A dispersion improved CABARET scheme for linear acoustic propagation problems

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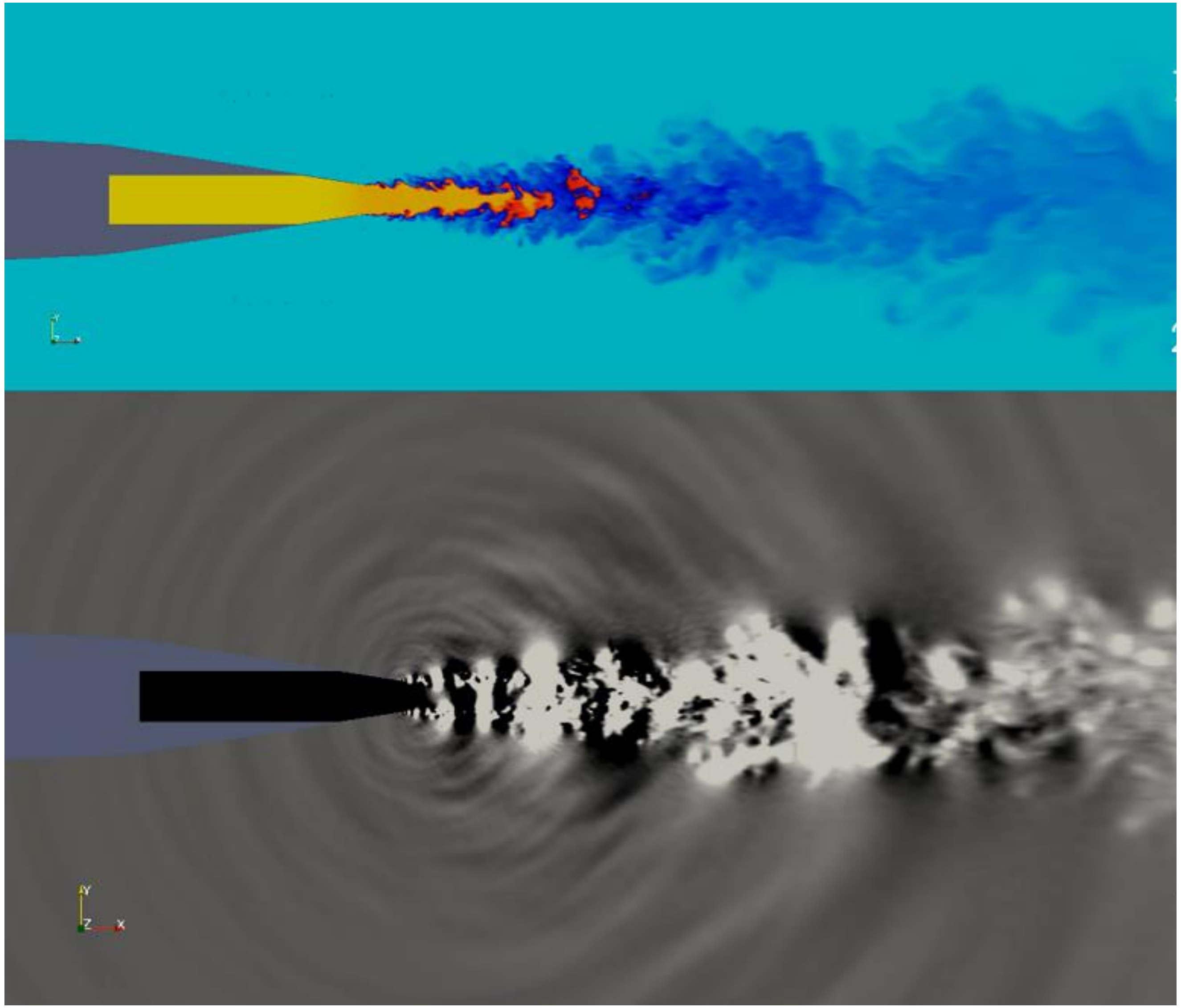
Acknowledgement:
Drs Anton Markesteijn and Vasily Semiletov

Summary

- Challenges of modelling in computational aeroacoustics
- CABARET solver for flow and noise modelling: our state-of-the art
- CABARET, dispersion improved version and flux correction possibilities
- Numerical examples: 1d and 2d test problems

Jet noise example: turbulent jet is a very non-efficient sound generator, less than ~ 1/100,000th of its mechanical energy transforms into sound: effect of sound cancellation



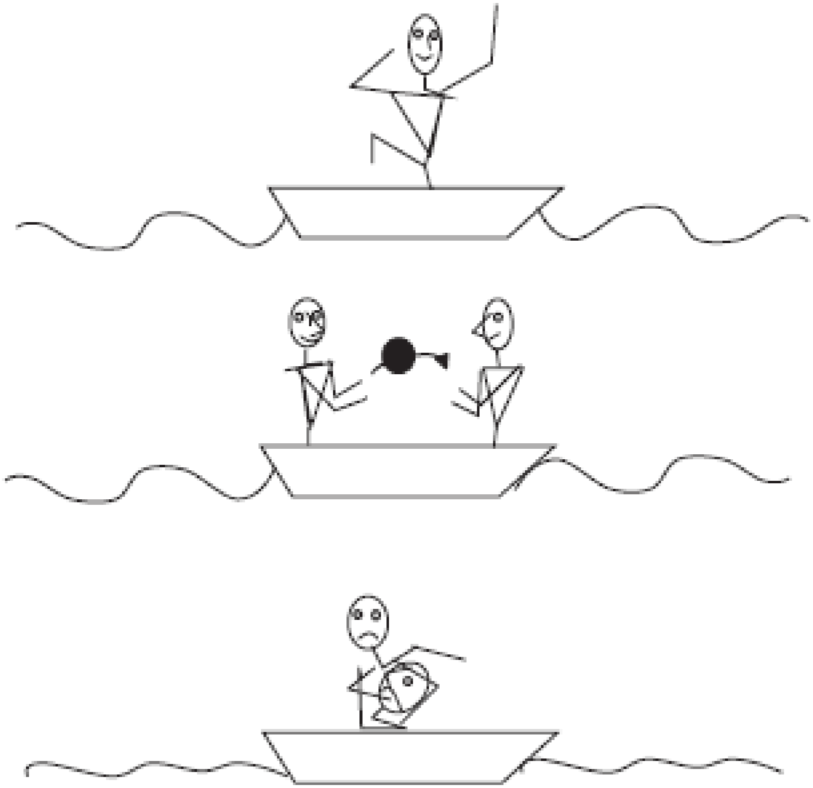


Figure 1: Monopole, dipole and quadrupole generating waves on the surface of the water around a boat.

Hirshberg and Rienstra, 2004

Von Neumann linear analysis for linear wave propagation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$$

Represent the solution by a Fourier harmonics

$$u(x,t) = \frac{1}{2\pi}\sum_k u_k(t)e^{-ik\cdot x}$$

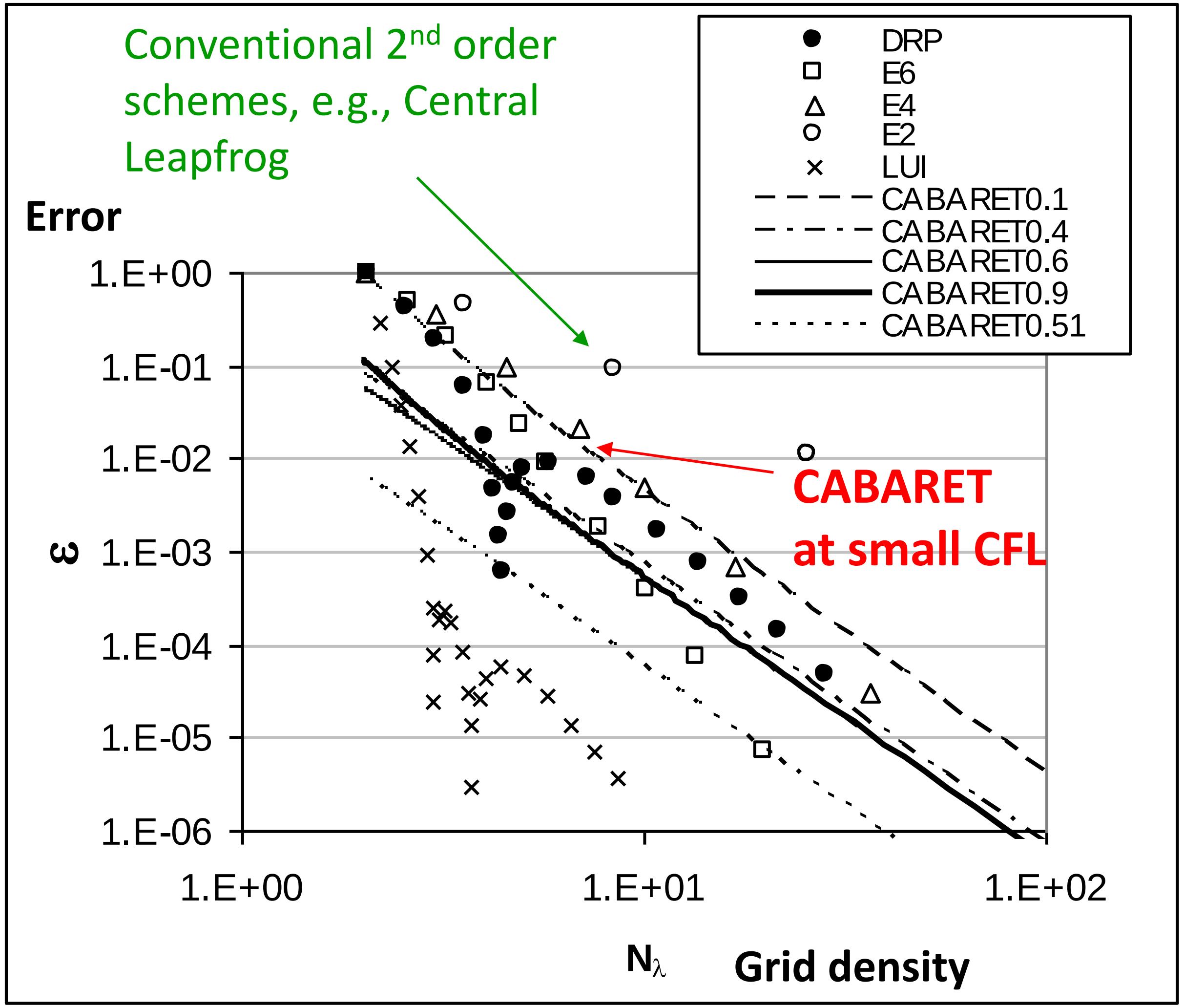
$$\frac{\partial u_k}{\partial t} + iA\cdot k\cdot u_k = 0 \Rightarrow u_k = C\cdot e^{i\cdot\omega\cdot t}, \underline{\omega = A\cdot k}$$ Dispersion relation

For real frequencies (A=real) the amplitude of simple waves is preserved

Comparison of linear dispersion errors of several finite-difference schemes in terms of grid points per wavelength (P.P.W.)

Dispersion error due to numerical discretisation

$$\varepsilon = \left| \frac{\omega}{Ak} - 1 \right|$$



Ex –central differences of the x-th order
LUI – 6th order penta-diagonal compact scheme by Lui and Lele
DRP – 4-th order DRP scheme by Tam and Webb
CABARETx, CABARET scheme at CFL=x

$$N_\lambda = \pi/(k \cdot h)$$

= Wavelength/ grid cell size

PS: Can do a similar analysis for dissipation error

Basic CABARET for linear advection using "active" conservation and flux variables

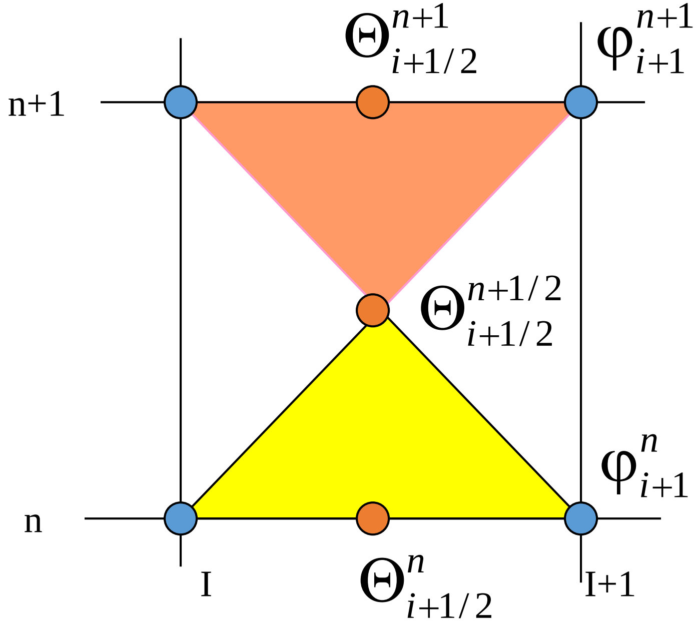
Goloviznin& Samarskii, 1998; Karabasov and Goloviznin, 2009

$$\frac{\partial \varphi}{\partial t} + c \cdot \frac{\partial \varphi}{\partial x} = Q$$

Calculate conservation variables at the mid-time step

$$\Theta_{i+1/2}^{n+1/2}$$

$$\frac{\Theta_{i+1/2}^{n+1/2} - \Theta_{i+1/2}^{n}}{\tau/2} + c \cdot \frac{\varphi_{i+1}^{n} - \varphi_{i}^{n}}{h} = Q \;;$$



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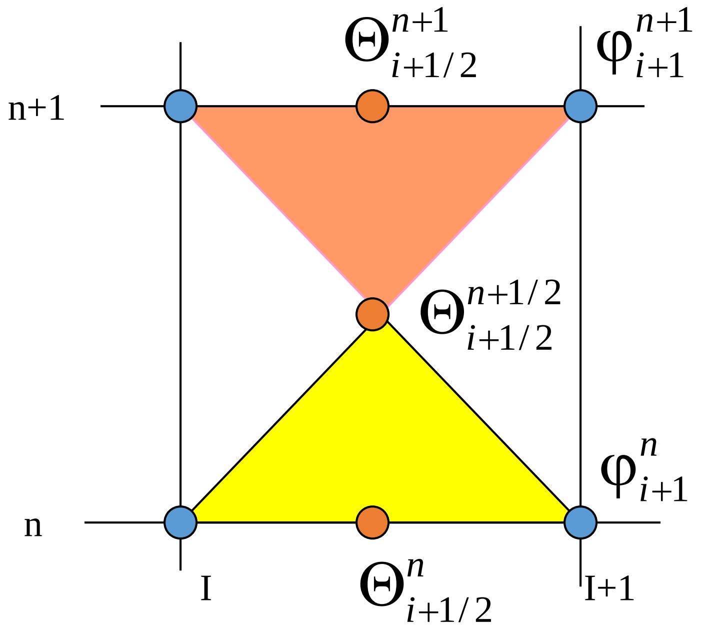
Upwind extrapolation of the flux variables

at the new time level φ_{i+1}^{n+1}

$$\varphi_{i+1}^{n+1} = 2 \cdot \Theta_{i+1/2}^{n+1/2} - \varphi_{i}^{n} \qquad C > 0$$



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Upwind extrapolation of the flux variables at the new time level φ_{i+1}^{n+1}

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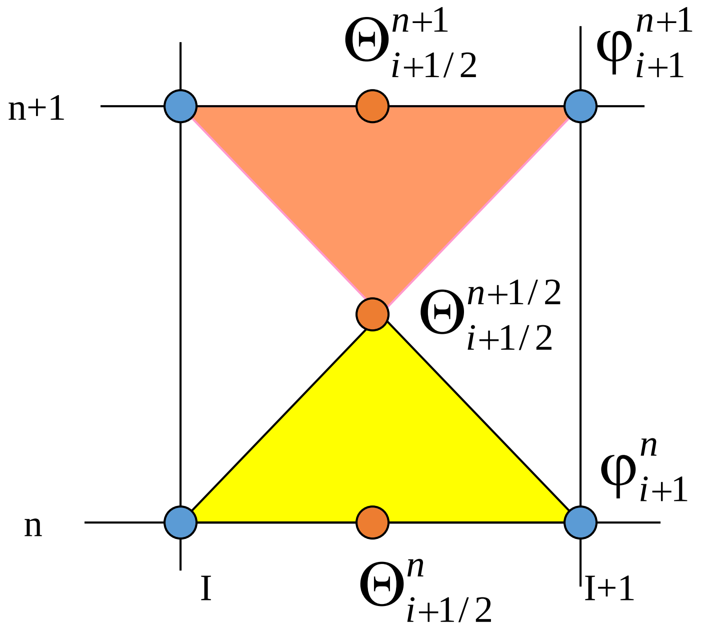
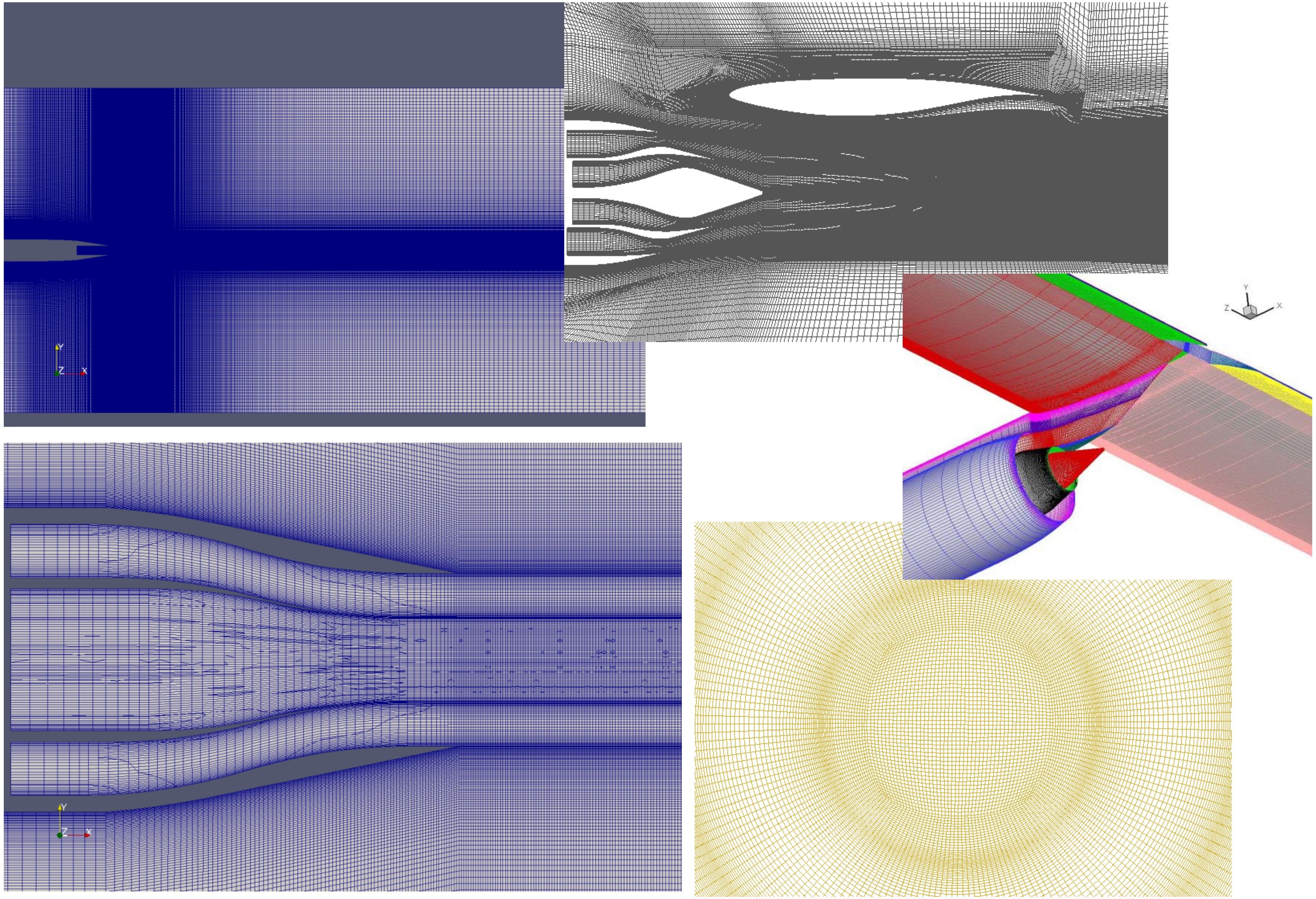


