*Continuously Variable Fidelity Adaptive Large Eddy Simulation**

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MULTI-SCALE MODELING & SIMULATION LABORATORY

Motivation

Where is aeroacoustics?

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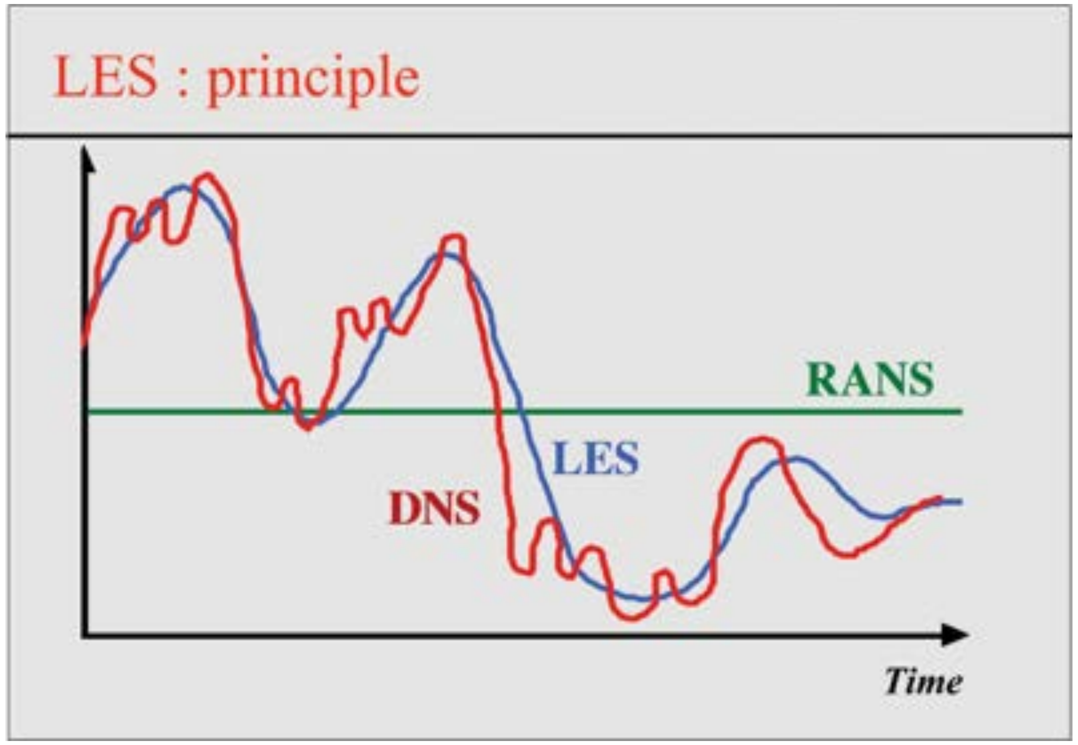
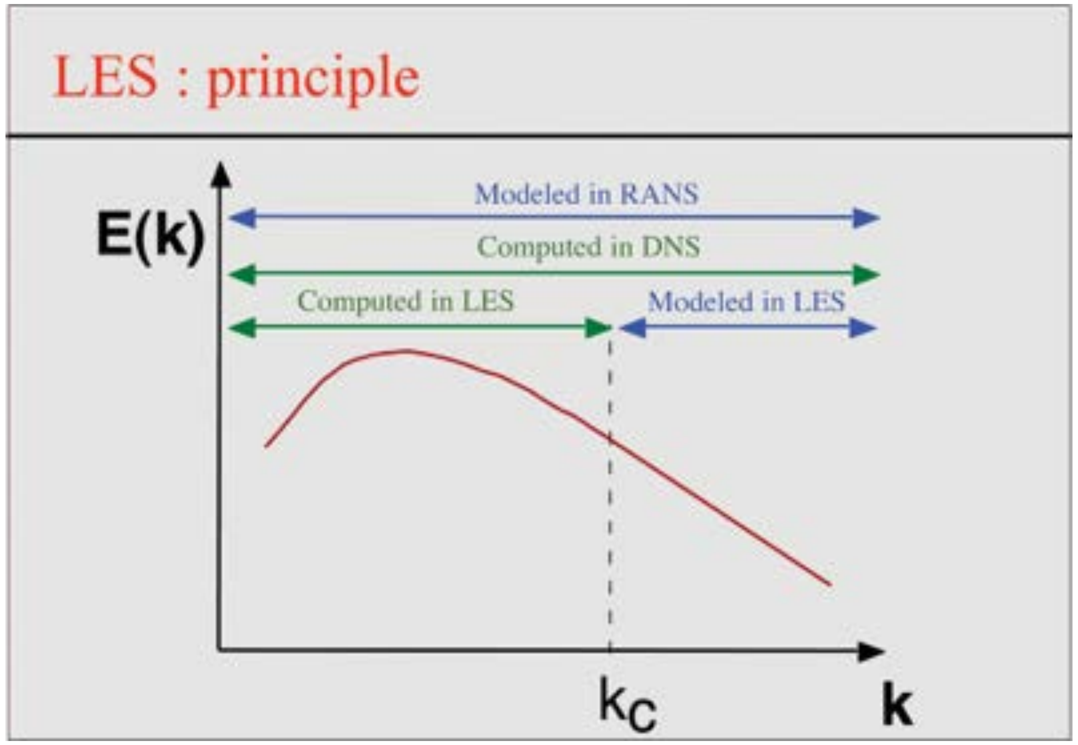
- Accuracy of aeroacoustic calculations depends how accurately the velocity field is predicted.
- Most of the flows of interest are turbulent.
- Modeling turbulence is the essential and often most important part of an aeroacoustic calculation.

Outline

- Motivation
  - New paradigm of direct physics-based coupling of adaptive numerical methods & turbulence models
- Wavelets and their basic properties
- Adaptive wavelet collocation method
- Hierarchy of Turbulence Modeling
  - Wavelet-Based Direct Numerical Simulations (WDNS)
  - Coherent Vortex Simulation
  - Stochastic Coherent Adaptive Large Eddy Simulations
  - Low-Fidelity Approaches
- Relationship of wavelet and other methods
- Computational Complexity of Turbulent Flows
- Examples
- Conclusions and Perspectives

At-a-Glance Comparison of DNS, LES, RANS





Taken From – Prof. D. Veynante Lecture Note – (without permission)
ERCOFTAC SIG4 Summer School on "Turbulence and Mixing in Compressible Flows", Strasbourg, France, 7-11 July 2005

