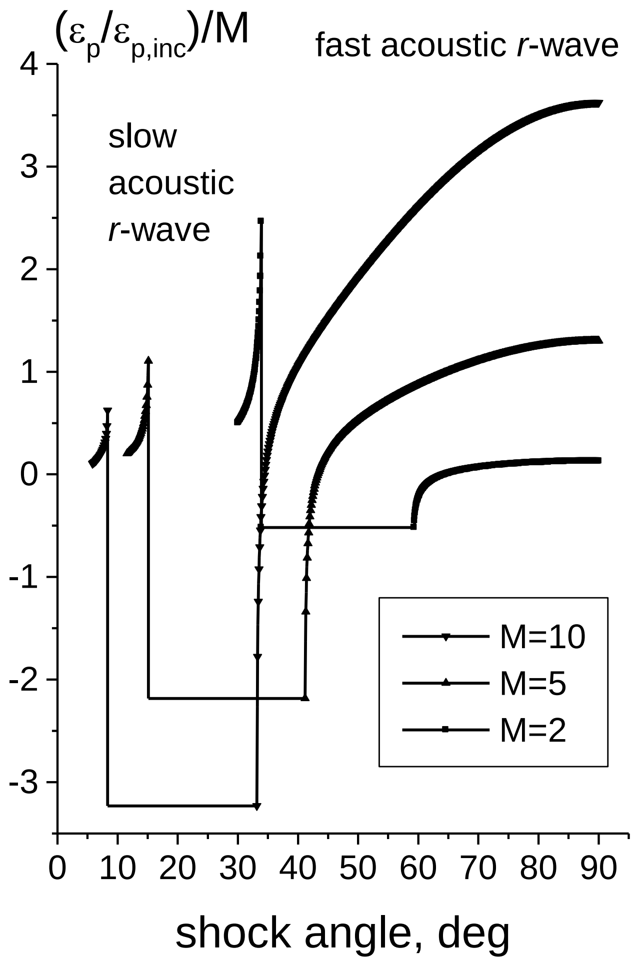
Generation coefficient
(fast wave generated)