Update conservation variables at the new time step

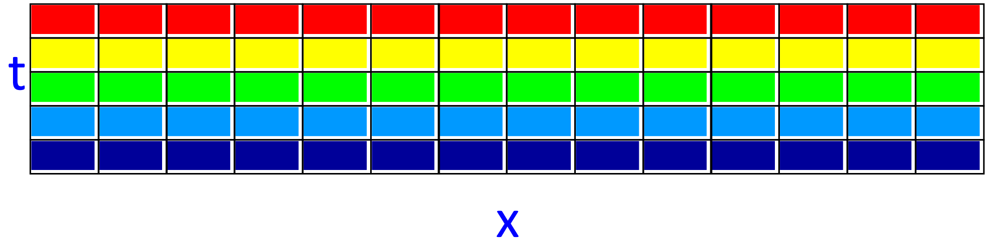
$$\frac{\Theta_{i+1/2}^{n+1} - \Theta_{i+1/2}^{n+1/2}}{\tau/2} + c \cdot \frac{\varphi_{i+1}^{n+1} - \varphi_{i}^{n+1}}{h} = Q \;;$$

Examples of grids

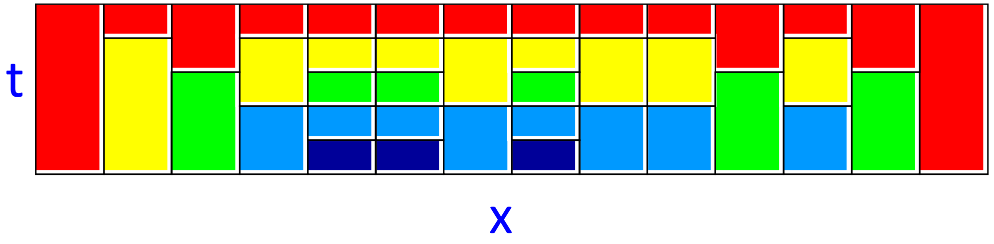


Time-stepping

Homogeneous



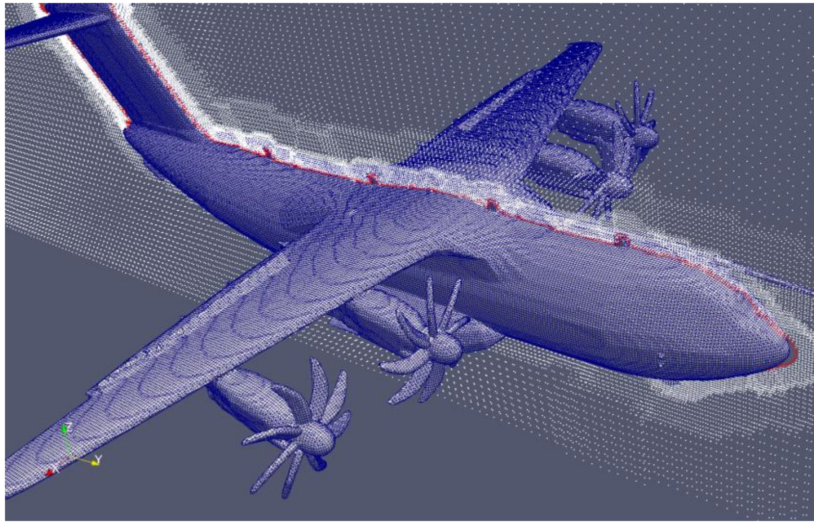
Asynchronous

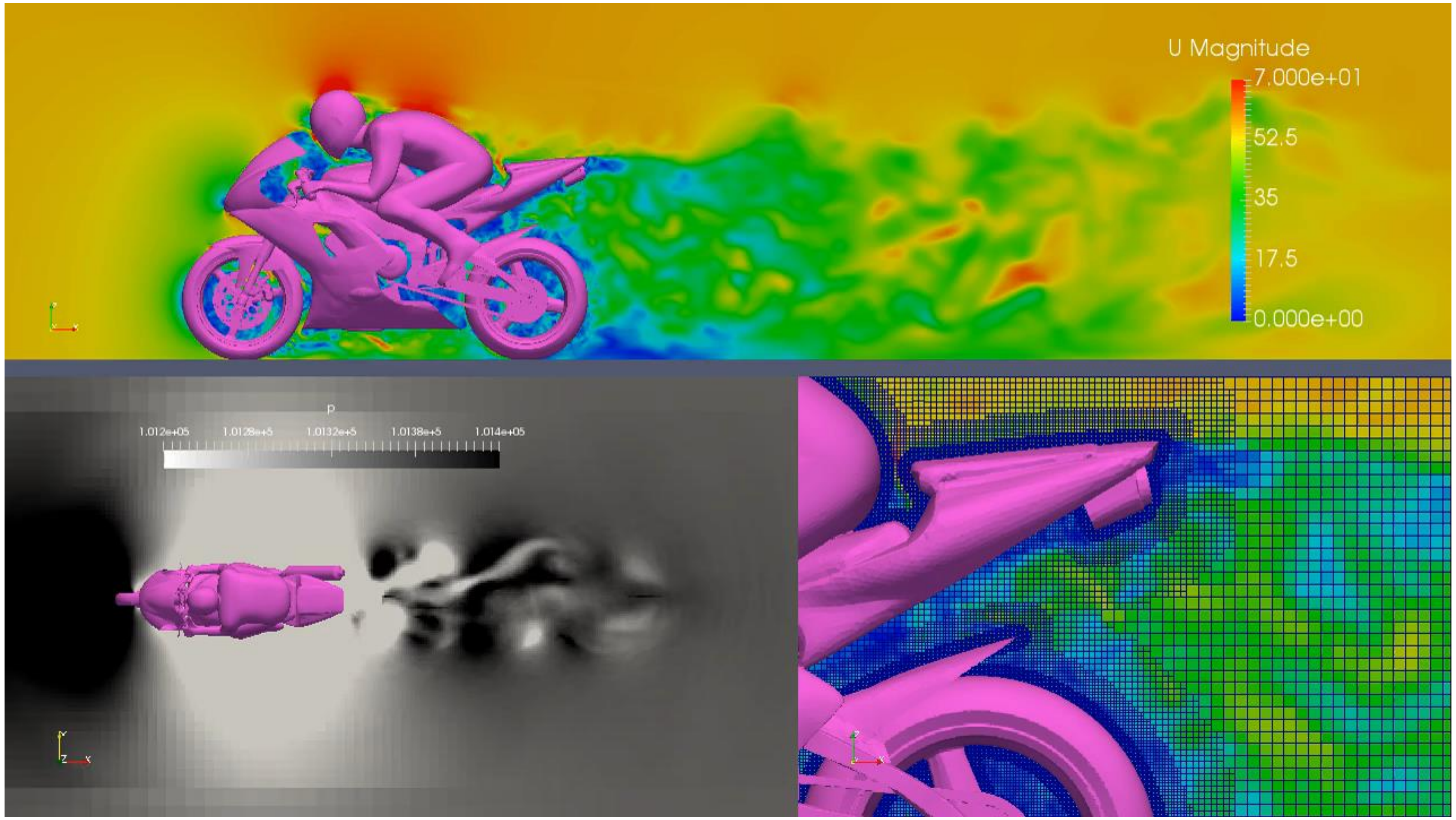


step

20 mln cell LES calculation on a snappy-hex mesh: 2GPU cards ~ 2 days simulate ~ 50 TUs

"per time step, 2 GPUs are about a factor 55 quicker than a code like OpenFOAM on a 16-core cluster node"





GPU Solver for Fast-Turn-Around Flow and Noise Calculations: our state-of-the art

"HiFi prediction for validation of designs is mandatory"

- Unstructured Hexagonal mesh + snappy-hexa + limited number of 1st order tetras is possible (OpenFOAM format)
- Some (old CPU) algorithms optimised for GPU usage
- Some algorithms completely rewritten from scratch
- Code is memory-optimised to fit on GPU and MPI-ed
  - 2.2 mln mesh per GB memory (6 GB~13mln)
- Asynchronous algorithm speeds-up computations
  - GPUs could do several magnitudes faster (>1000) than CPU
  - Same convergence rate of the original CABARET method
  - Decrease absolute error (CABARET's optimal CFL condition)
- Typical grids we can handle (2 K80 GPUs) =80-100 mln cells, typical run times (jet noise) = 2-4 days
- FWH acoustic modelling post-processor
- Goldstein acoustic analogy computation
- Results easily visualised with ParaView

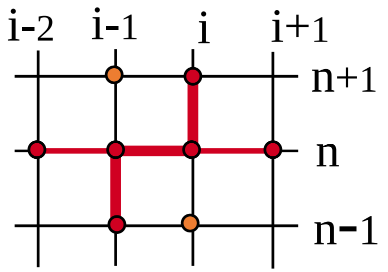
Goal: optimise CABARET for linear dispersion properties at small CFL
(ok to tolerate a few new calibration parameters so long as they are well defined!)
while preserving conservation & flux correction

CABARET with an artificial dispersion term ~ d^3u/dx^3

$$\frac{1}{2}\left(\frac{\hat{\varphi}_i-\varphi_i}{\tau}+\frac{\varphi_{i-1}-\breve{\varphi}_{i-1}}{\tau}\right)+c\,\frac{\varphi_i-\varphi_{i-1}}{h}=\frac{c\mu}{h}\left[(\varphi_{i+1}-2\varphi_i+\varphi_{i-1})-(\varphi_i-2\varphi_{i-1}+\varphi_{i-2})\right];$$

Goloviznin and Samarskii, 1998

Dispersion coeffient: $\mu=\varepsilon\,(1-r)(1-2r);$



Characteristic equation:

$$q_k^{\ 2}-q_k\left(1-e^{ikh}\right)\left[(1-2r)-8\mu r\left(1-\cos(kh)\right)\right]-e^{ikh}=0;$$

Non-dissipation (and stability) condition:

$$|q_1(kh,r)|\equiv 1\ ;\ |q_2(kh,r)|\equiv 1\ ;\ r\in[0,1]\ ;\ kh\in[0,2\pi];$$

satisfied for $(-2<\varepsilon<0.29)$

$\varepsilon=-0.08$

Dispersion improved CABARET in the flux form

Goloviznin and Samarskii, 1998

$$\frac{1}{2}\left(\frac{\hat{\varphi}_i-\varphi_i}{\tau}+\frac{\varphi_{i-1}-\breve{\varphi}_{i-1}}{\tau}\right)+\frac{f_i-f_{i-1}}{h_i}=0$$

F F*

$$f_i=\underbrace{c\varphi_i}_{F}+\underbrace{c\frac{\mu_i}{\hbar_{i+1}}\left(\frac{\varphi_{i+1}-\varphi_i}{h_{i+1}}-\frac{\varphi_i-\varphi_{i-1}}{h_i}\right)}_{F^*};$$

where

$$\hbar_i=0.5\left(h_i+h_{i-1}\right);$$

$$\mu_i=\varepsilon\left(h_i h_i-3c\tau\hbar_i+2c^2\tau^2\right);$$

$$\varepsilon=0.08$$

Dispersion improved CABARET in the flux form

Goloviznin and Samarskii, 1998

$$\frac{1}{2}\left(\frac{\hat{\varphi}_i-\varphi_i}{\tau}+\frac{\varphi_{i-1}-\breve{\varphi}_{i-1}}{\tau}\right)+\frac{f_i-f_{i-1}}{h_i}=0$$

F F*

$$f_i = c\varphi_i + c\frac{\mu_i}{\hbar_{i+1}}\left(\frac{\varphi_{i+1}-\varphi_i}{h_{i+1}}-\frac{\varphi_i-\varphi_{i-1}}{h_i}\right);$$

where

$$\hbar_i = 0.5\left(h_i+h_{i-1}\right);$$

$$\mu_i = \varepsilon\left(h_i h_i - 3c\tau\hbar_i + 2c^2\tau^2\right);$$

$$\varepsilon = 0.08$$

Extension to the vector case: (F=flux vector, U=vector variable, A=quasi-linear matrix, W=local Riemann invariants, L= left eigen matrix)