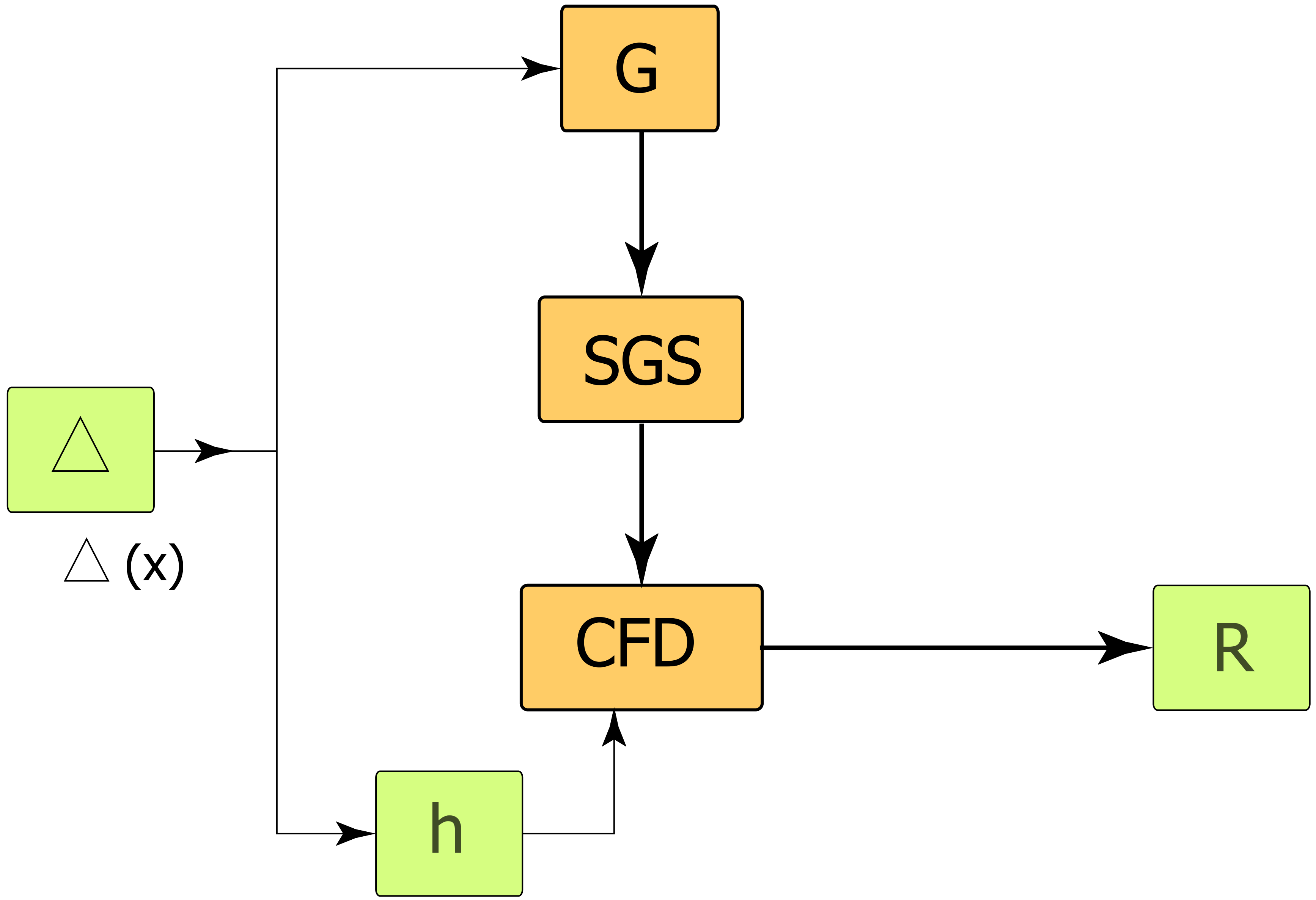
- Use a low pass filter to separate the large scale eddies from the small subgrid scales.

$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}(\mathbf{x},t) + \mathbf{u}'(\mathbf{x},t)$$

Simulate the evolution of the large scale vorticies, while modeling the effect of the small subgrid scales.

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j}$$

Dependency Diagram – LES



Defficiencies of Classical LES:

- Does not take advantage of spatial/temporal intermittency of turbulent flows
- Inhomegeneous fidelity
  - *a-priori* large/small scale separation
  - under-resolves energetic structures
  - over-resolves in between them

New **direction/philosophy/paradigm**:

Direct physics-based **coupling** of

adaptive high order **numerical methods**
&
turbulence models

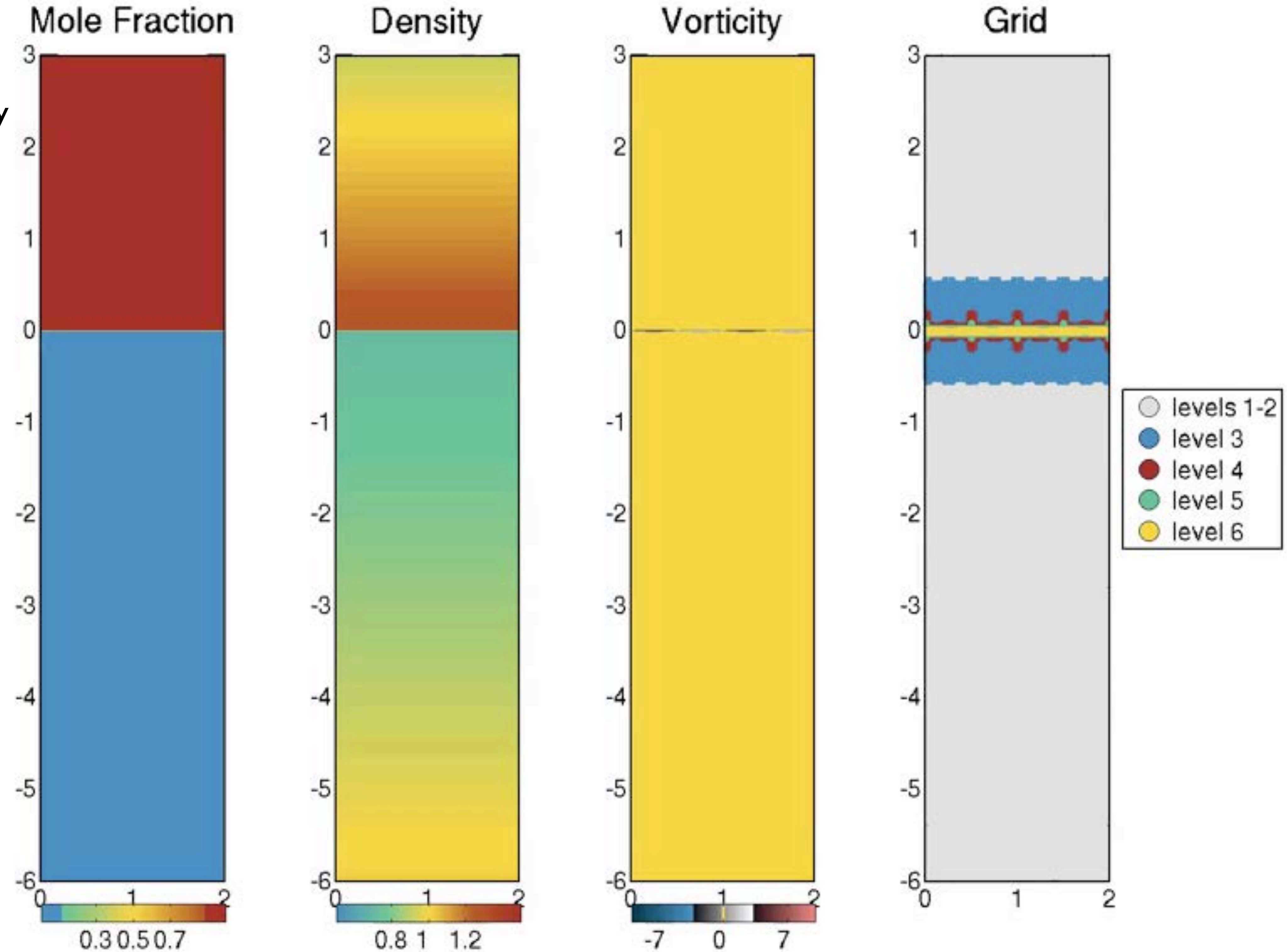
that takes advantage of spatio-temporal intermittency of turbulent flows

What does direct coupling bring?