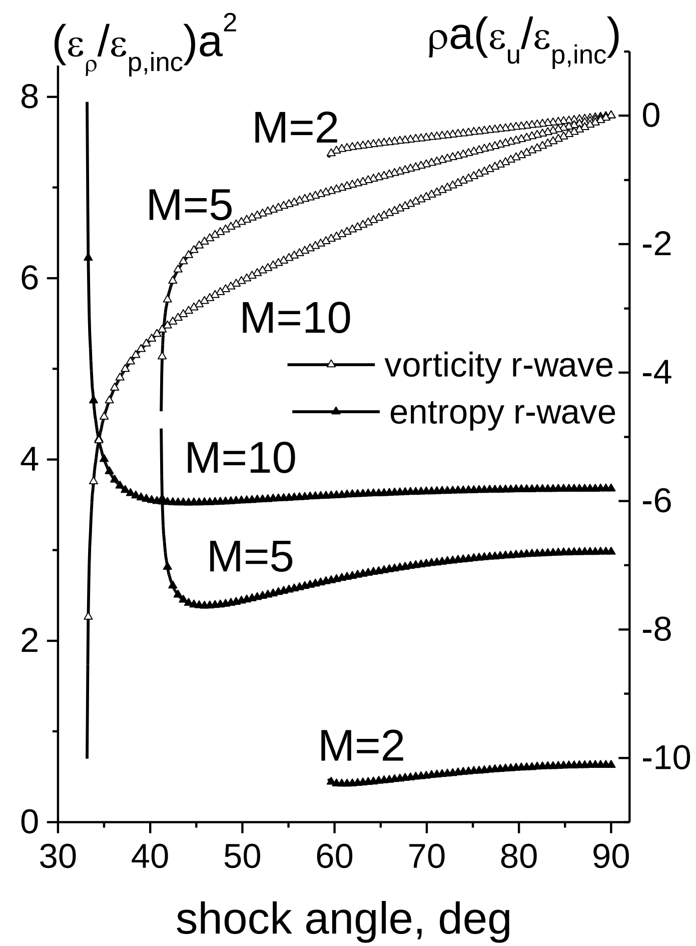
Generation/transmission
coefficients

Transmission and generation coefficients

Incidence of slow acoustic wave



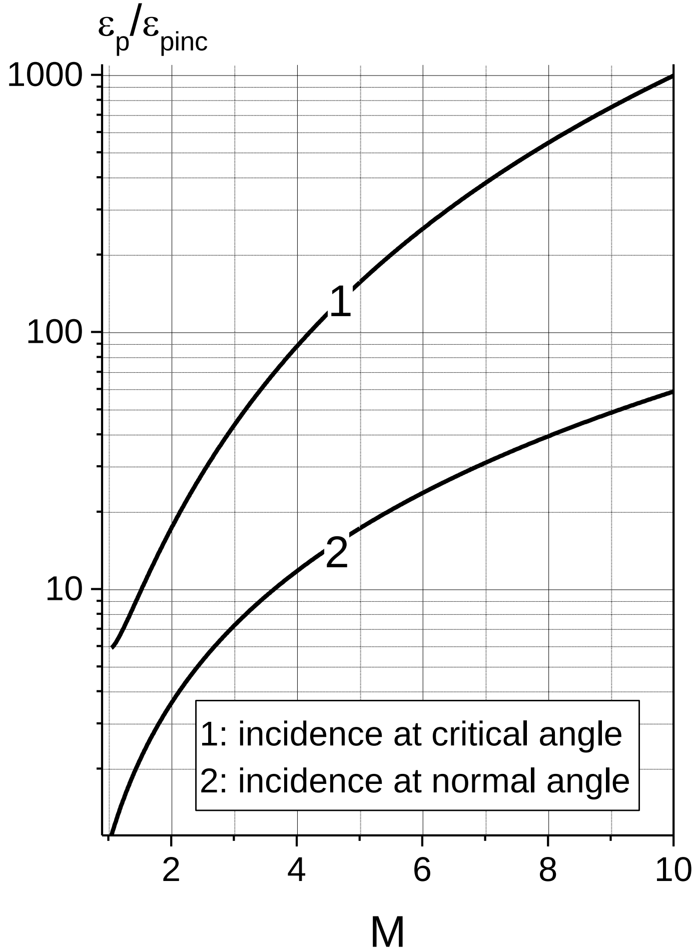
Transmission coefficient



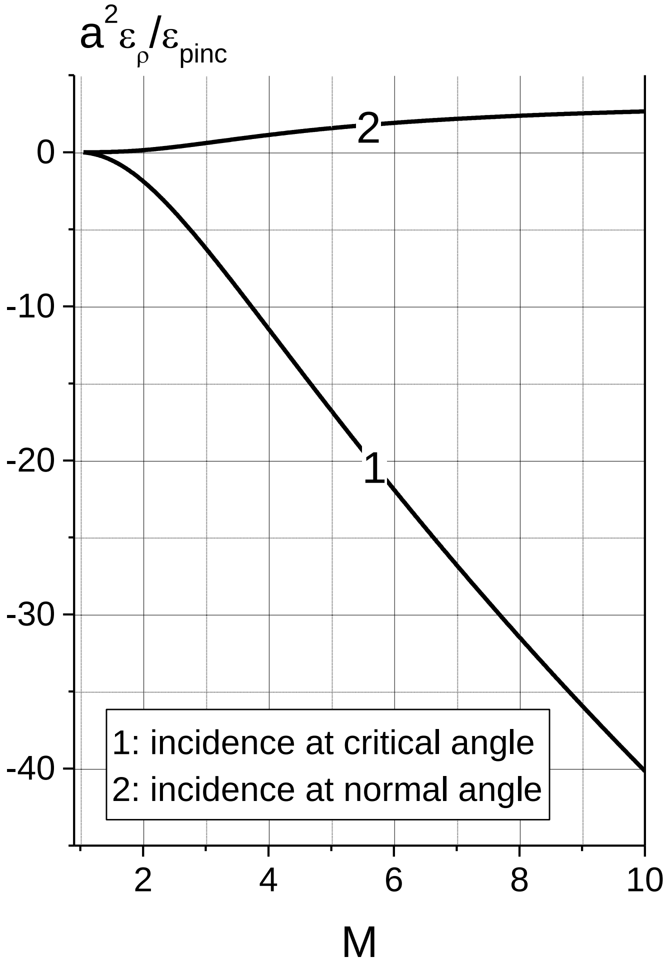
Generation coefficient

Waves at critical and normal incidence

Fast wave incidence from ahead of the normal shock



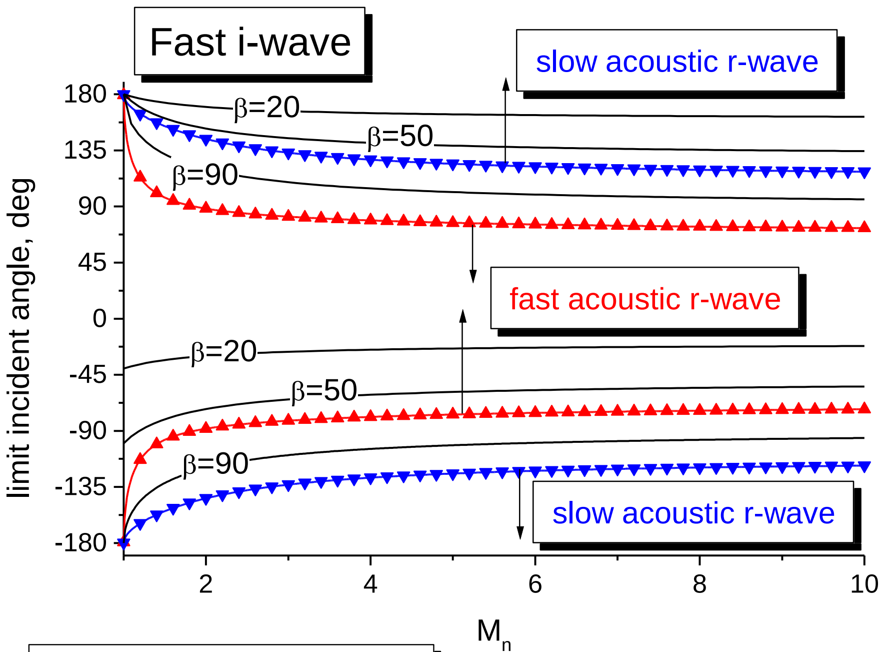
Transmission pressure coefficient