F*= A⊗*U; W* =L ⊗*U**; Λ=L ⊗A⊗ L^{-1}**

L ⊗F*= (L ⊗A⊗ L^{-1})⊗ L ⊗ *U =Λ ⊗ W***

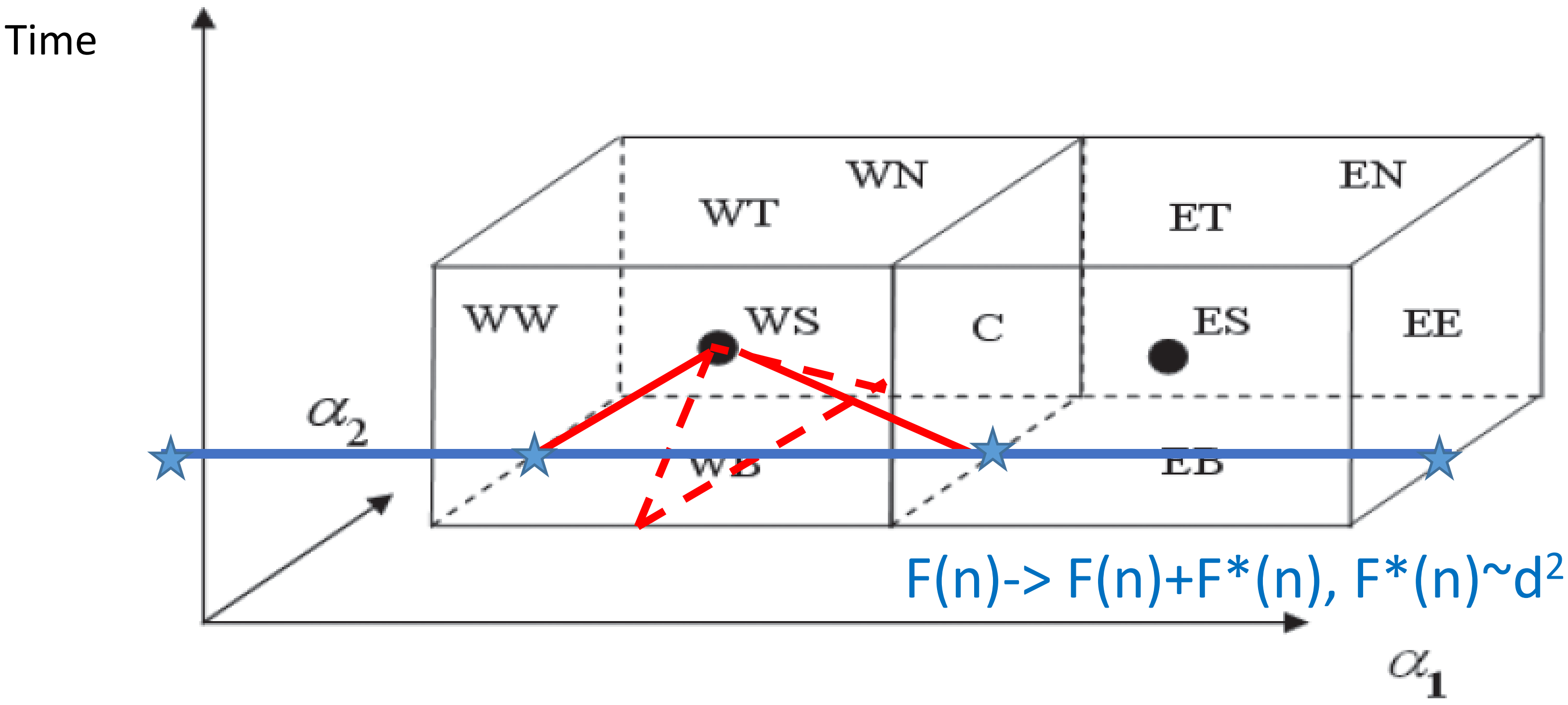
W* ~d^2W/dx^2; L ⊗F*~ Λ ⊗ d^2W/dx^2

F*= L^{-1} ⊗(L ⊗ F*)~ L^{-1} ⊗ (L ⊗A⊗ L^{-1}) ⊗ d^2W/dx^2

F*~ A⊗d^2U/dx^2= d^2F/dx^2

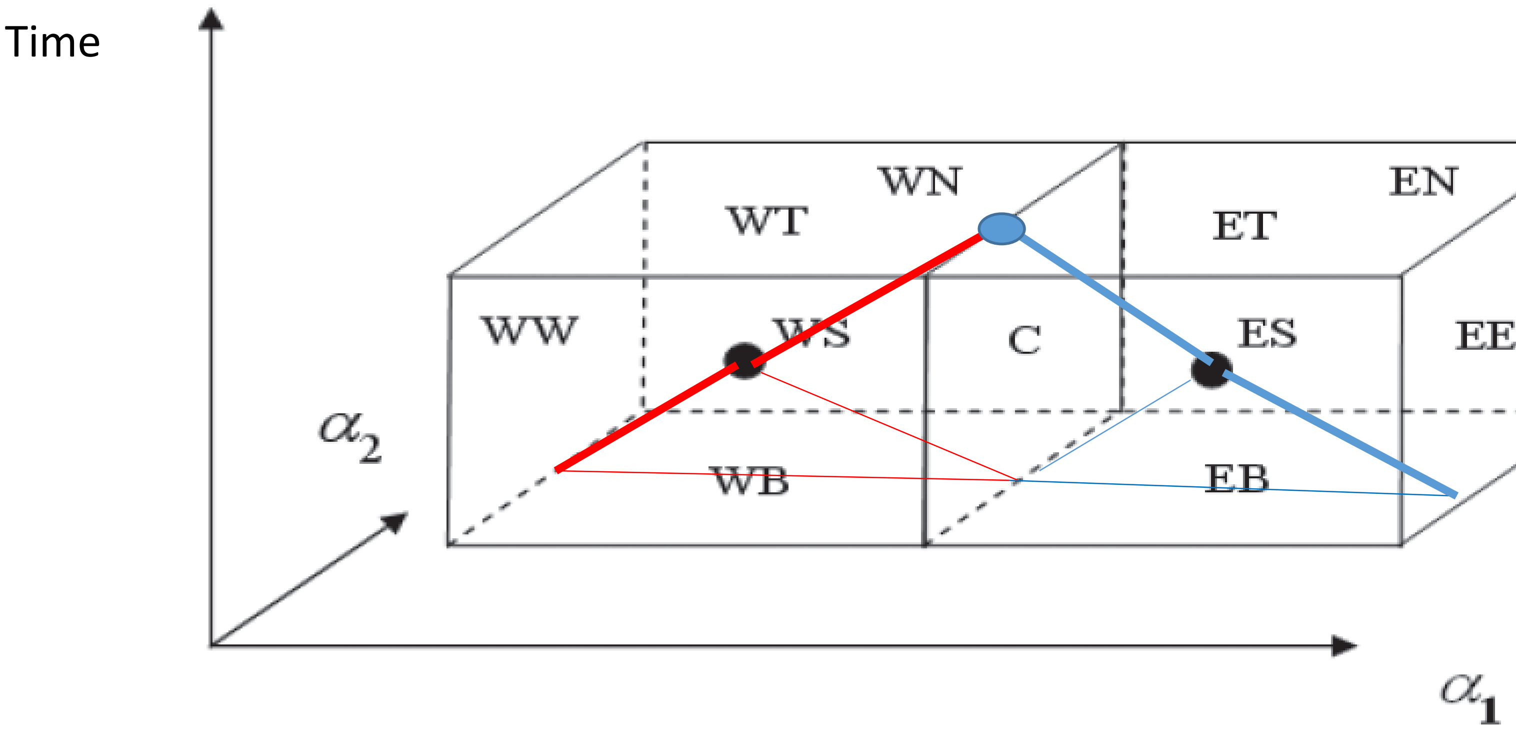
2D CABARET: predictor step

cell centre + cell face variables -> mid time level variables



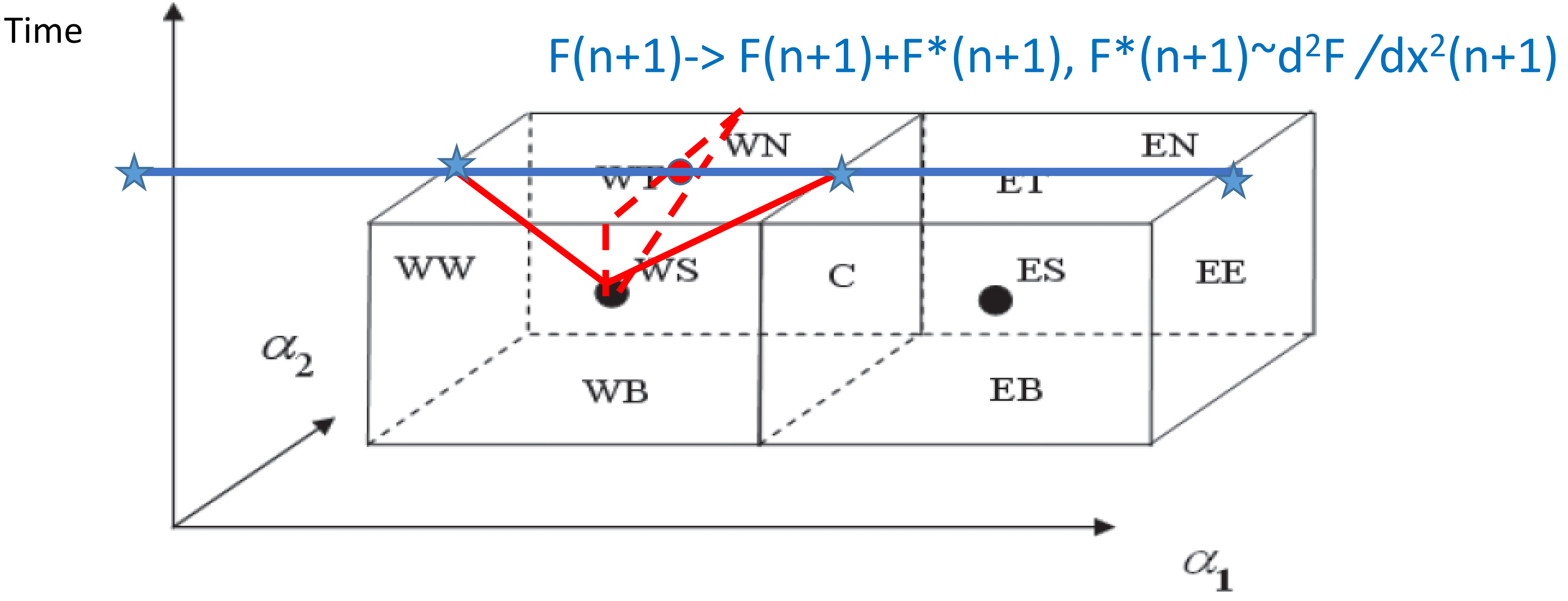
F(n)-> F(n)+F*(n), F*(n)~d^{2}F /dx^2(n)

CABARET characteristic decomposition step, extrapolation and solution of the Riemann problem at the cell face in each grid direction =the same!



2D CABARET: corrector step

cell centre + cell face variables -> new time level variables



Flux correction: Full Flux Corrected CABARET

- The P-CAB scheme is not monotone and allows non-physical oscillations.
- This is mitigated by applying a correction based on the maximum principle at the extrapolation stage

$$if\,(u_{i+\frac{1}{2}}^{n+1} > M)\,u_{i+\frac{1}{2}}^{n+1} = M \qquad M = max(u_{i+\frac{1}{2}}^{n}, u_{i}^{n}, u_{i-\frac{1}{2}}^{n})$$

$$if\,(u_{i+\frac{1}{2}}^{n+1} < m)\,u_{i+\frac{1}{2}}^{n+1} = m \qquad m = min(u_{i+\frac{1}{2}}^{n}, u_{i}^{n}, u_{i-\frac{1}{2}}^{n})$$

Flux correction: Relax Flux Corrected CABARET

- The maximum principle limits are relaxed using a tunable relaxation parameter (epsilon).

$$Mnew = (1+\varepsilon)M \qquad mnew = (1-\varepsilon)m$$

$$if(M<0)Mnew = (1-\varepsilon)M \qquad if(m<0)mnew = (1+\varepsilon)m$$

Flux correction: Modified Relax Flux Corrected CABARET

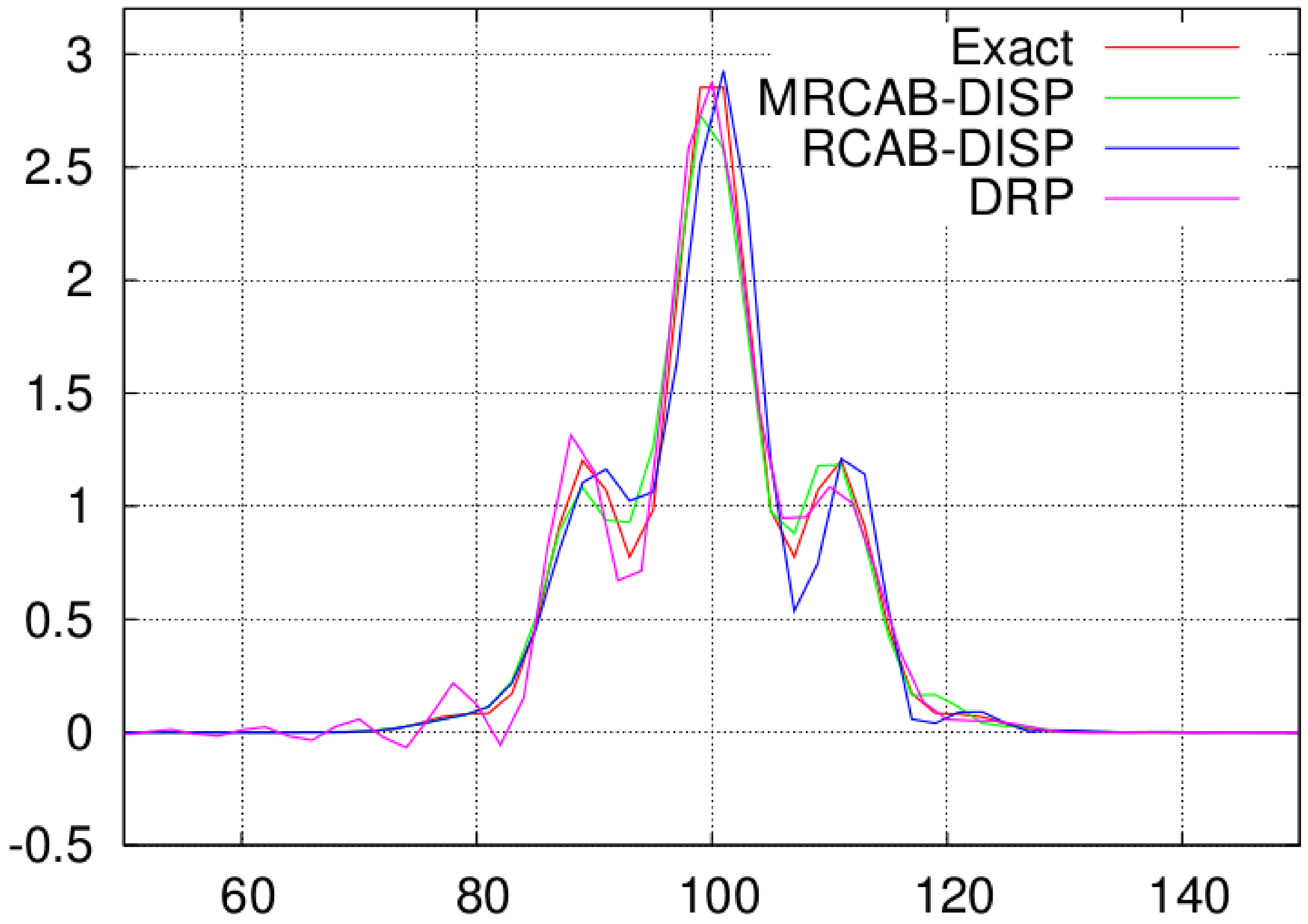
- The maximum principle limits are relaxed using a tunable relaxation parameter (epsilon) and also the local maxima and mimina.