- the active control of the fidelity/accuracy of the simulation
- near optimal spatially adaptive computational mesh
- the “desired” flow-physics is captured by considerably smaller number of spatial modes
- considerably smaller Reynolds scaling exponent, $Re^{\alpha},\ \alpha < 9/4$
- robust general mathematical framework for spatial/temporal *model*-refinement (*m*-refinement) that can be extended to LES with AMR approach
- new mathematical framework for epistemic uncertainty quantification

Adaptive Wavelet Collocation Method

Single-mode
Rayleigh-Taylor Instability
(incompressible limit)



Single-mode
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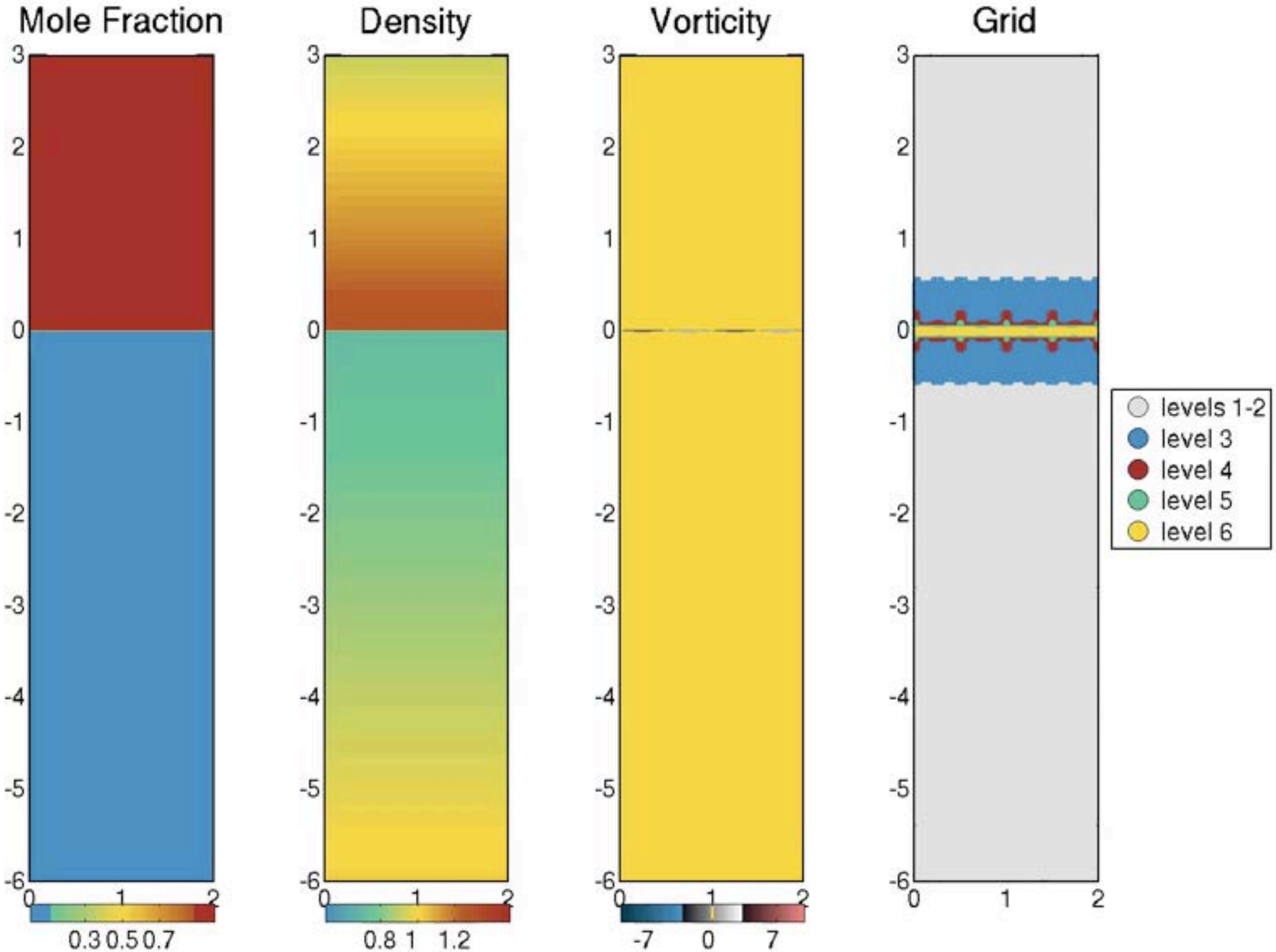