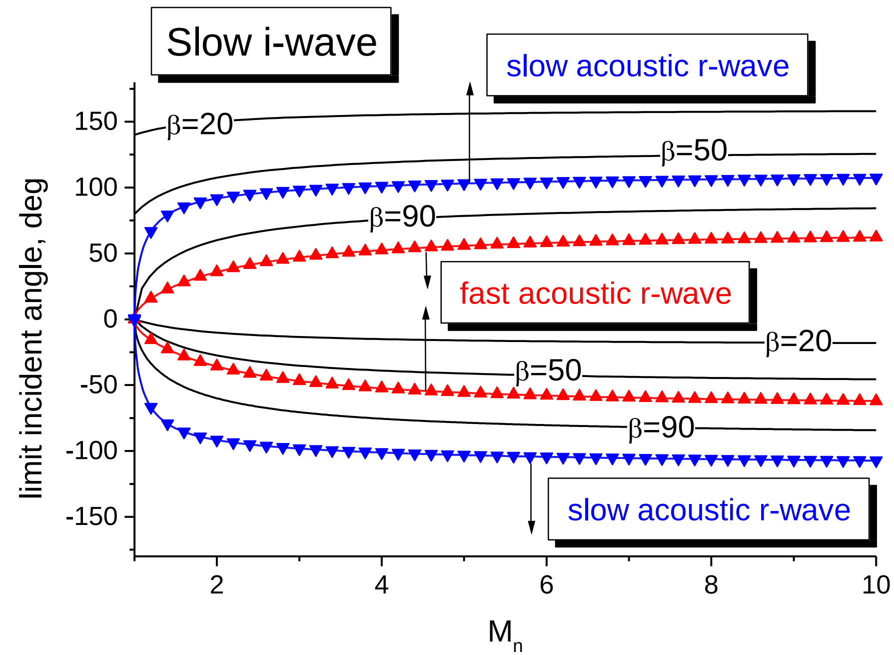


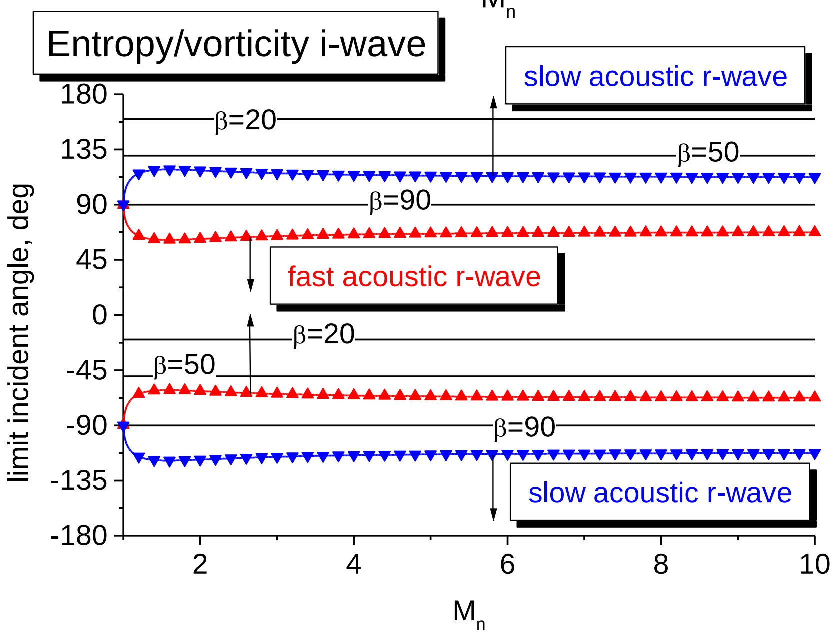
Generation density coefficient

Conclusion

Critical incidence angle in the refraction problem







Black lines depict Mach cone angles ($\mathbf{n}_i$,$\mathbf{l}$)>ε/M:

$$\arcsin(\varepsilon / M) - \beta < \psi_{inc} <$$

$$< \pi - \beta - \arcsin(\varepsilon / M)$$

Incidence angle in the reflection problem

In the ***reflection*** problem, the ***incidence angle condition*** yields