$$\delta_{rx} = \frac{(M - m)}{2}$$

$$Mnew = M + \varepsilon\delta_{rx}$$

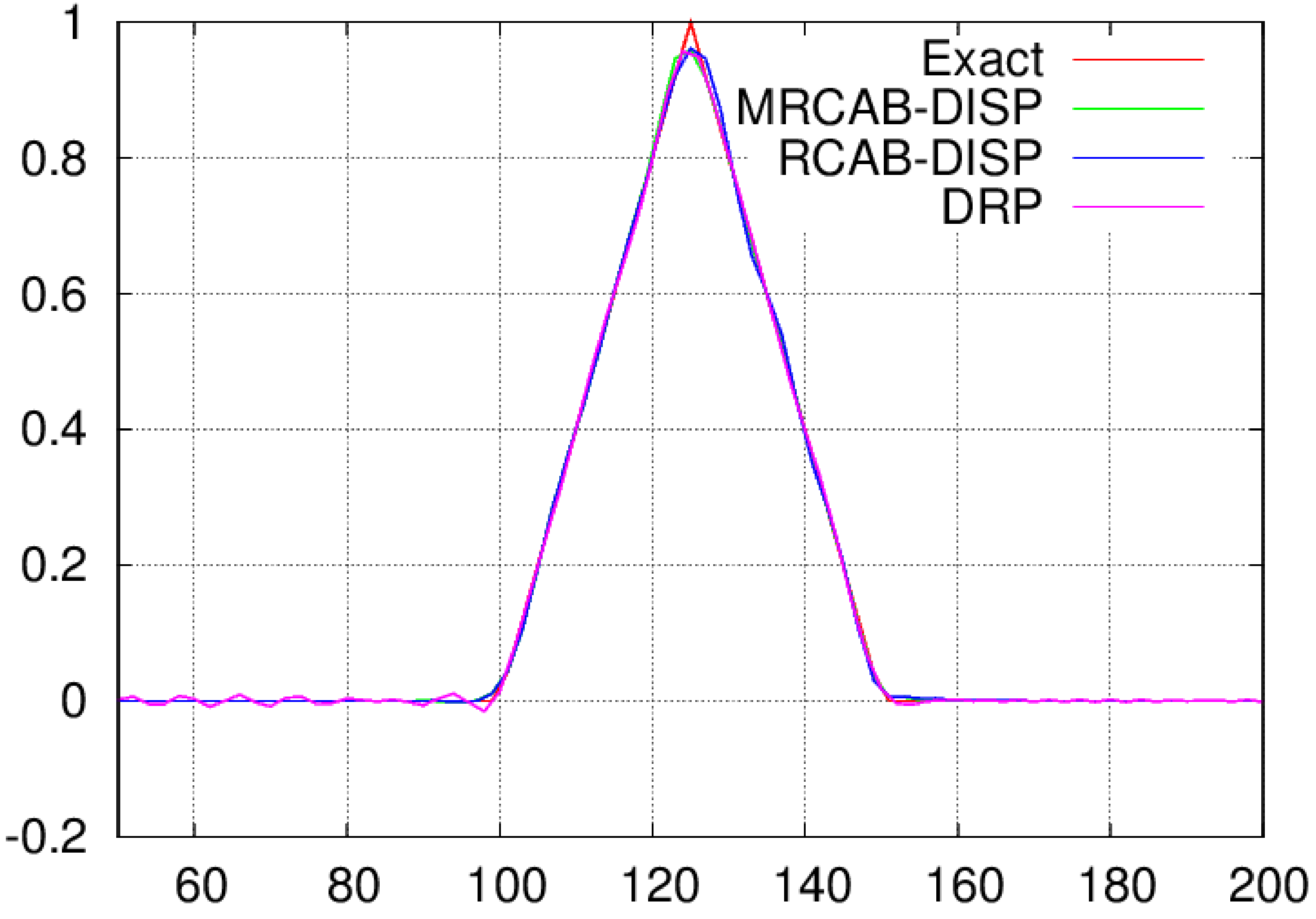
$$mnew = m - \varepsilon\delta_{rx}$$

Linear advection test#1: R-CAB,MR-CAB and DRP



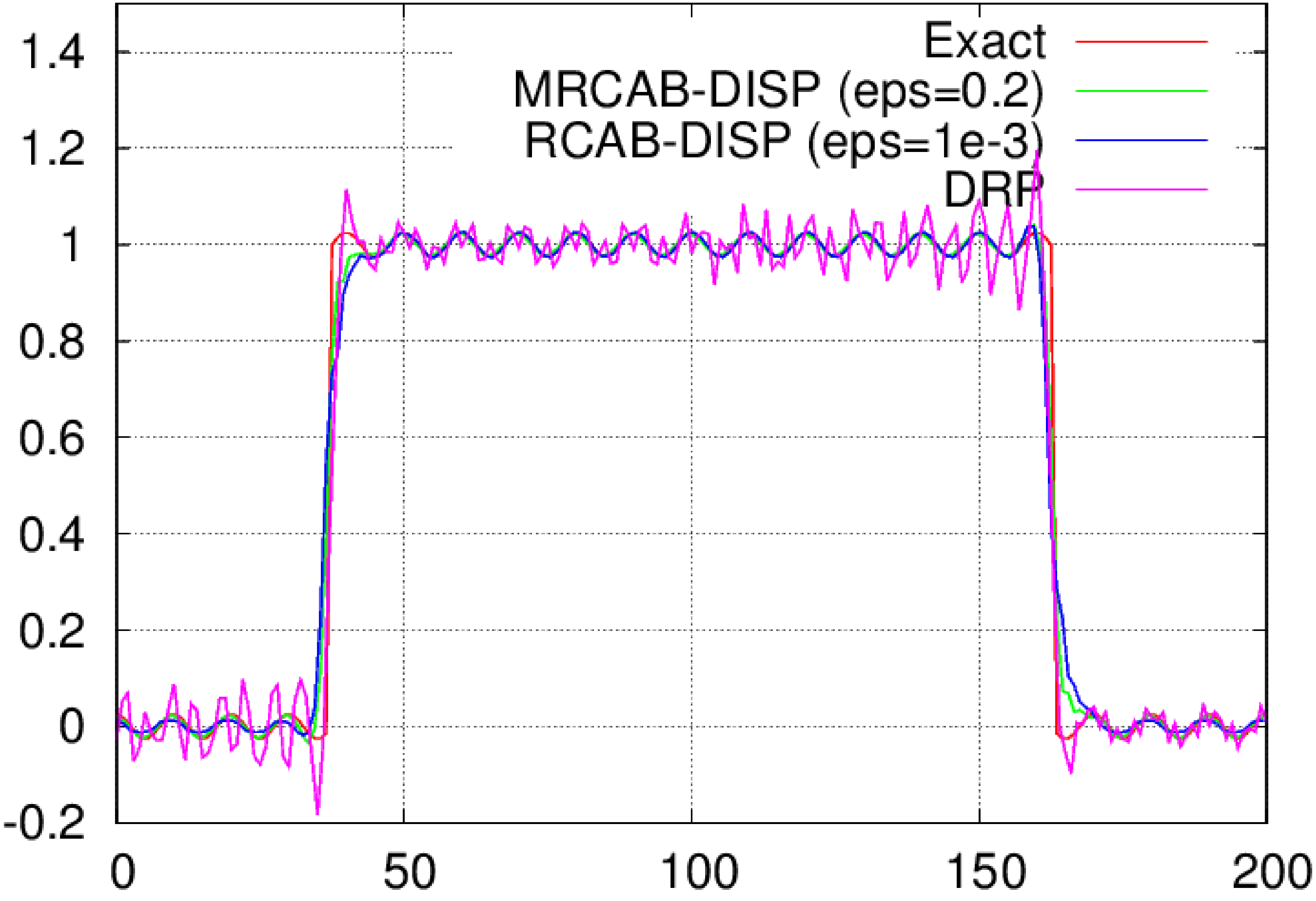
DRP is the winner for most grids

Linear advection test#2: R-CAB,MR-CAB and DRP

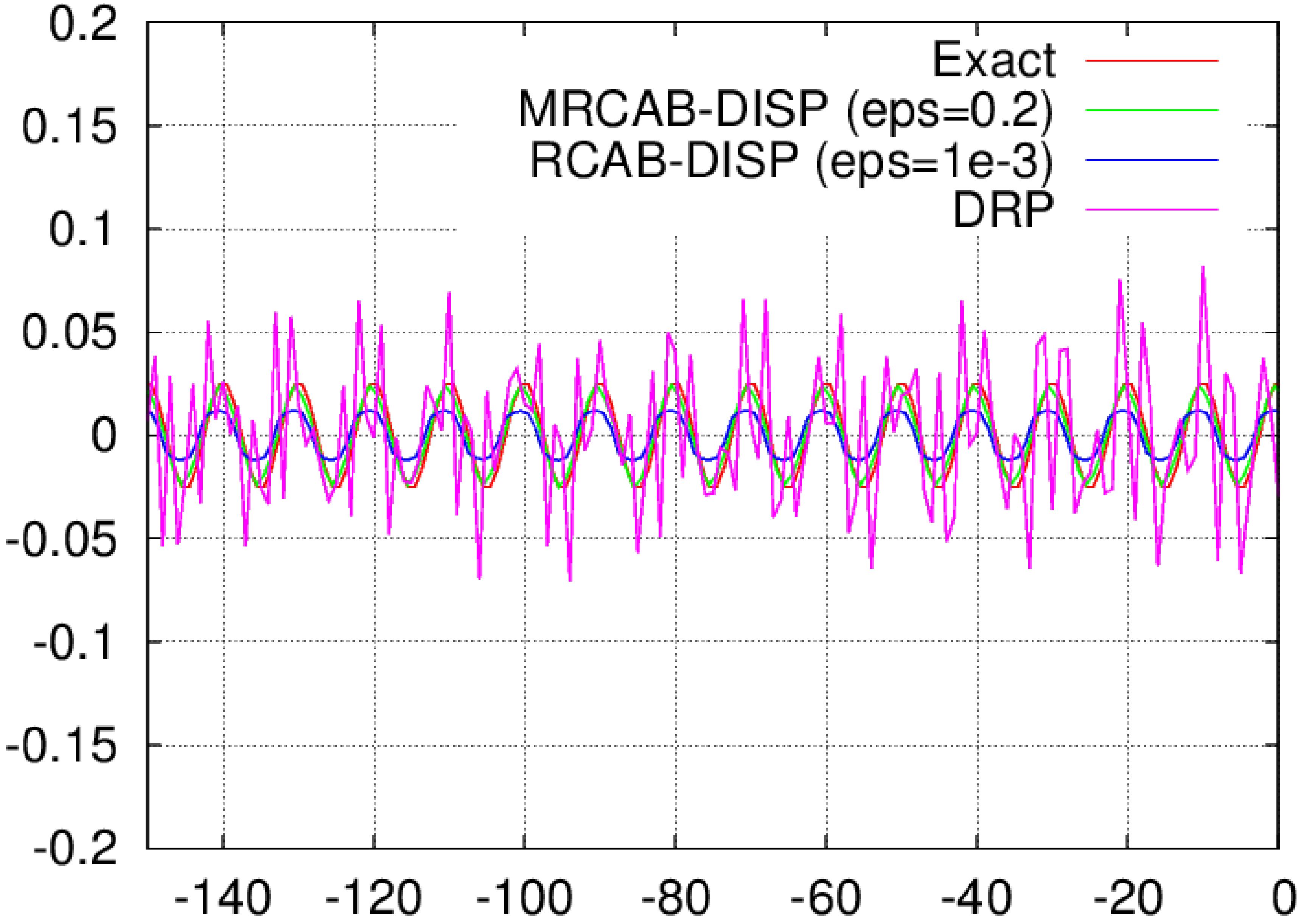


MRCAB-DISP ~ DRP for all grids

Linear advection test#3: R-CAB, MR-CAB and DRP schemes



Test case#3 – floor



MRCAB-DISP is the winner for all grids

Analytical test problem for CAA methods

Physics: acoustic wave propagation in a channel with periodic side walls and uniform flow at M=0.5;
Model: polytropic & isothermal inviscid gas flow equations in 1D and 2D

Linear wave equation

$$\frac{\partial^2}{\partial t^2}\rho' - c_0{}^2 \frac{\partial^2}{\partial x \partial x}\rho' - c_0{}^2 \frac{\partial^2}{\partial y \partial y}\rho' = 0$$

Linear wave equation with constant flow

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\rho' - c_0{}^2 \frac{\partial^2}{\partial x \partial x}\rho' - c_0{}^2 \frac{\partial^2}{\partial y \partial y}\rho' = 0$$

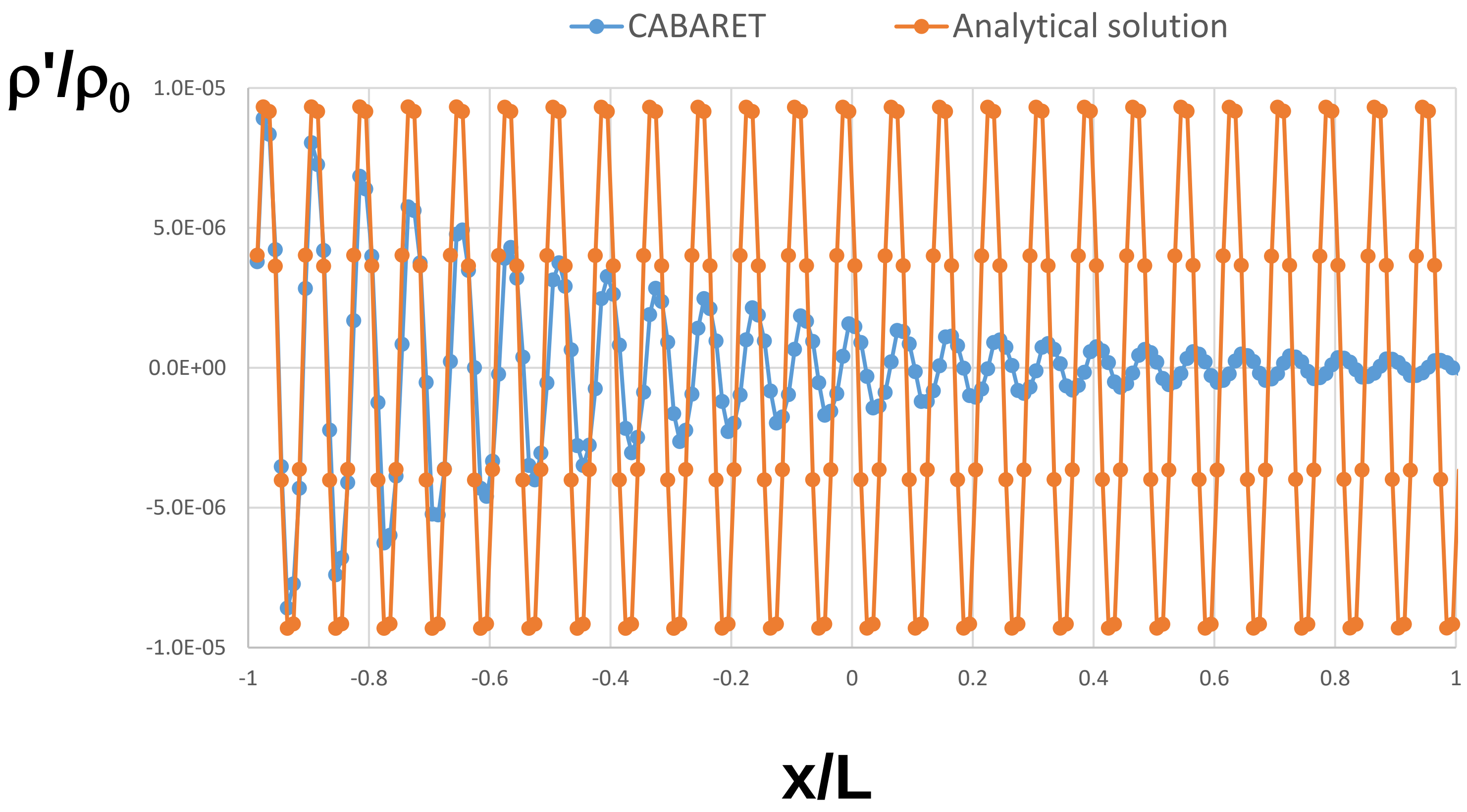
Travelling wave solution $\rho' \sim \rho'_k \, e^{i\left(\omega \cdot t - k_x \cdot x - k_y \cdot y\right)}$

Dispersion relations

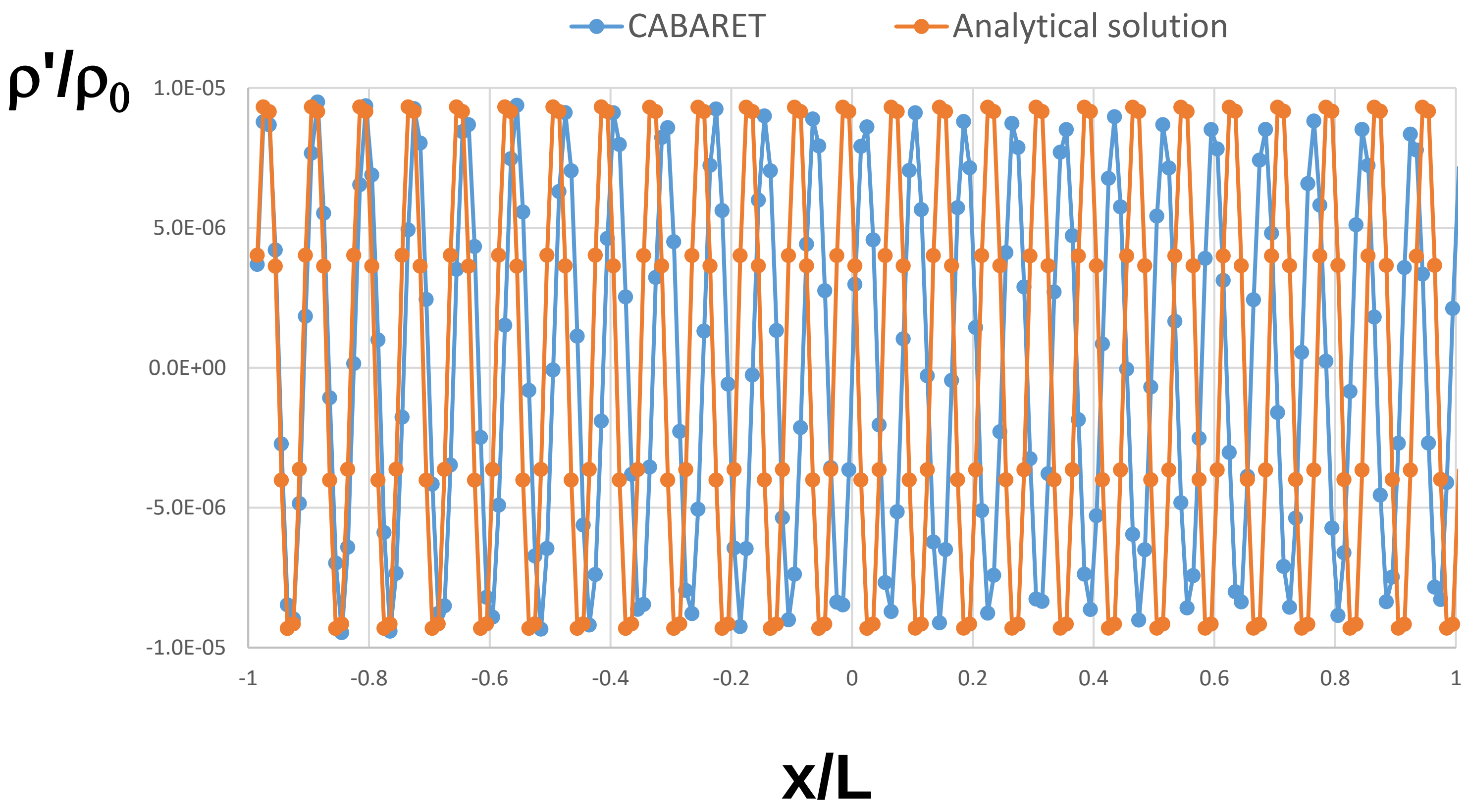
$$\omega^2 - c_0{}^2 k_x{}^2 - c_0{}^2 k_y{}^2 = 0$$

$$\omega^2 + 2k_x U + k_x{}^2 U^2 - c_0{}^2 k_x{}^2 - c_0{}^2 k_y{}^2 = 0$$

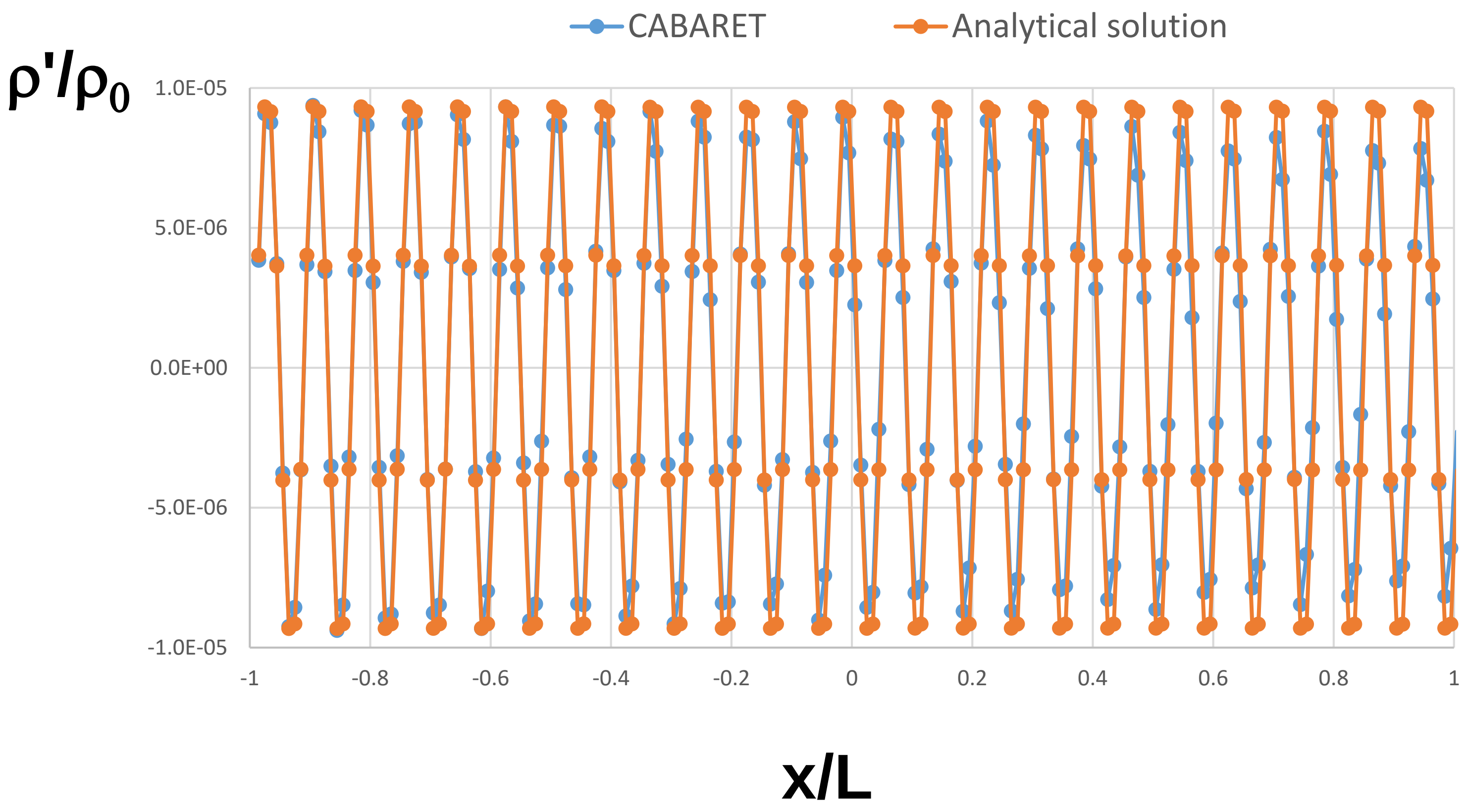
Planar wave (1D), 8ppw, CFL=0.1
Full flux correction, original CABARET