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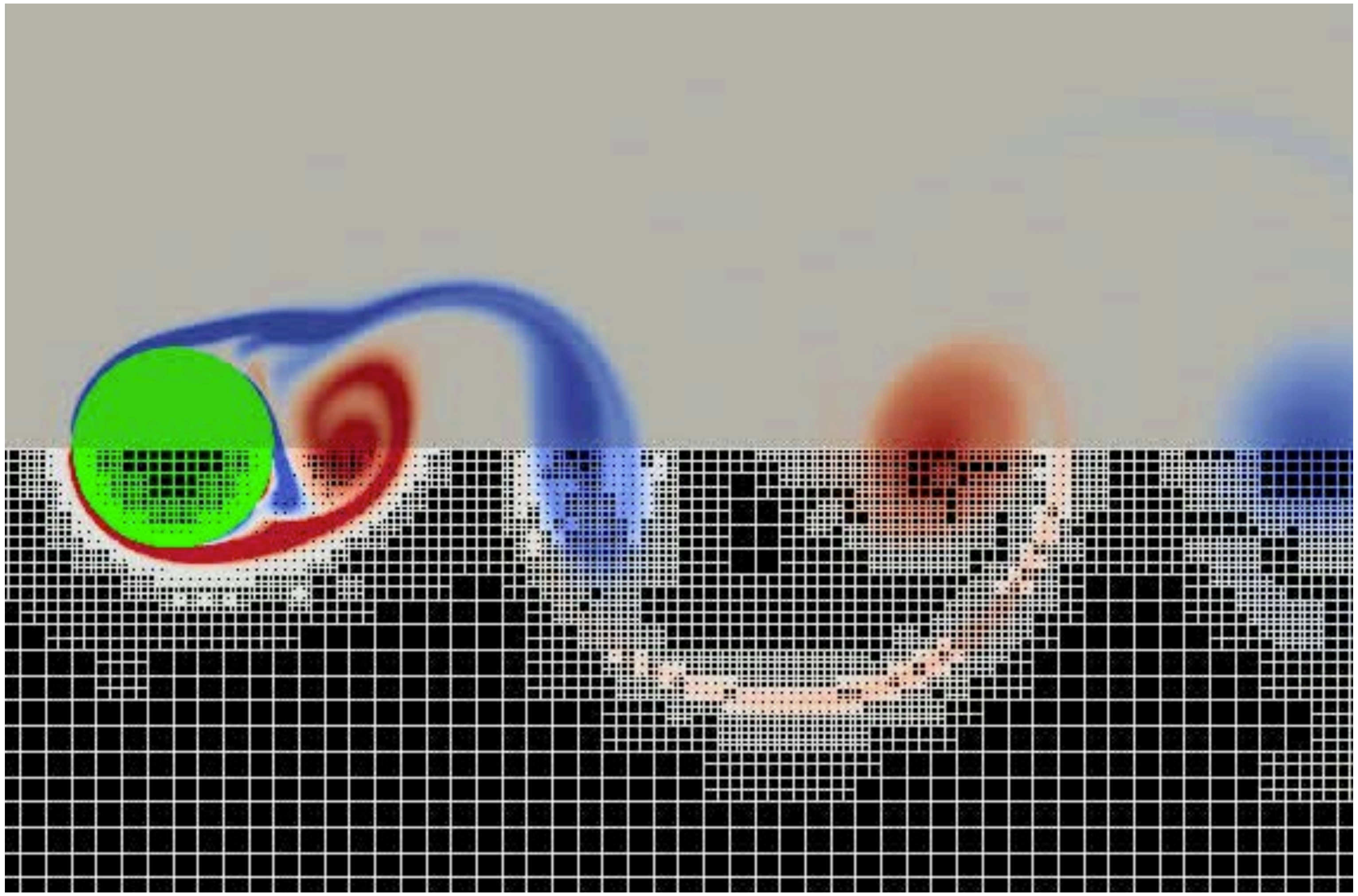
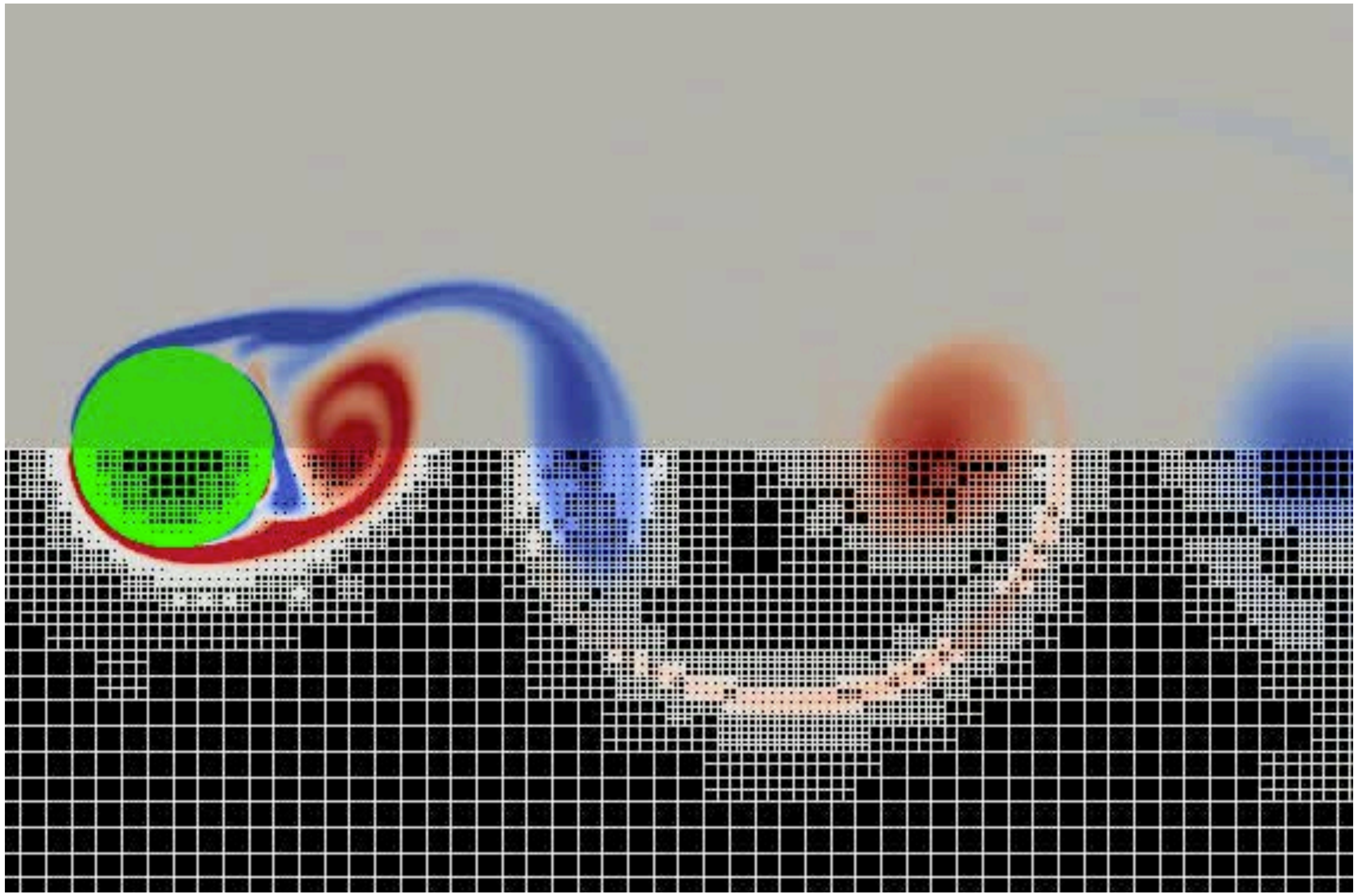
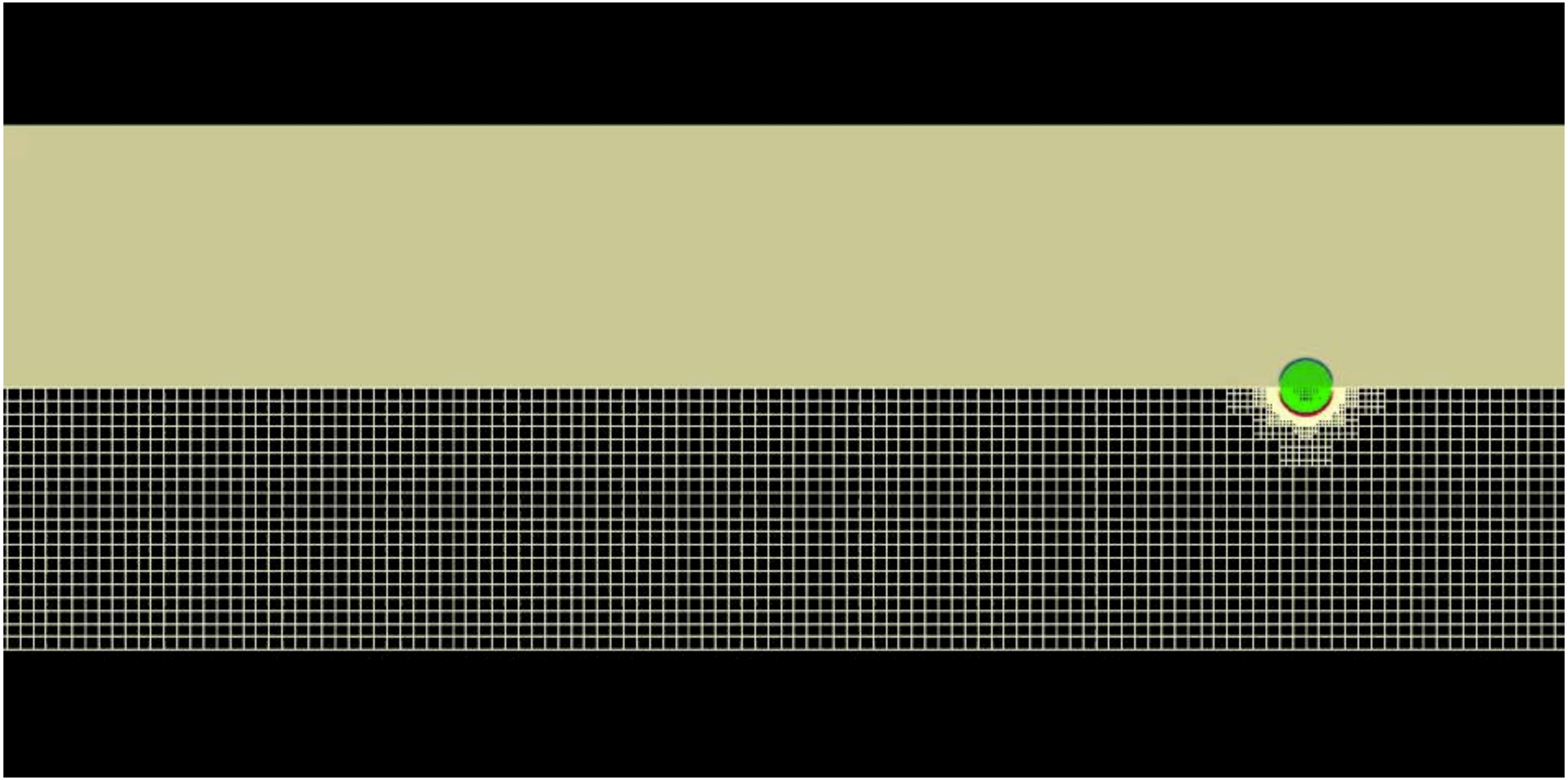
Vorticity