$$\pm(M_{sn}z-\varepsilon)\geq\sqrt{(1-M_{sn}^2)(1-z^2)}$$

with ε=-1,+1 for fast and slow acoustic i-wave

This shows that ***fast i-wave always reflects in a fast r-wave, while slow i-wave - in a slow r-wave***.

The limits of the incidence angle are given by two conditions:

Mach cone: $(\mathbf{n}_i,\mathbf{l})>\varepsilon/M$

i-wave: $(\mathbf{n},\mathbf{n}_i)<\varepsilon/M_{sn}$

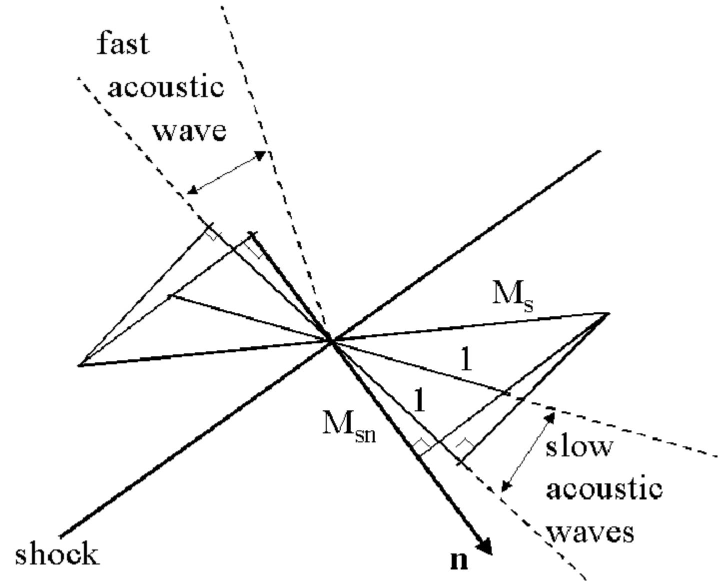


Diagram for the incidence angle in reflection problem