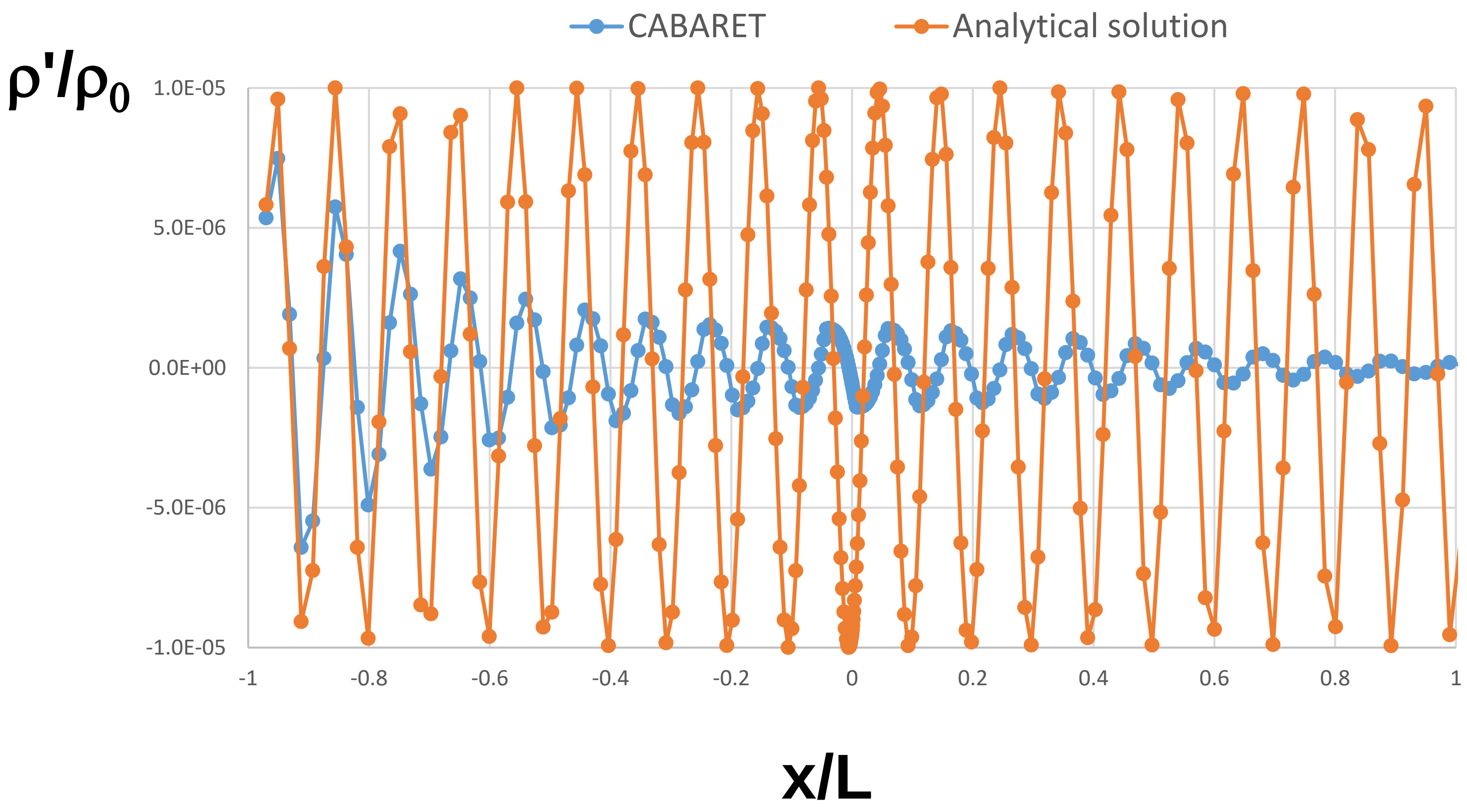
Planar wave (1D), 8ppw, CFL=0.1
New relaxed flux correction (ε=0.2), original CABARET



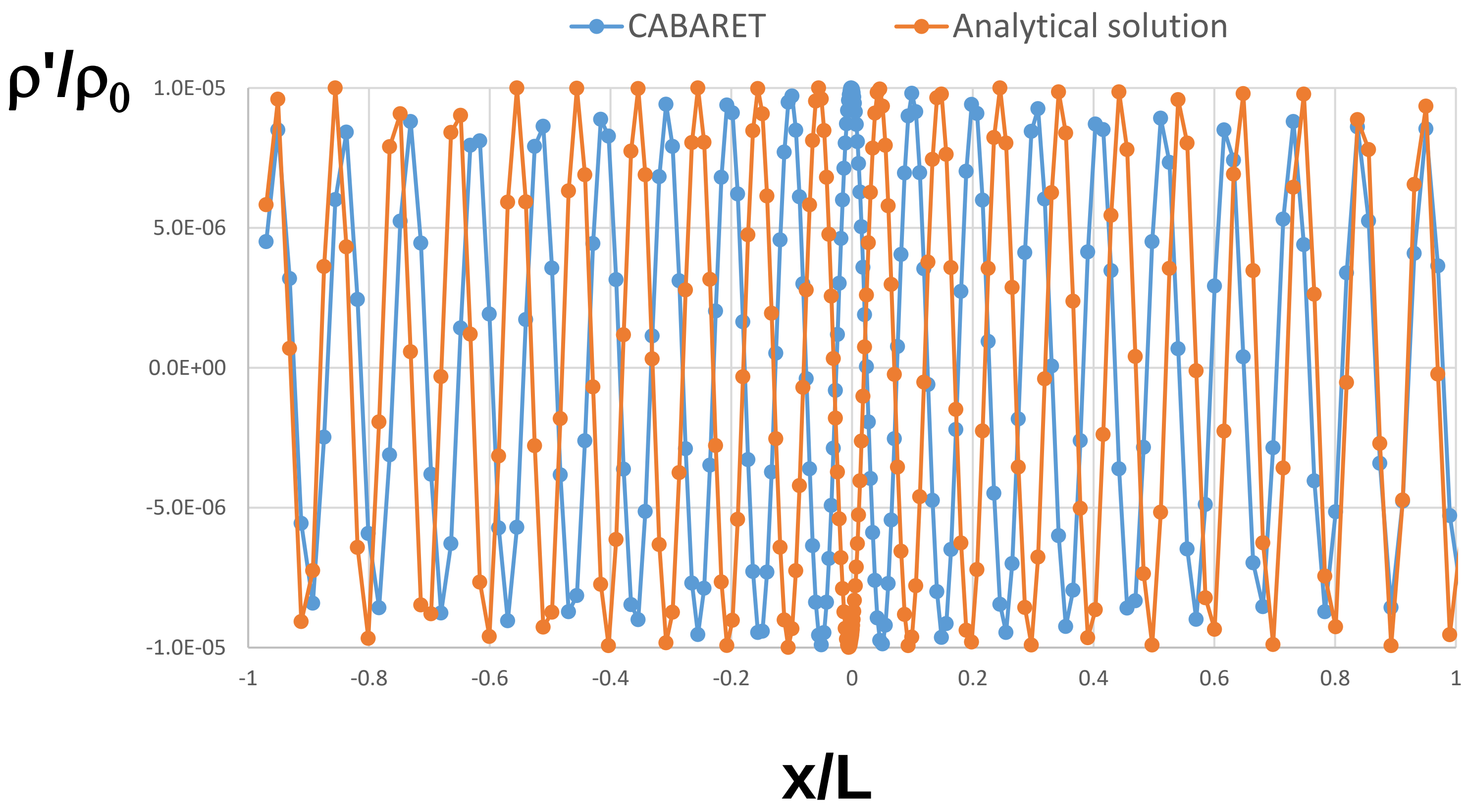
Planar wave (1D), 8ppw, CFL=0.1
New relaxed flux correction (ε=0.2), dispersion improved CABARET



Planar wave (1D), non-uniform grid, CFL_{max}=0.8
Full flux correction, original CABARET



Planar wave (1D), non-uniform grid, $CFL_{max}=0.8$
New relaxed flux correction ($\varepsilon=0.2$), original CABARET



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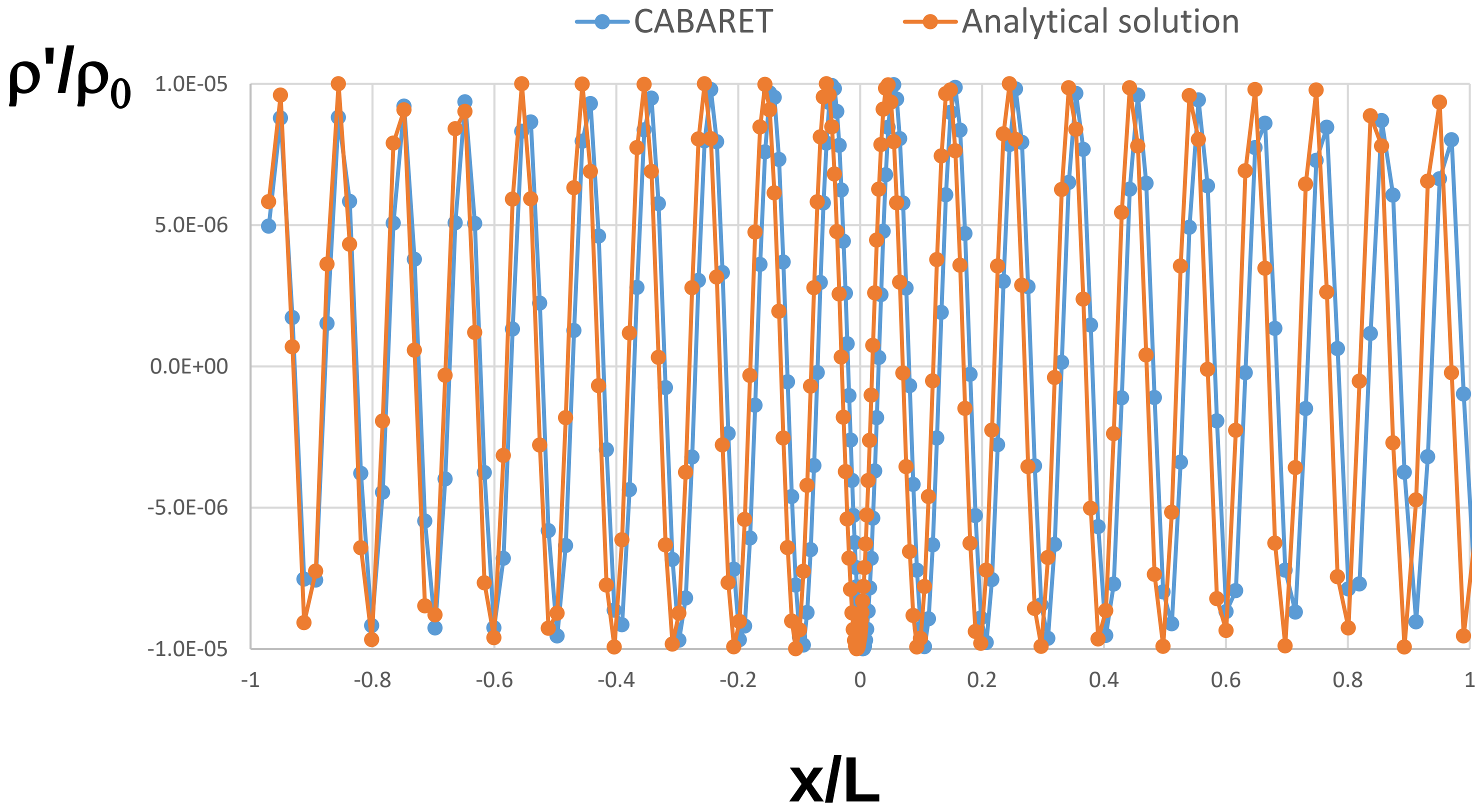
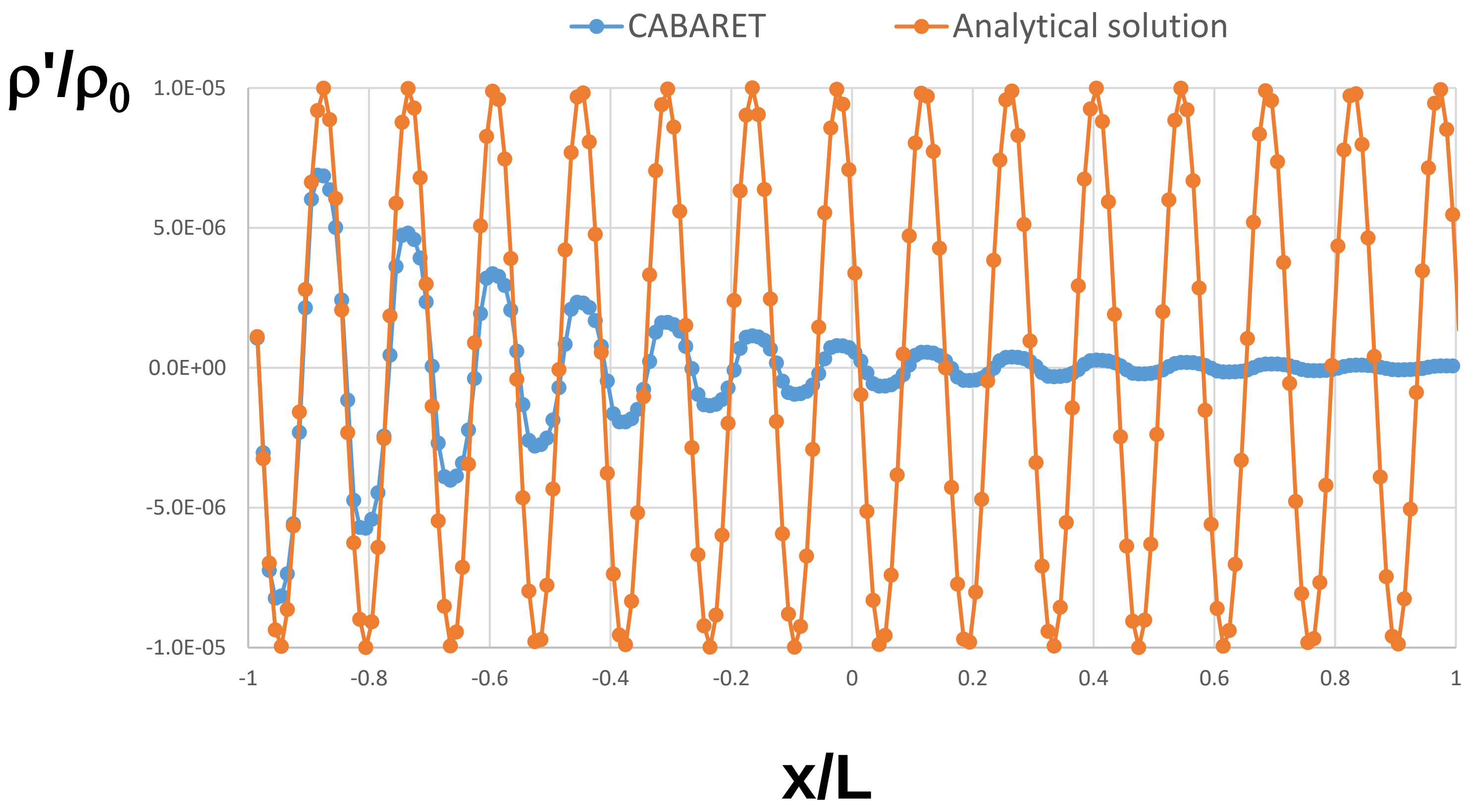


2D wave, CFL=0.1, $ppw_{x,y}$={15,9}, $atan(k_y/k_x)=58^0$ Full flux correction, original CABARET