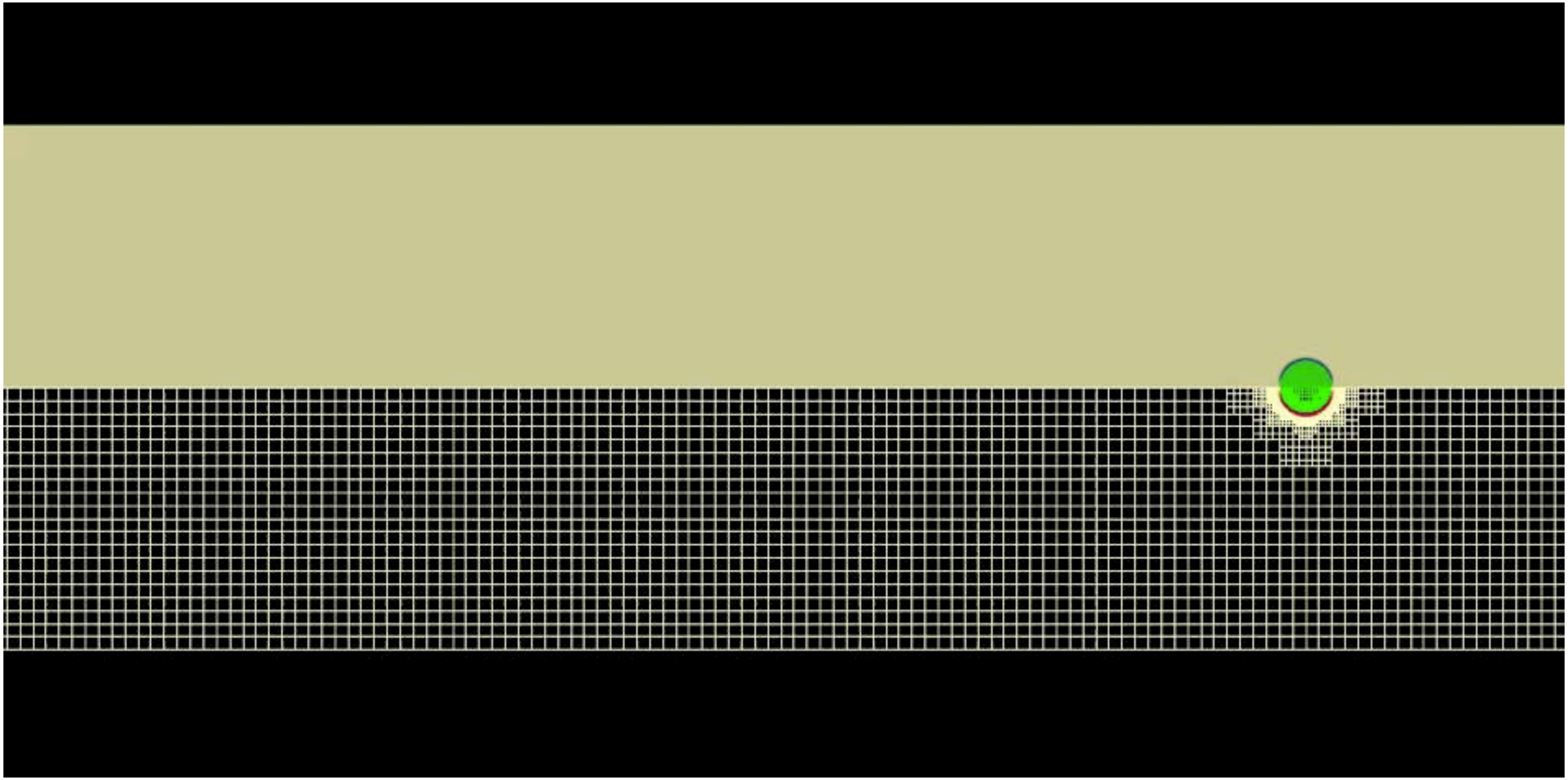
Vorticity



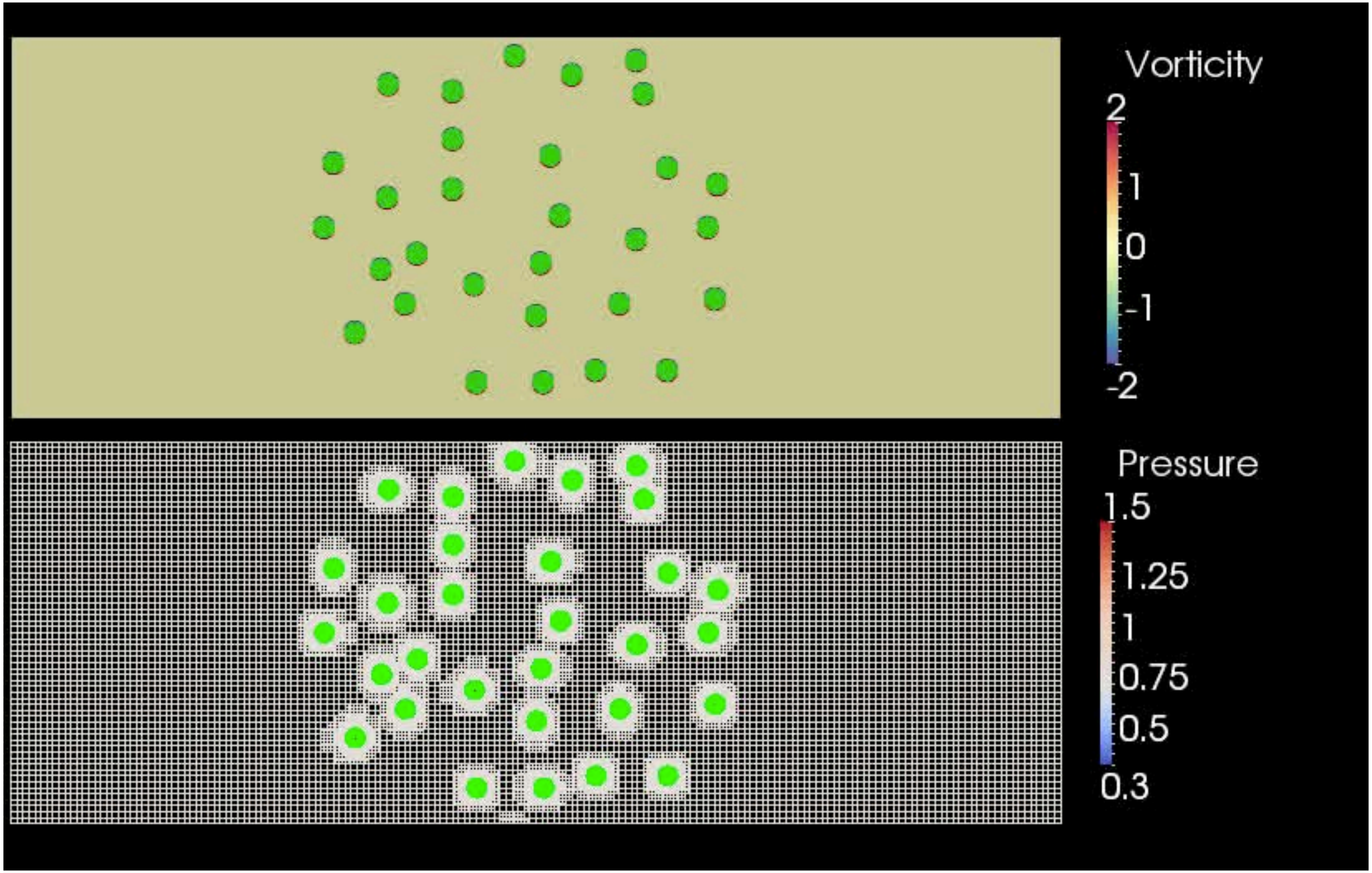
Vorticity



Vorticity



Multiple obstacles with prescribed motion



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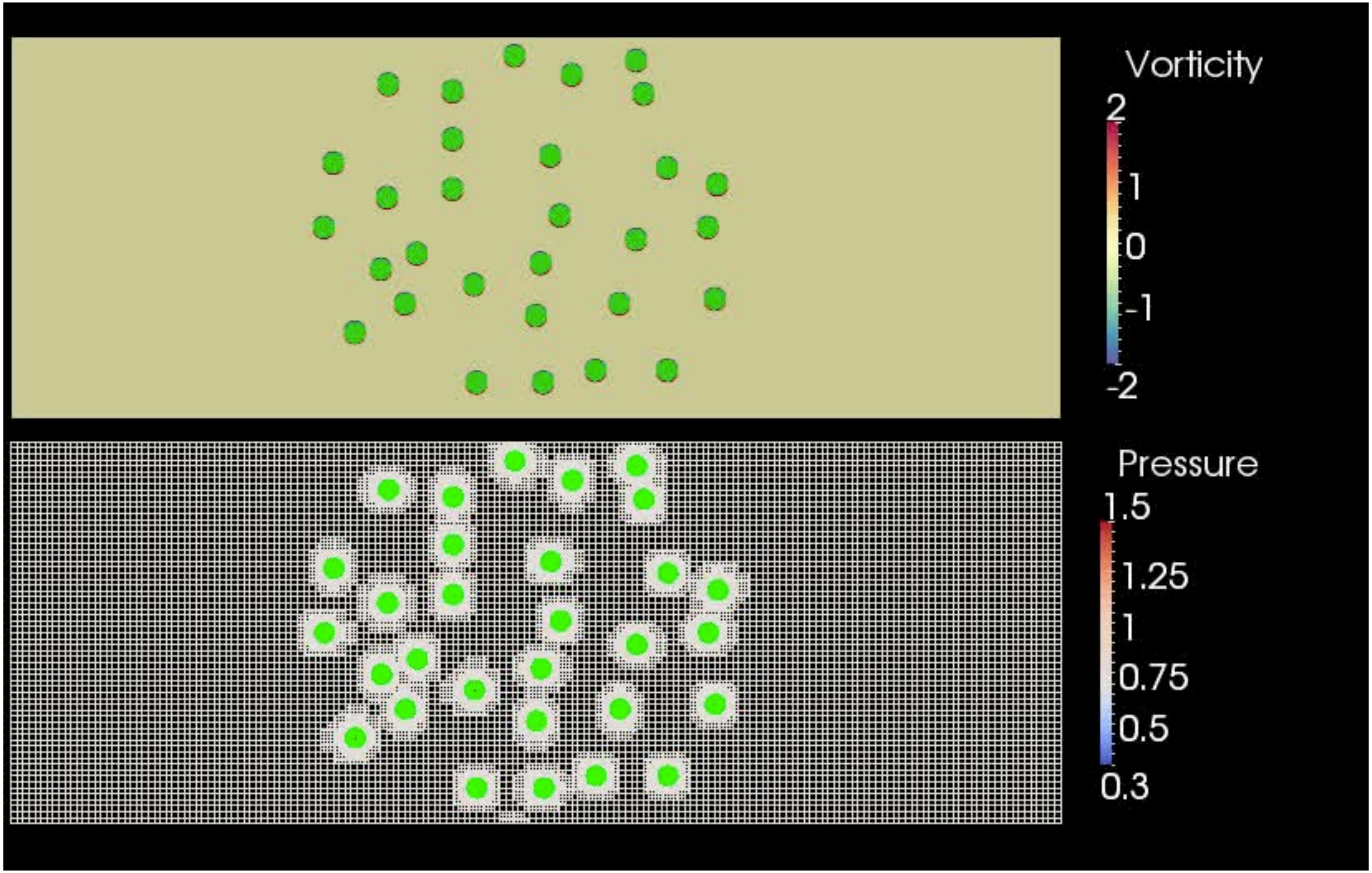
Multiple obstacles with prescribed motion