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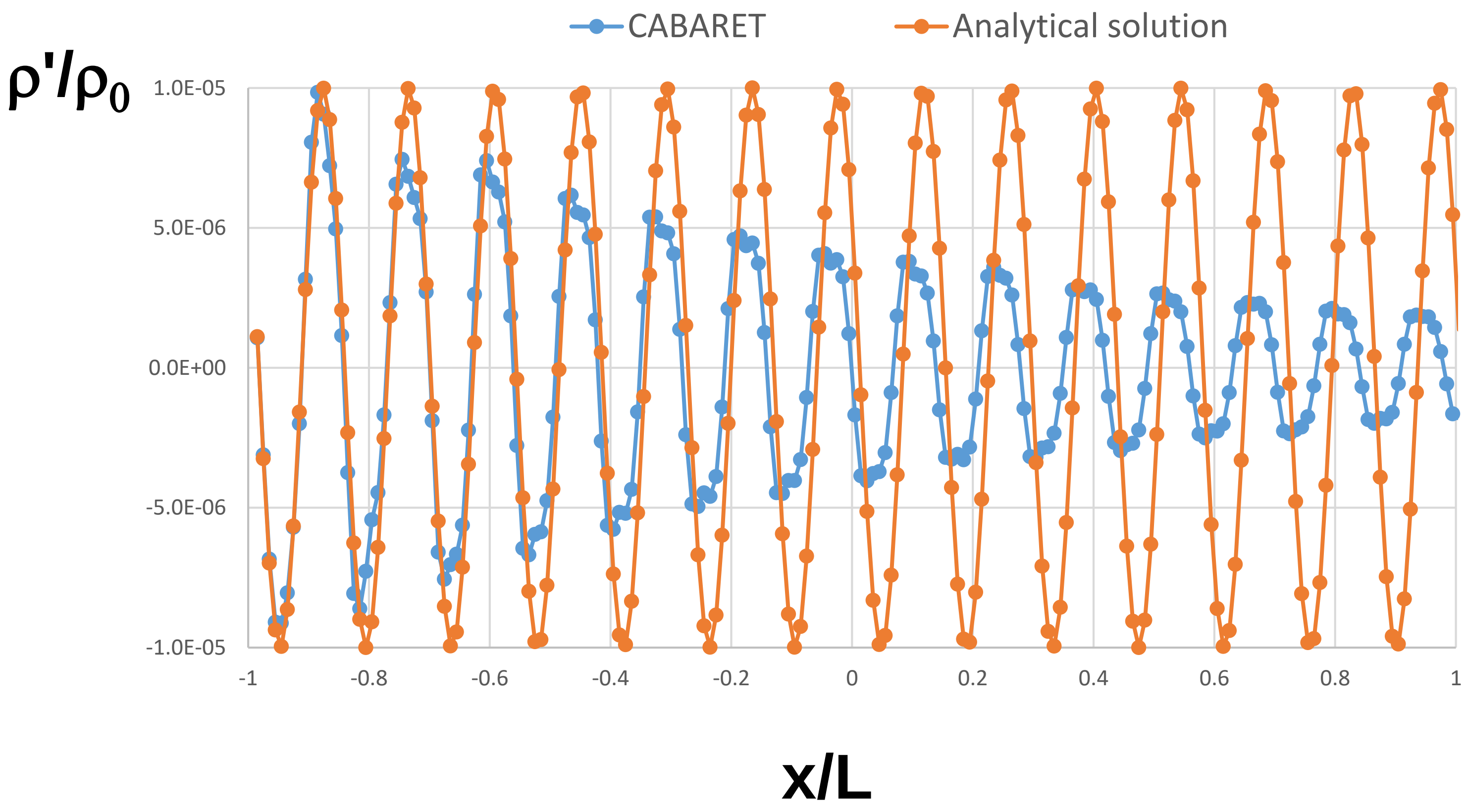
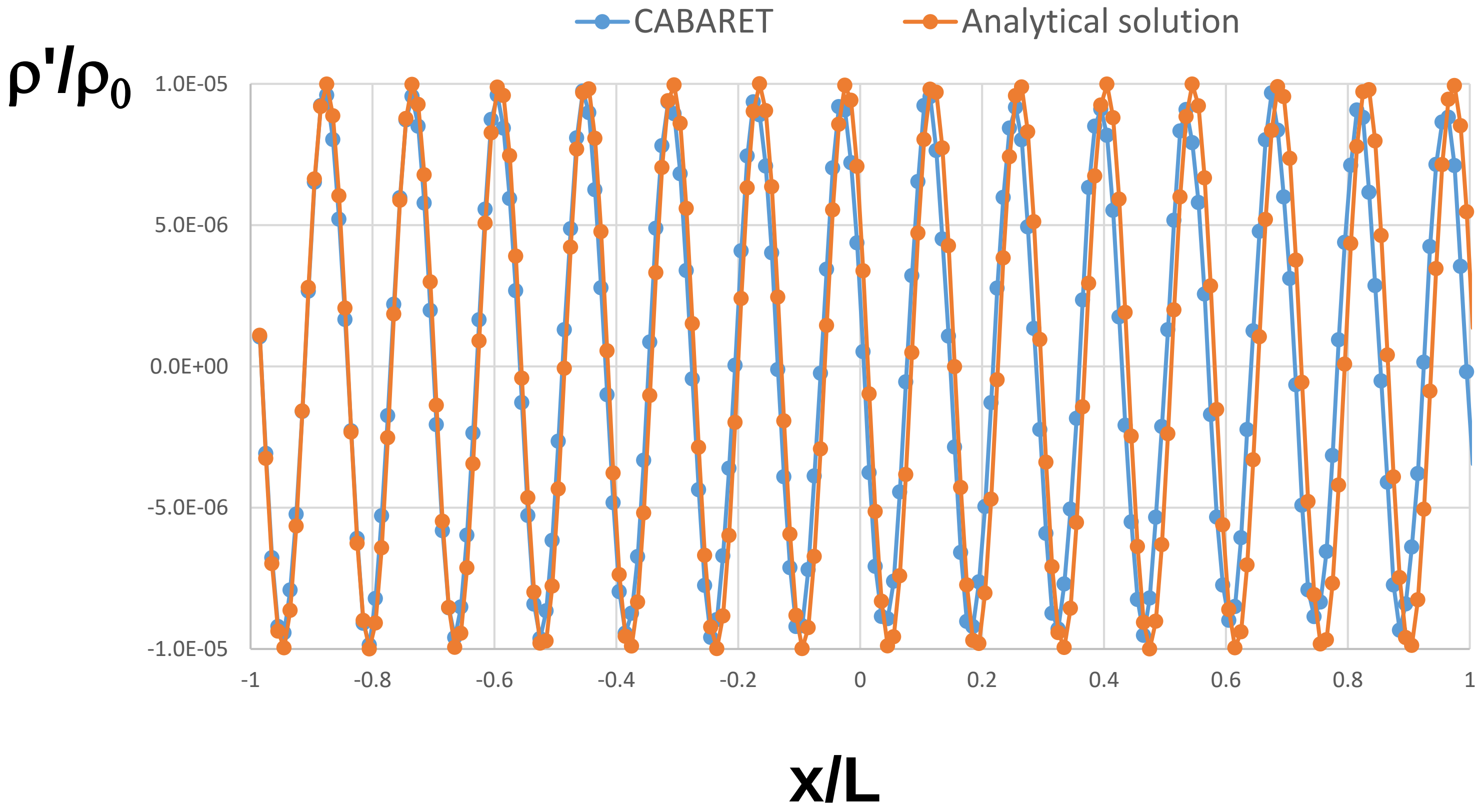
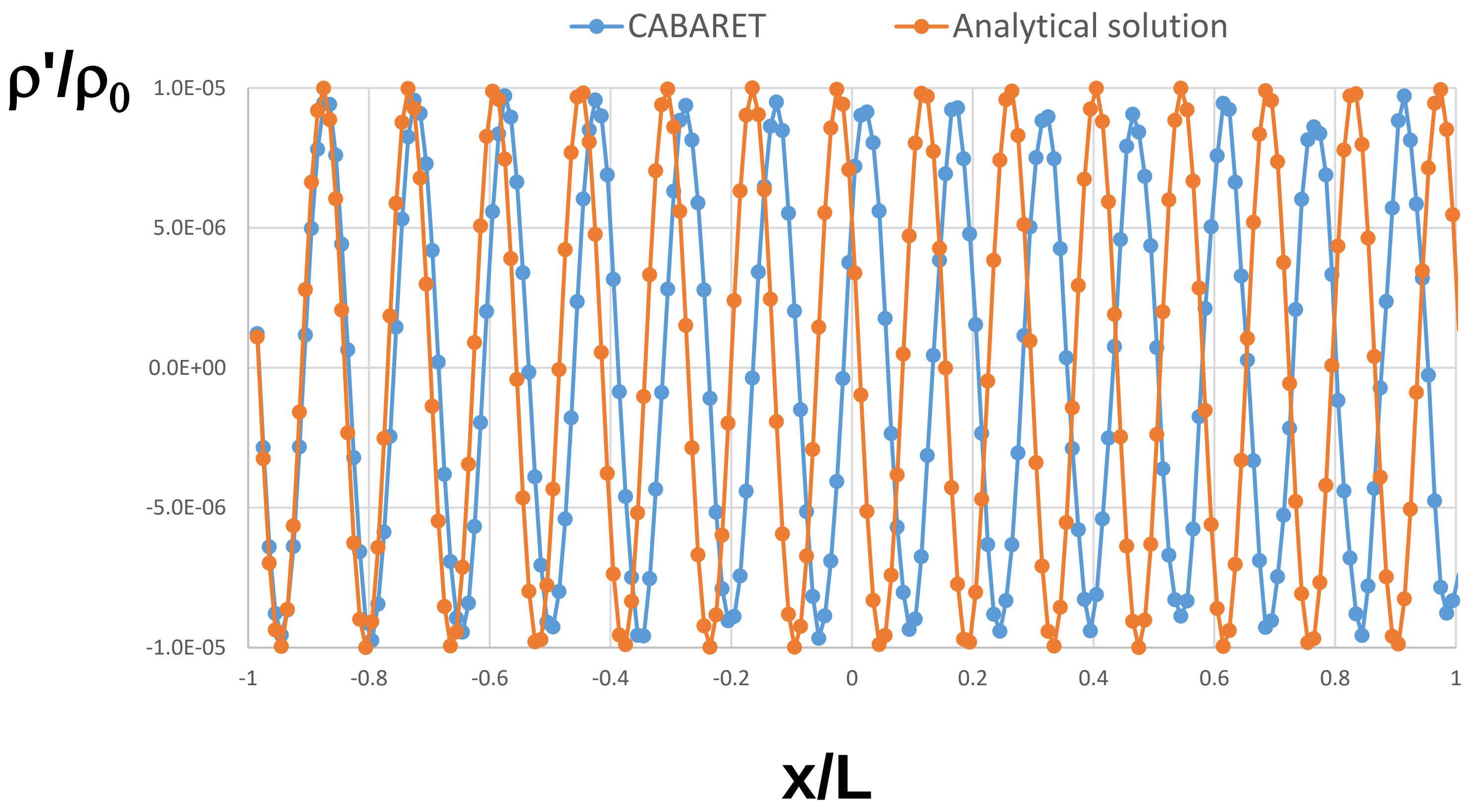


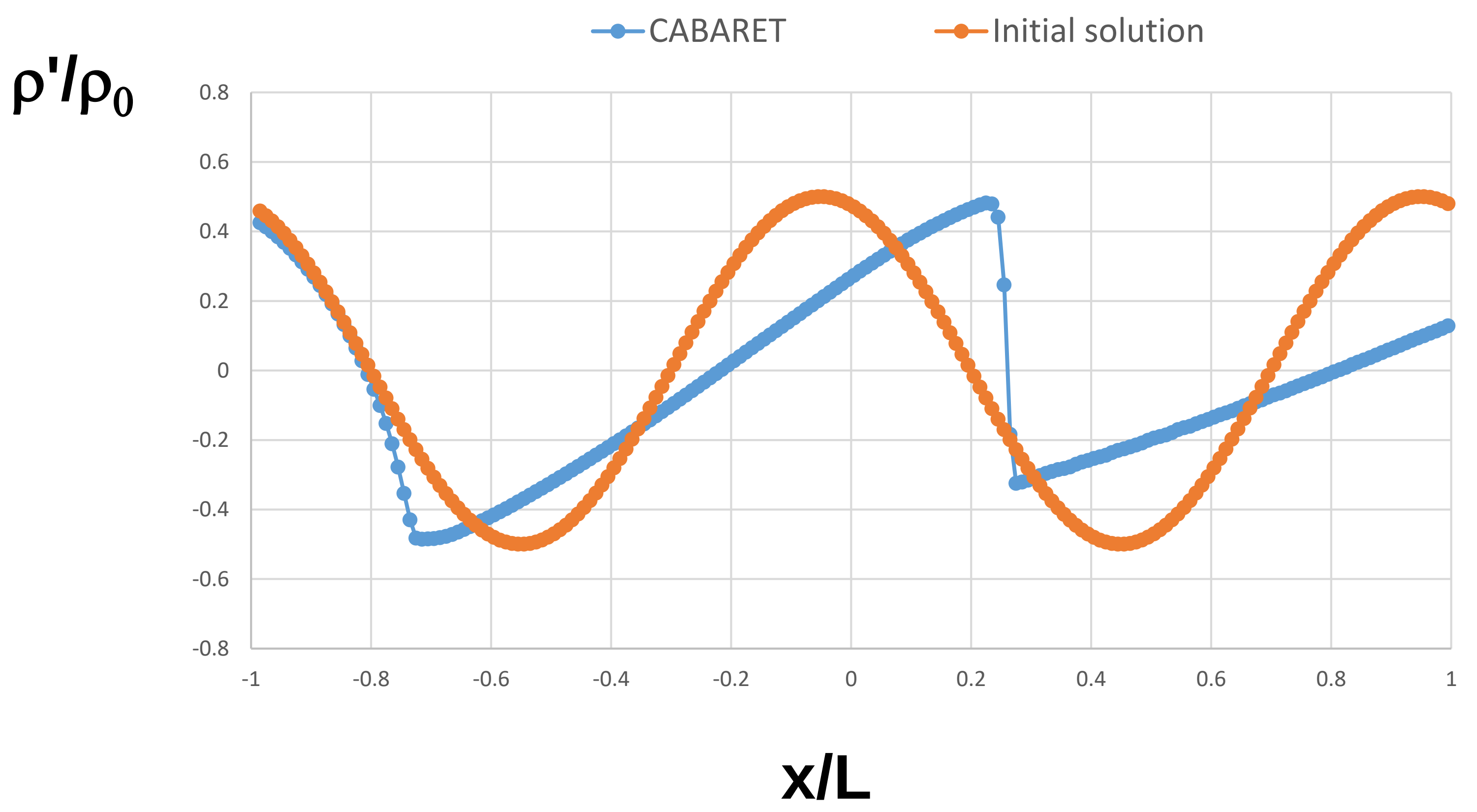
2D wave, CFL=0.1, $ppw_{x,y}$={15,9}, $atan(k_y/k_x)=58^0$ New relaxed flux correction (ε=0.4), dispersion improved CABARET



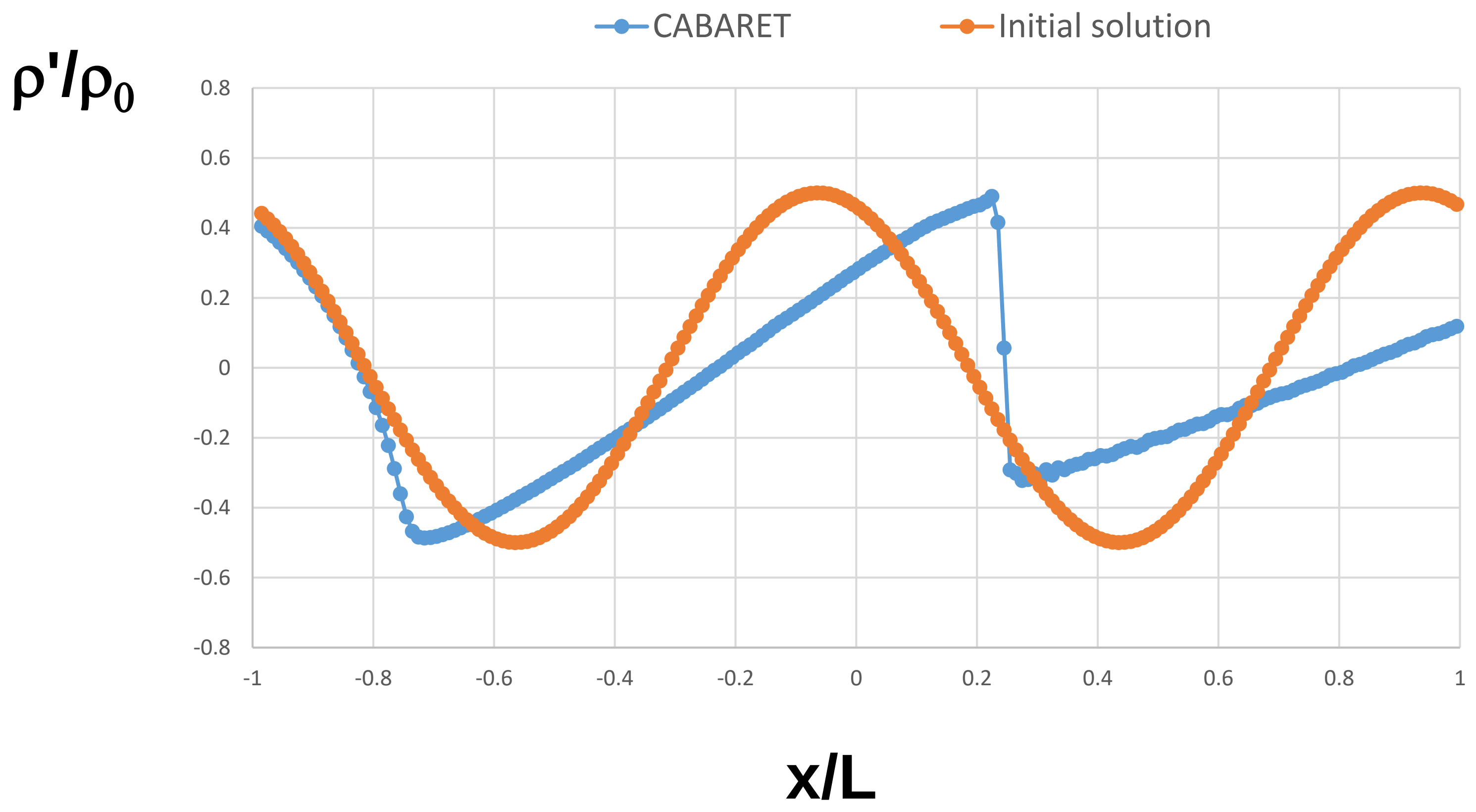
2D wave, CFL=0.1, $ppw_{x,y}$={15,9}, atan(k_y/k_x)=58^0 New relaxed flux correction (ε=0.4), standard CABARET



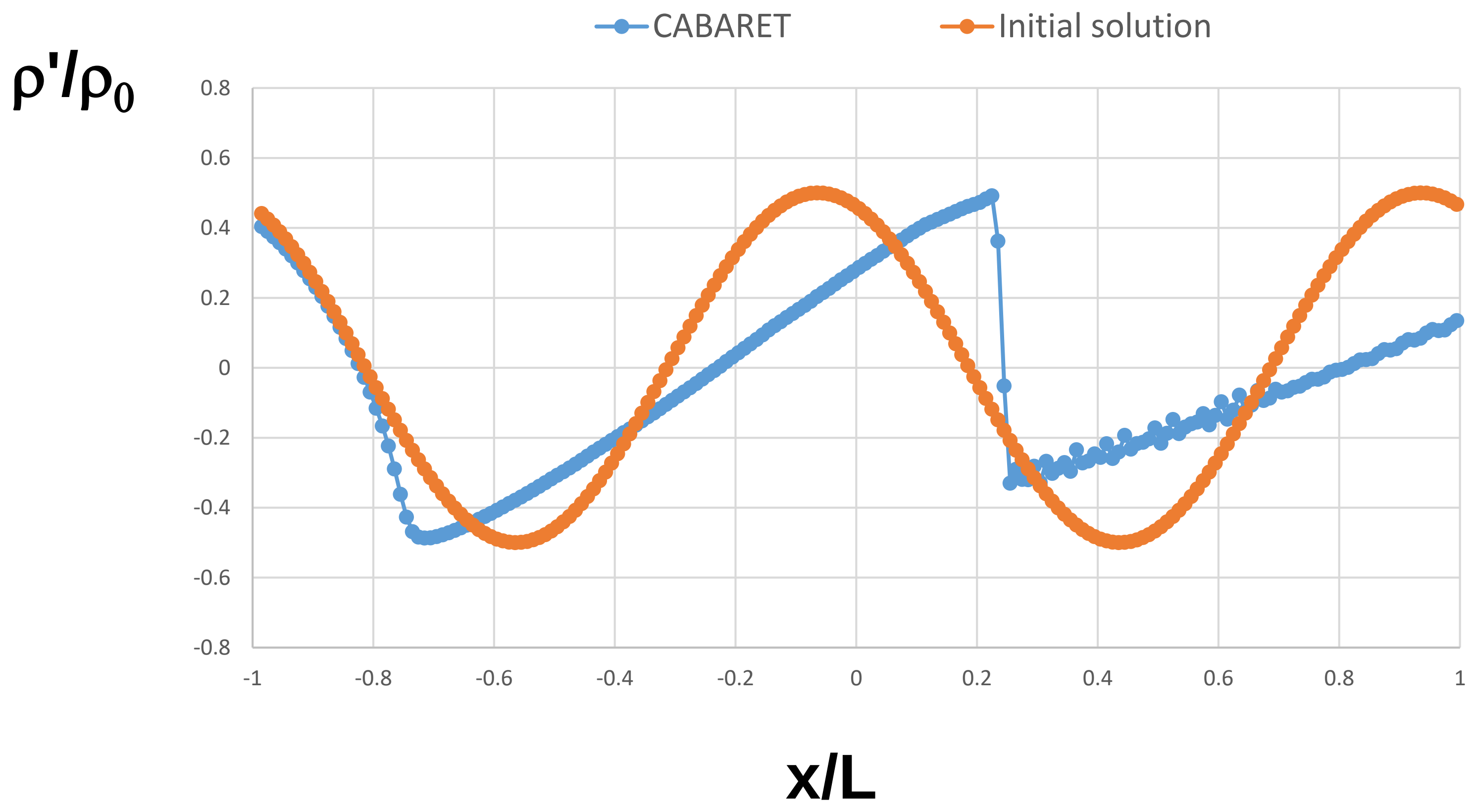
Nonlinear planar wave (1D), 100ppw, CFL=0.1
Full flux correction, original CABARET



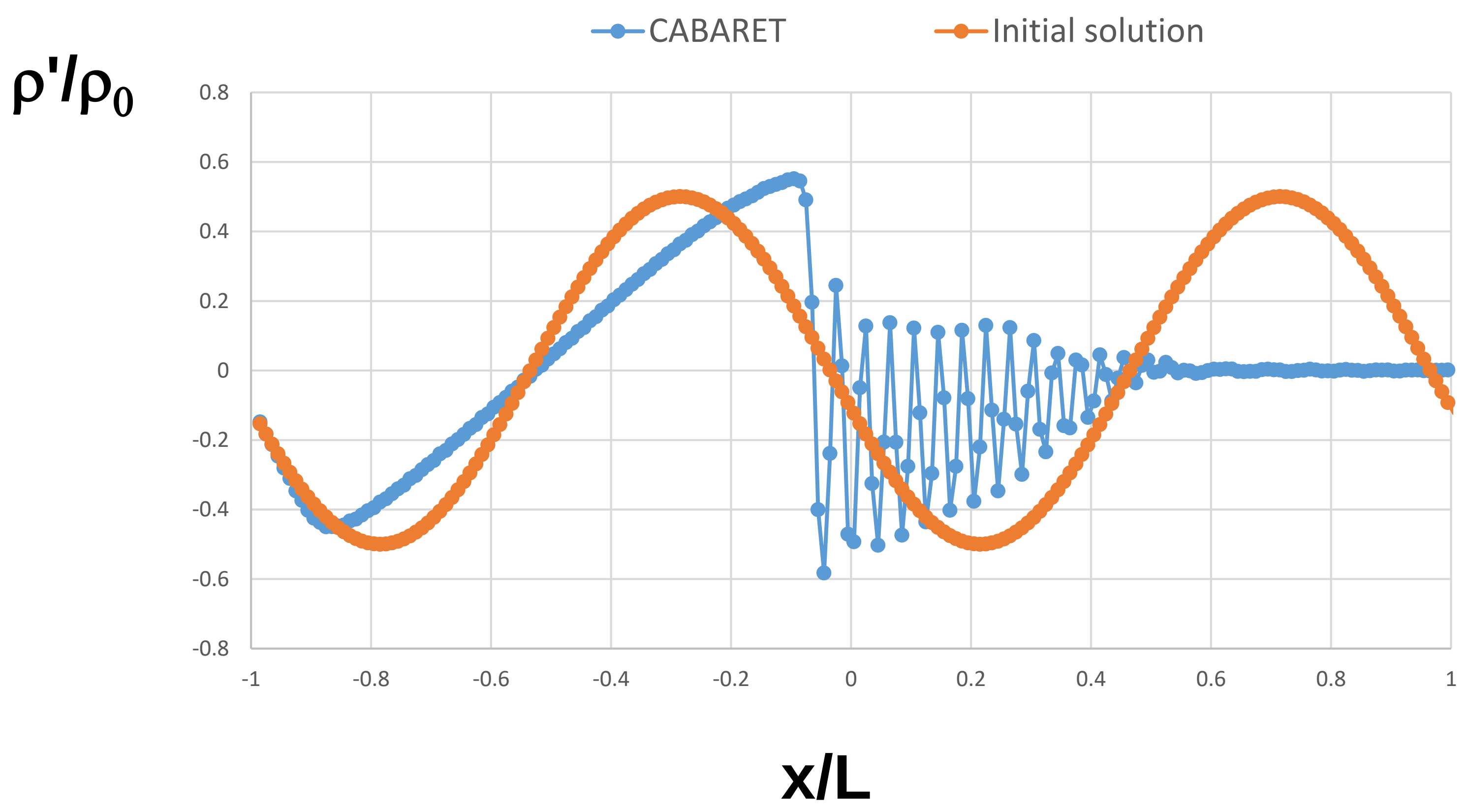
Nonlinear planar wave (1D), 100ppw, CFL=0.1 New relaxed flux correction (ε=0.2), dispersion improved CABARET



Nonlinear planar wave (1D), 100ppw, CFL=0.1 New relaxed flux correction (ε=0.4), dispersion improved CABARET



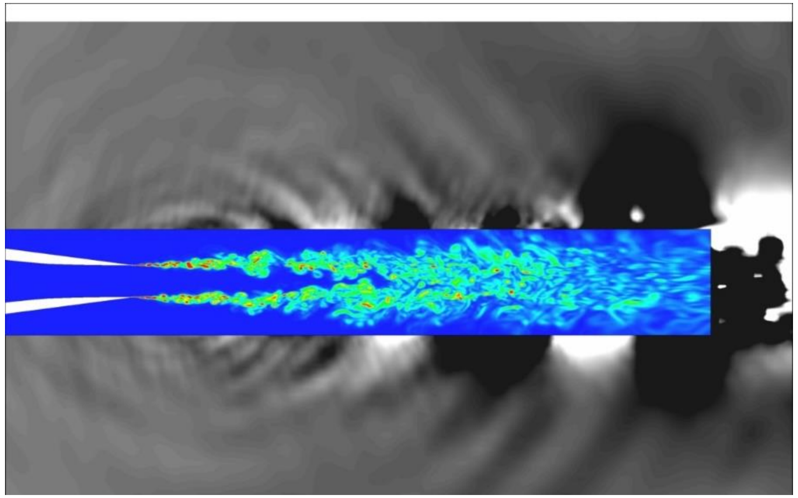
Nonlinear planar wave (1D), 100ppw, CFL=0.1
No flux correction, ‘pure’ CABARET



Conclusion

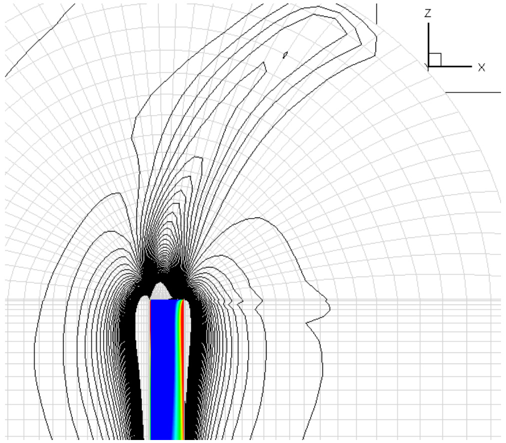
- Dispersion improved CABARET of Goloviznin and Samarskii (1998) is extended to the compact two-time-level predictor-corrector form
- The compact formulation allows further generalisations to include non-linear flux correction (including a new relaxed correction method) and extensions to multiple dimensions with gas dynamics.
- Numerical solutions for 1d linear advection, 1d and 2d linear acoustic propagation, and 1d non-linear z-wave propagation are considered
- A considerable increase in accuracy over the standard CABARET (not to mention DRP;) is reported. In combination with the new relaxed correction algorithm within a small range of calibration parameter ε=0.2-0.4, superior results are reported for both linear and nonlinear problems.
- Further work will be towards implementing the improved CABARET in the current 3D GPU CABARET NS solver

Aeroacoustics of engineering flows: challenges



Faranosov et al. 2013

Vorticity and acoustic pressure field of a turbulent jet



Morgans et a. 2005

Acoustic pressure contours of a high-speed helicopter blade

Aerodynamic scales << Acoustic scales

δ b.layer/D~ 0.01-0.001

L/D~100-1000

Acoustic fluctuations are so tiny that they are typically measured at log scale ($dB=10\log_{10}(r.m.s.(p')^2/p_{ref}^2)$) -- the same as the human hearing works!

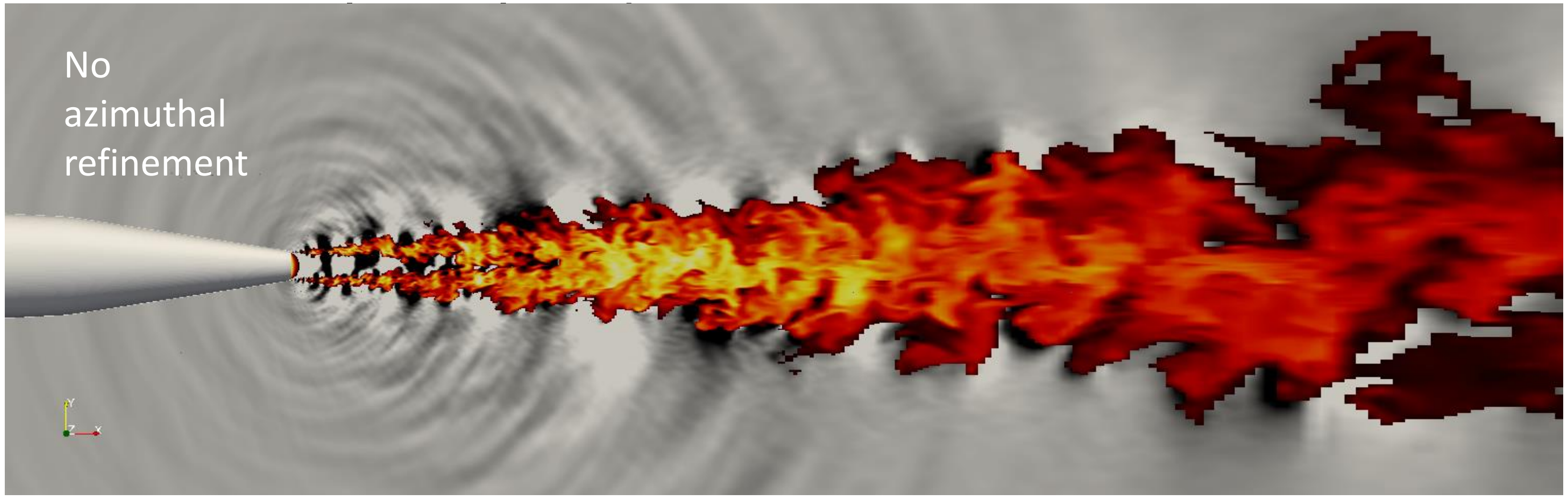
GPU CABARET: CPU VS GPU – 1 Billion challenge

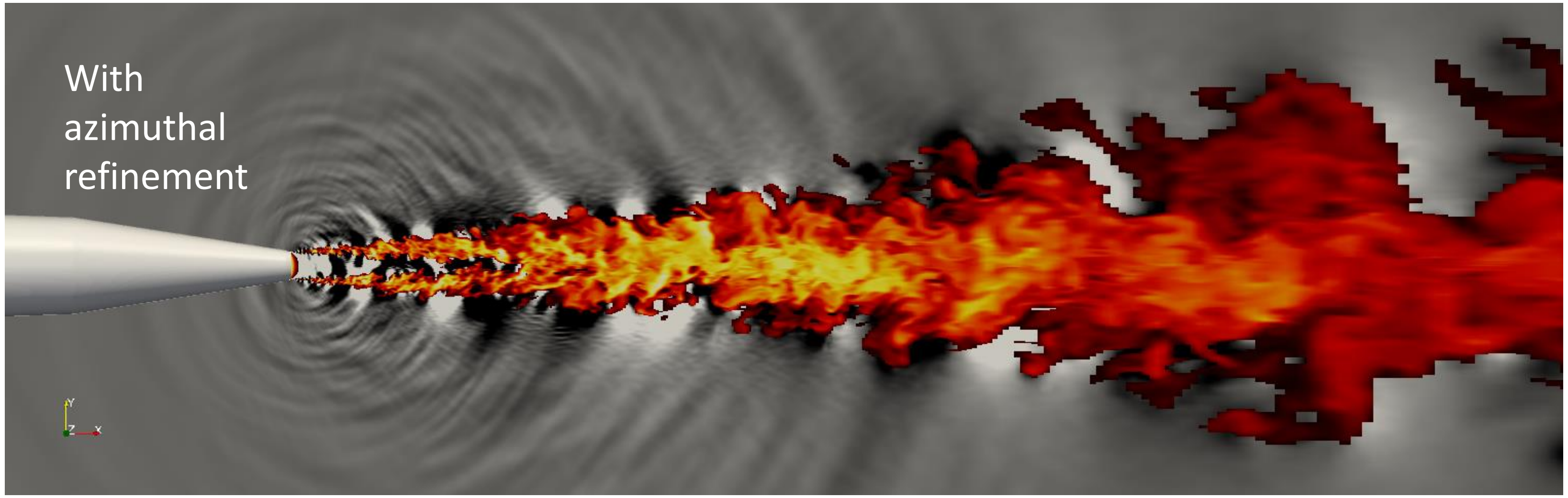
- CPU only:
  - Simple up-scaling: ~50000 cores, ~0.6 MW, many "core hours"
  - Heavily resource bounded (power requirements, communication infrastructure, only possible on a "national" level, multi-million Dollar/Pound/Euro investment, or as part of a large research grant

- GPU only:
  - Likely memory bounded, GPUs have limited on-board memory
  - Current GPU-CABARET code can handle ~2 million cells per GB
  - Possible GPUs: AMD W9100 32GB / NVIDIA K80 24GB
    - Both sell for "only" $5,000 and we need 1e9/2e6=500 GB
  - GPUs needed: ~16 W9100 or ~20 K80s == $80,000-$100,000
  - This is a similar amount of money spent in a medium-sized company on software licenses only!