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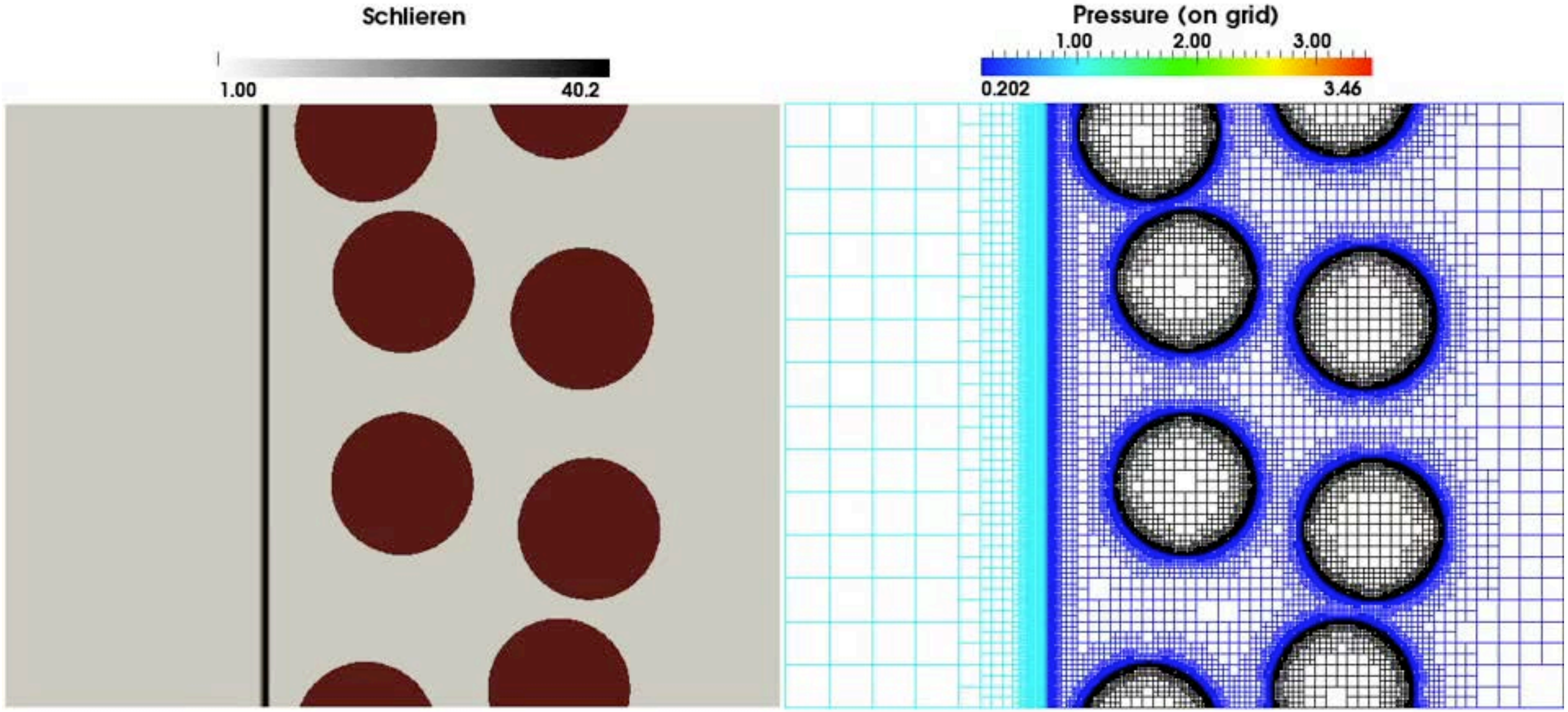