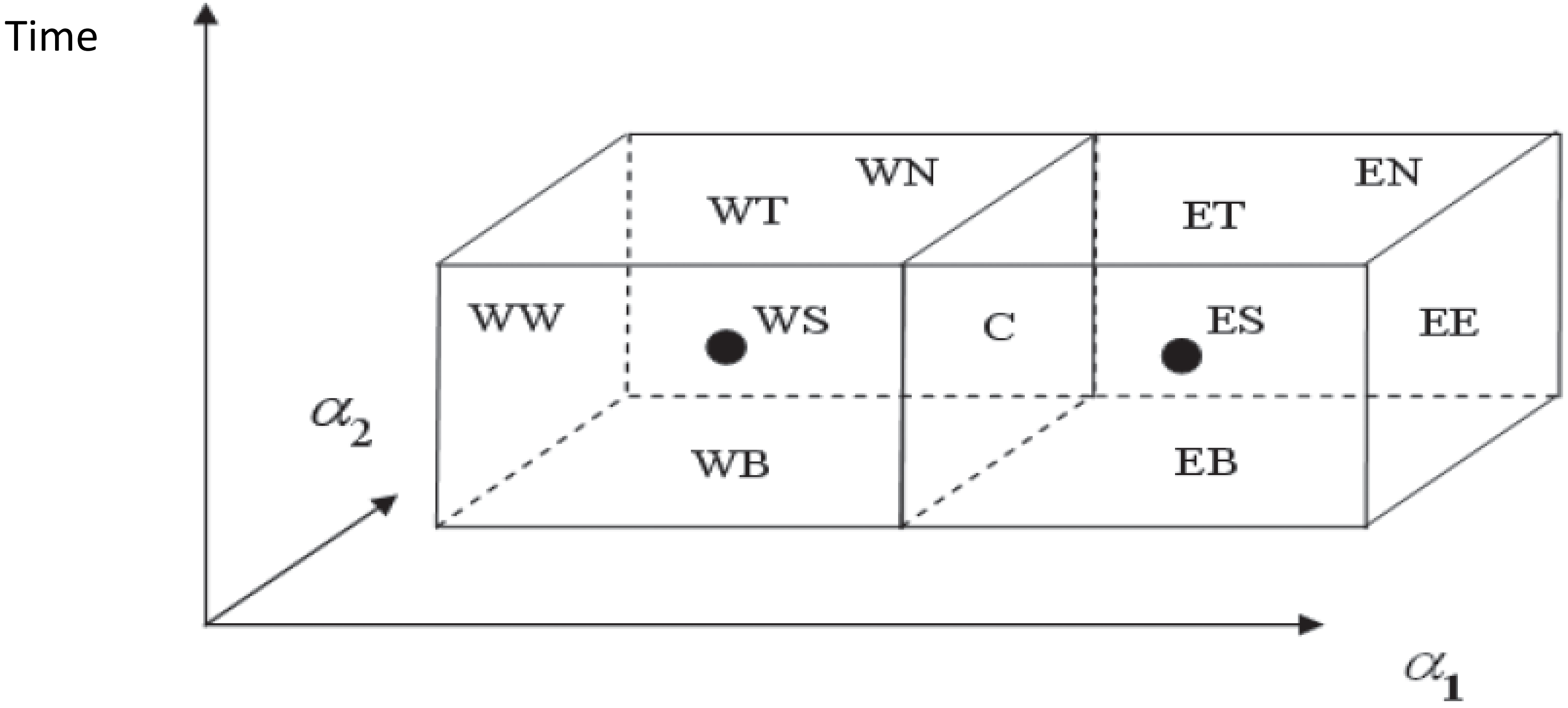
Isothermal Static SILOET Jet

- Two $20x10^6$ cells grids: effect of the azimuthal resolution



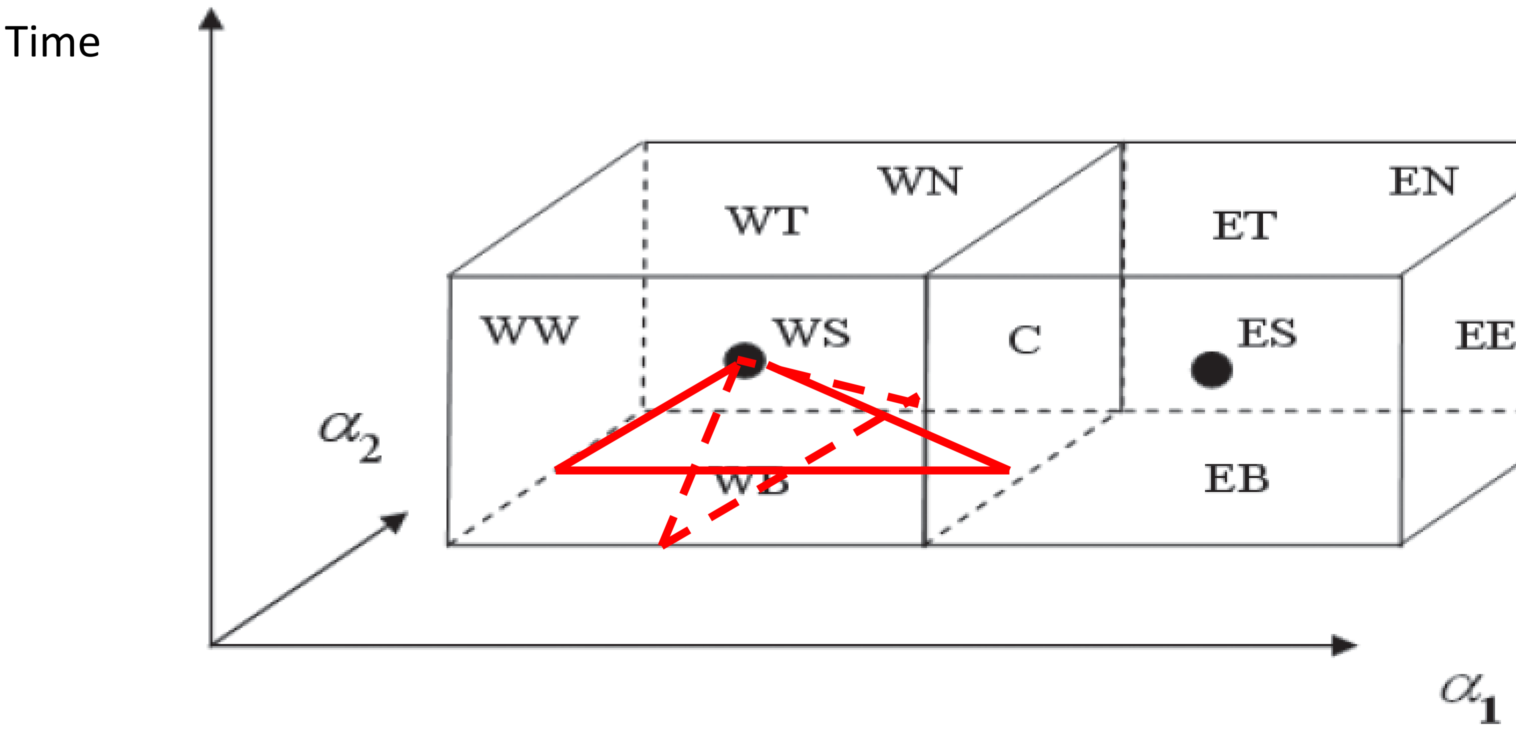


2D CABARET space stencil in grid coordinates

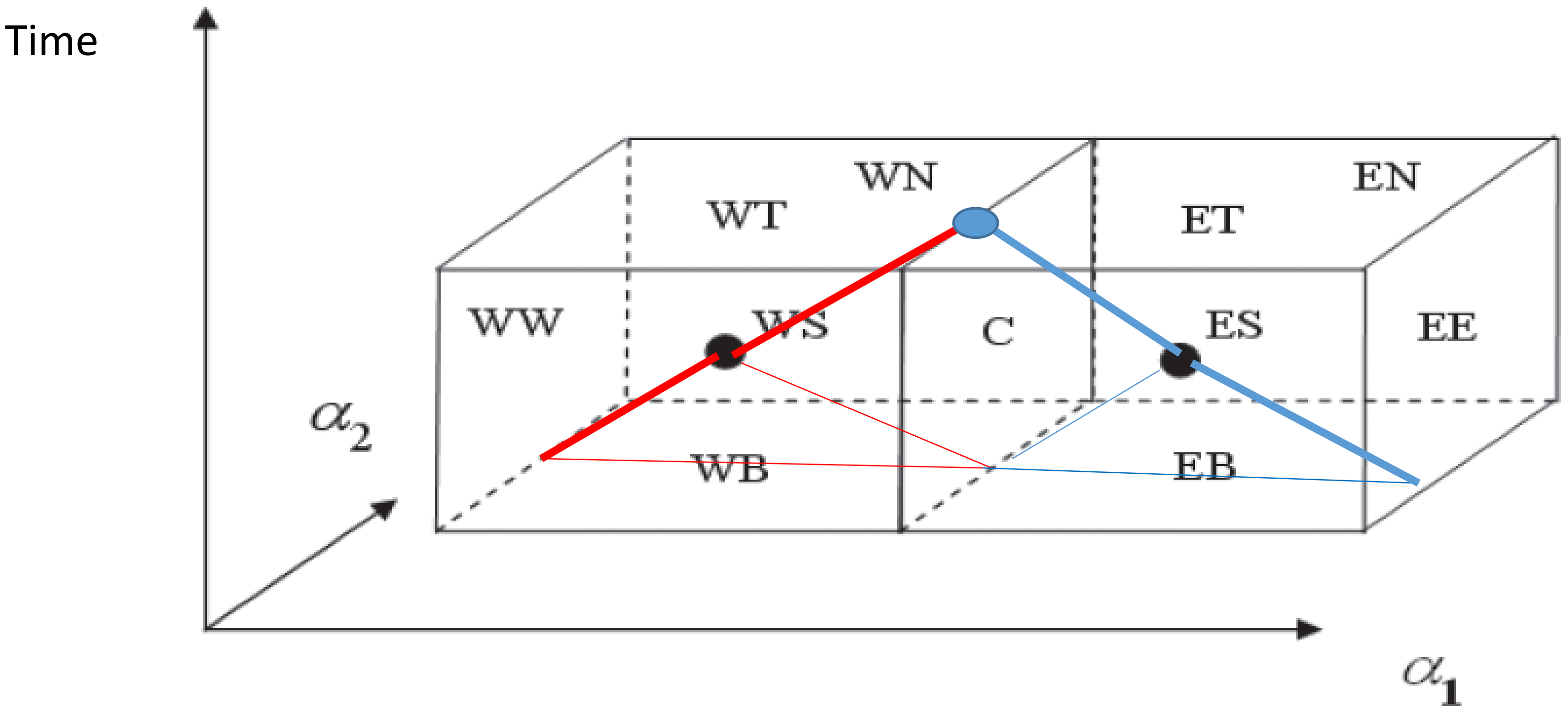


2D CABARET: predictor step

cell centre + cell face variables -> mid time level variables

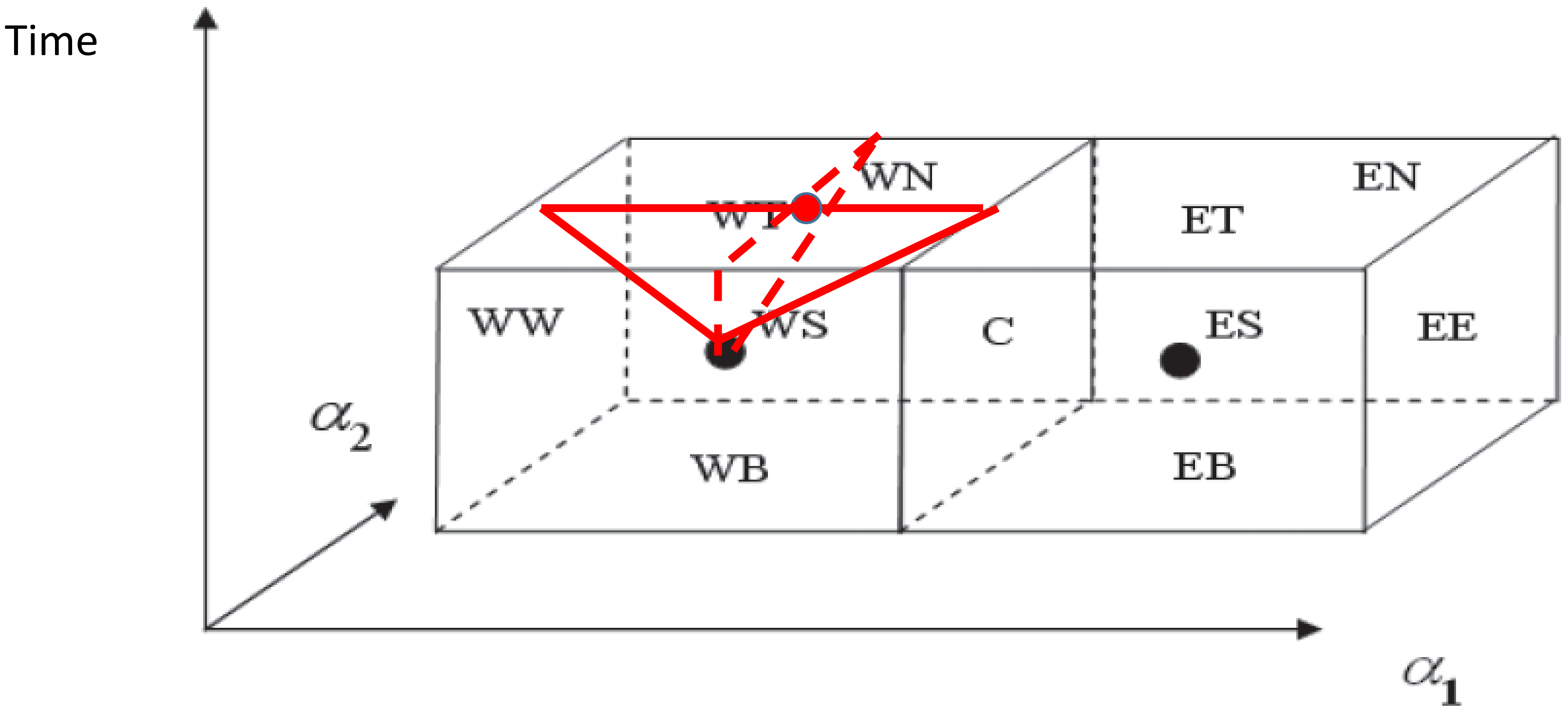


CABARET characteristic decomposition step, extrapolation and solution of the Riemann problem at the cell face in each grid direction



2D CABARET: corrector step

cell centre + cell face variables -> new time level variables



Flux Correction directly based on the maximum principle

$$\tilde{\varphi}_{i+1}^{n+1} = 2 \cdot \psi_{i+1/2}^{n+1/2} - \varphi_i^n$$

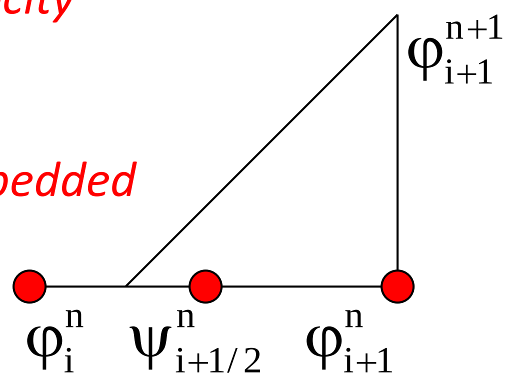
Maximum principle

$$\varphi_{i+1}^{n+1} \le \max(\varphi), \quad x \in [x_i, x_{i+1}], t = t_n$$

$$\varphi_{i+1}^{n+1} \ge \min(\varphi), \quad x \in [x_i, x_{i+1}], t = t_n$$

MILES approach: the capability of capturing sharp velocity gradients for smallest resolved scales emulates high-wavenumber end of the inertial subrange region characterised by thin filaments of intense vorticity embedded in a weak background vorticity

Fureby et al, Phys. Fluids 1997

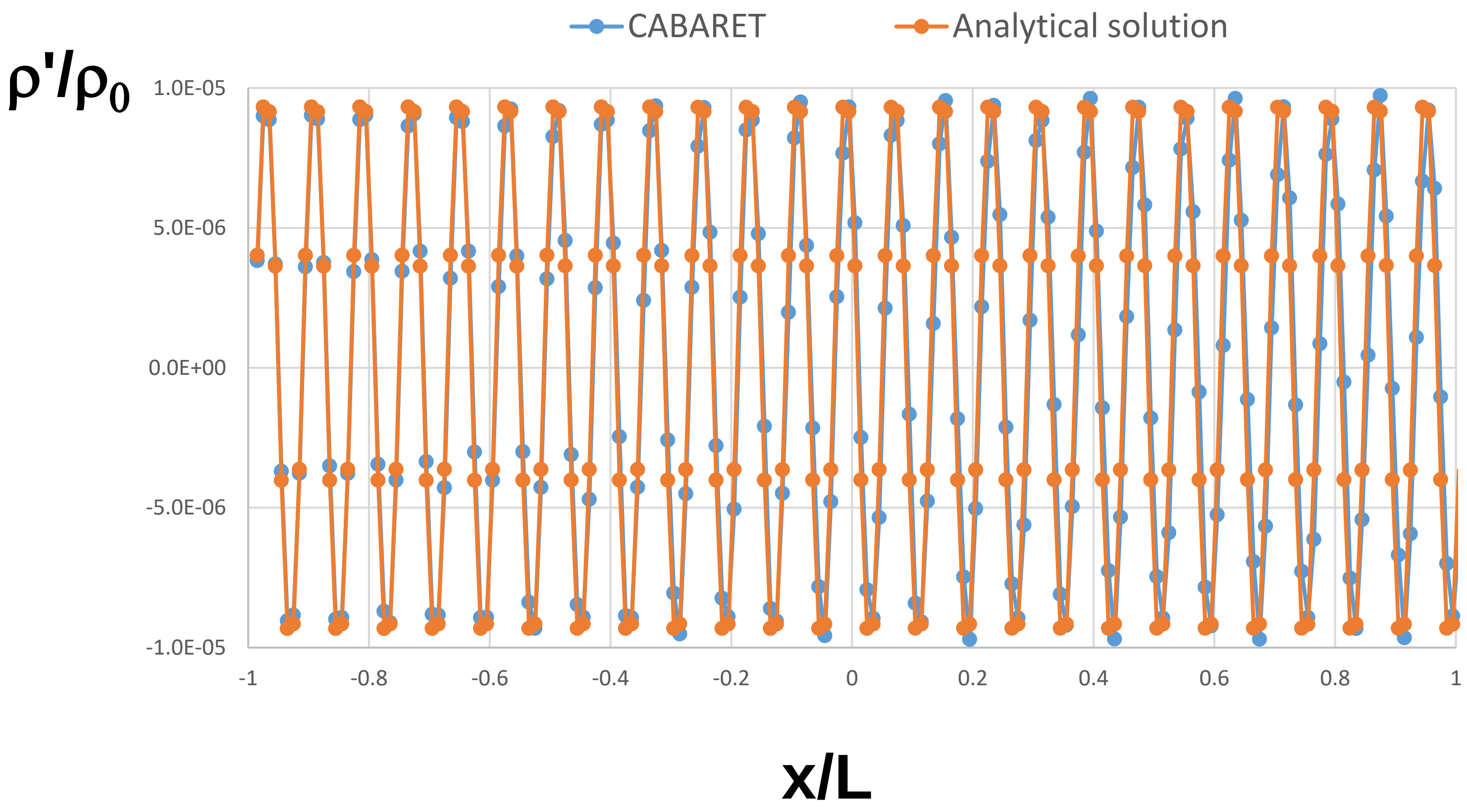


We constrain the solution so that

$$\varphi_{i+1}^{n+1} \le \max_{x \in [x_i, x_{i+1}], t=t_n} (\varphi) = \left[\max(\varphi)\right]_{i+1/2}^n = \max\left(\varphi_{i+1}^n, \psi_{i+1/2}^n, \varphi_i^n\right)$$

$$\varphi_{i+1}^{n+1} \ge \min_{x \in [x_i, x_{i+1}], t=t_n} (\varphi) = \left[\min(\varphi)\right]_{i+1/2}^n = \min\left(\varphi_{i+1}^n, \psi_{i+1/2}^n, \varphi_i^n\right)$$

Planar wave (1D), 8ppw, CFL=0.1
New relaxed flux correction (ε=0.4), dispersion improved CABARET



Planar wave (1D), non-uniform grid, CFL_{max}=0.8 New relaxed flux correction (ε=0.4), dispersion improved CABARET