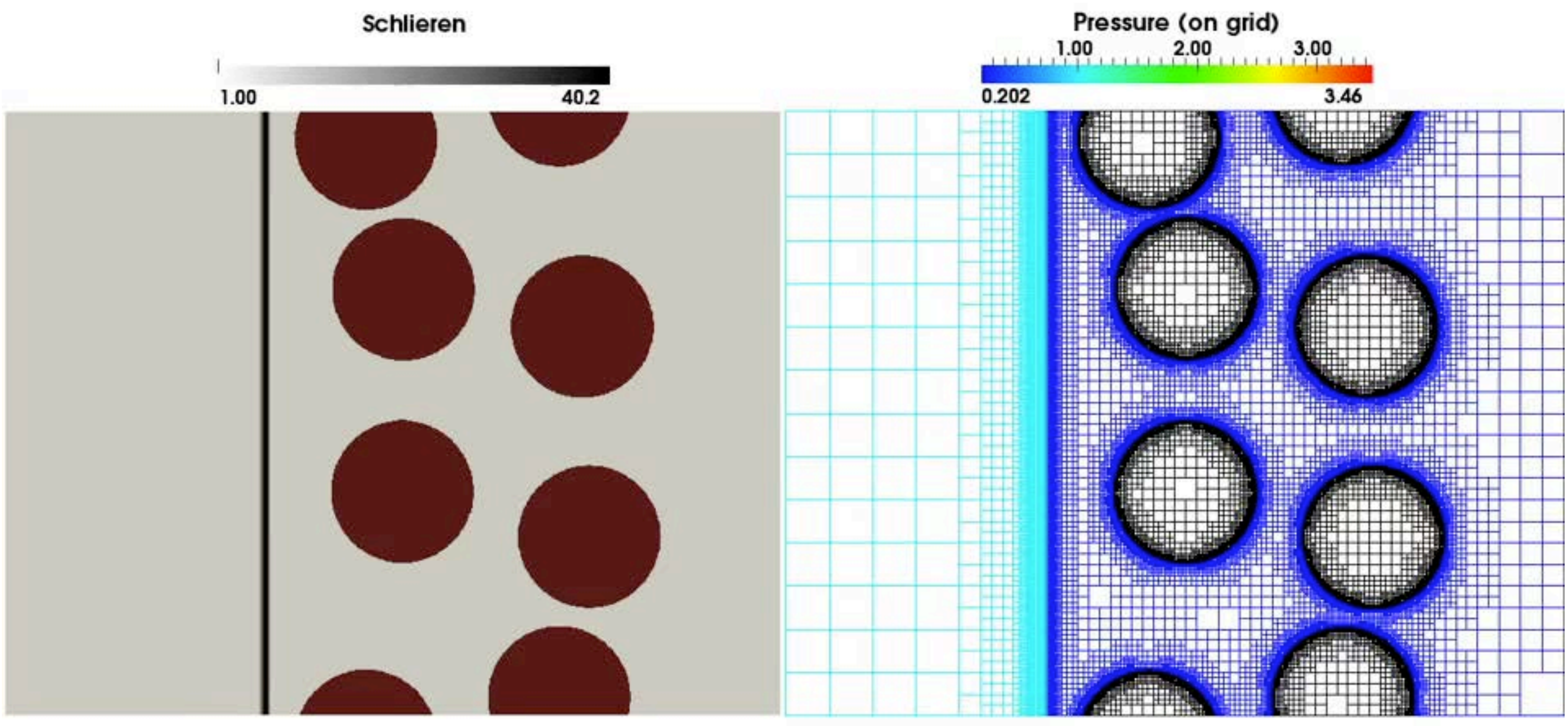
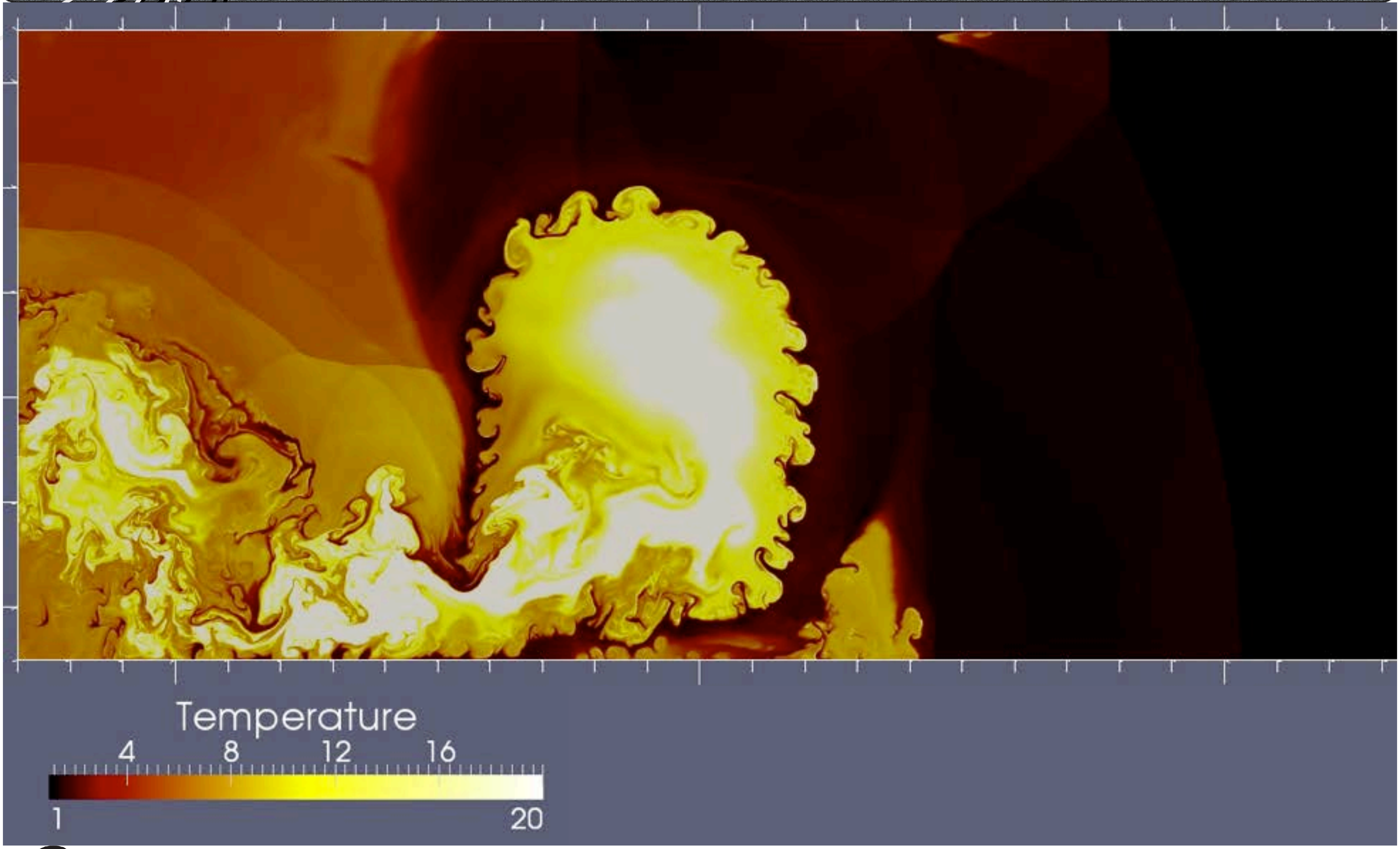
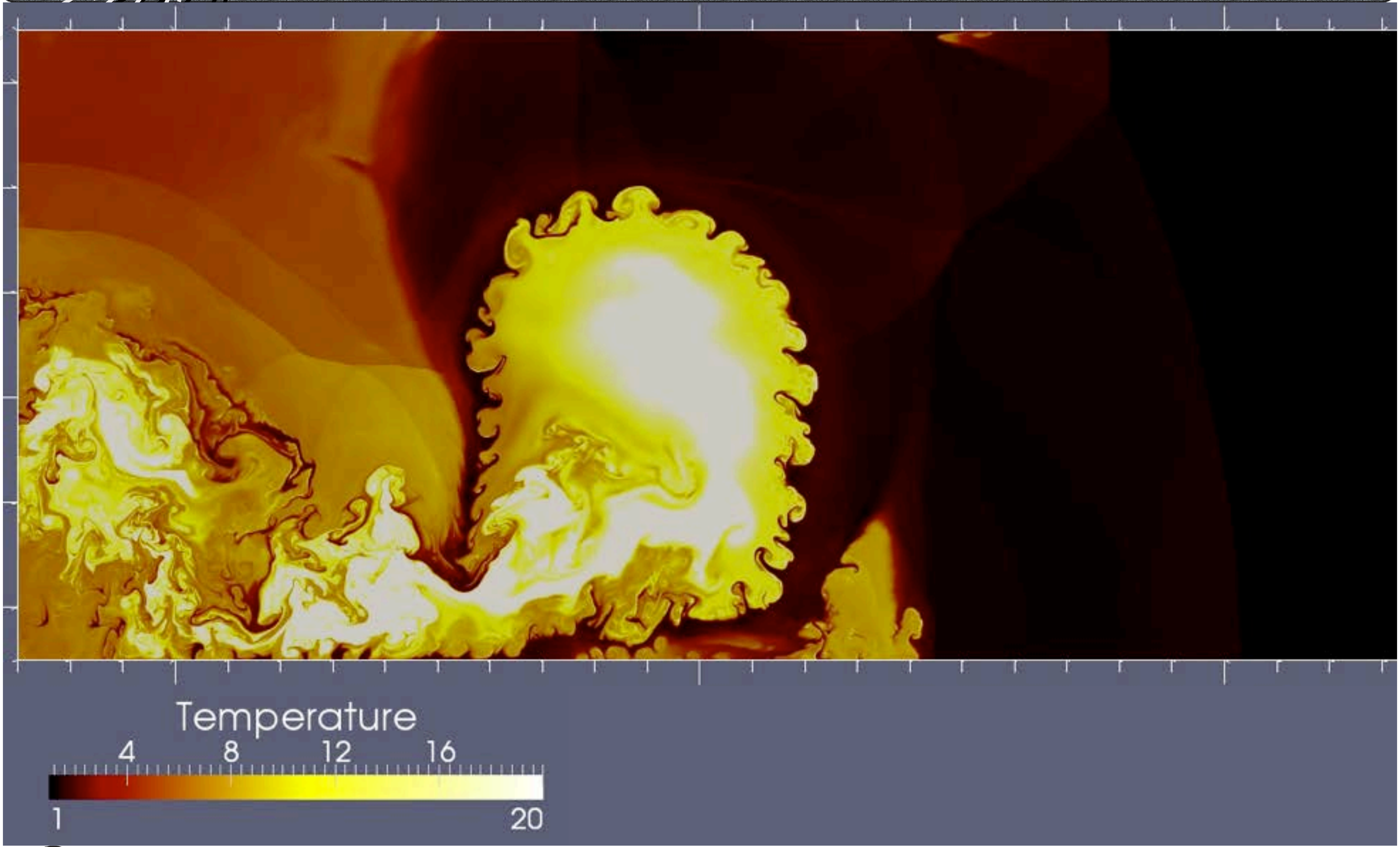

Schlieren
1.00
40.2
Pressure (on grid)
1.00
2.00
3.00
0.202
3.46

Shock Wave Propagation through the Cylinder Array




Temperature
4
8
12
16
1
20


Temperature
4
8
12
16
1
20

Hierarchical Variable Fidelity Multiscale Turbulence Modeling

Coherent Vortex Extraction*

$$\omega = \omega_C + \omega_I$$

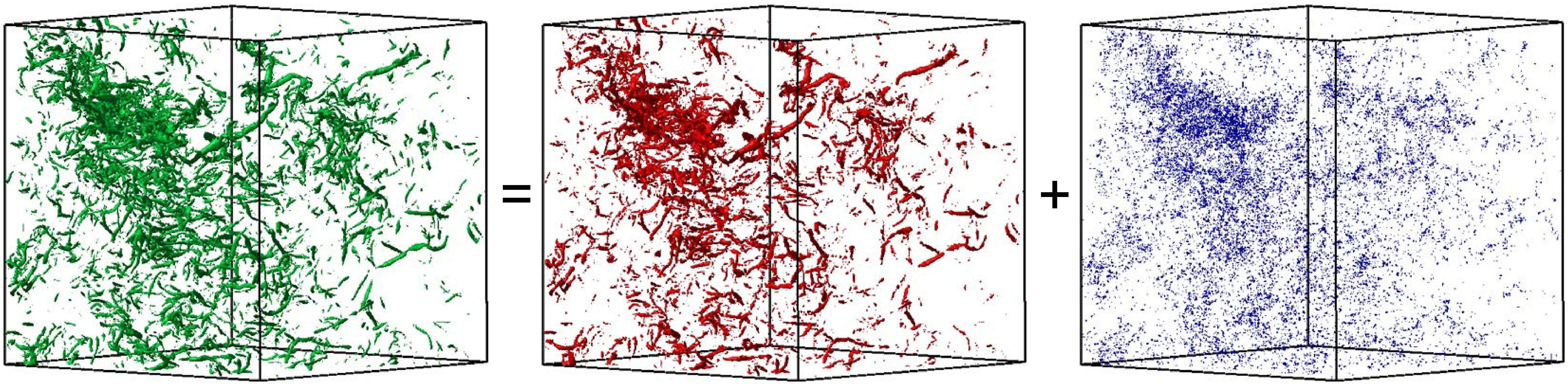
$$\omega_C = \sum_{j=0}^{j_{\max}} \sum_{\substack{\mathbf{k} \in \mathcal{K}^j \\ |d_{\mathbf{k}}^j| \geq \epsilon_{\text{opt}} \|\omega\|}} d_{\mathbf{k}}^j \psi_{\mathbf{k}}^j(\mathbf{x}) \qquad \omega_I = \sum_{j=0}^{j_{\max}} \sum_{\substack{\mathbf{k} \in \mathcal{K}^j \\ |d_{\mathbf{k}}^j| < \epsilon_{\text{opt}} \|\omega\|}} d_{\mathbf{k}}^j \psi_{\mathbf{k}}^j(\mathbf{x})$$

- optimal threshold from denoising theory†: $\epsilon_{\text{opt}} = \sigma_n \sqrt{2 \ln N}$
- σ_n is variance of incoherent vorticity

*Farge M, Schneider K, Kevlahan N. 1999. Phys. Fluids 11:2187–201
†Donoho DL, Johnstone IM. 1994. Biometrika 81:425–55

Coherent Vortex Extraction*

$$\omega = \omega_C + \omega_I$$



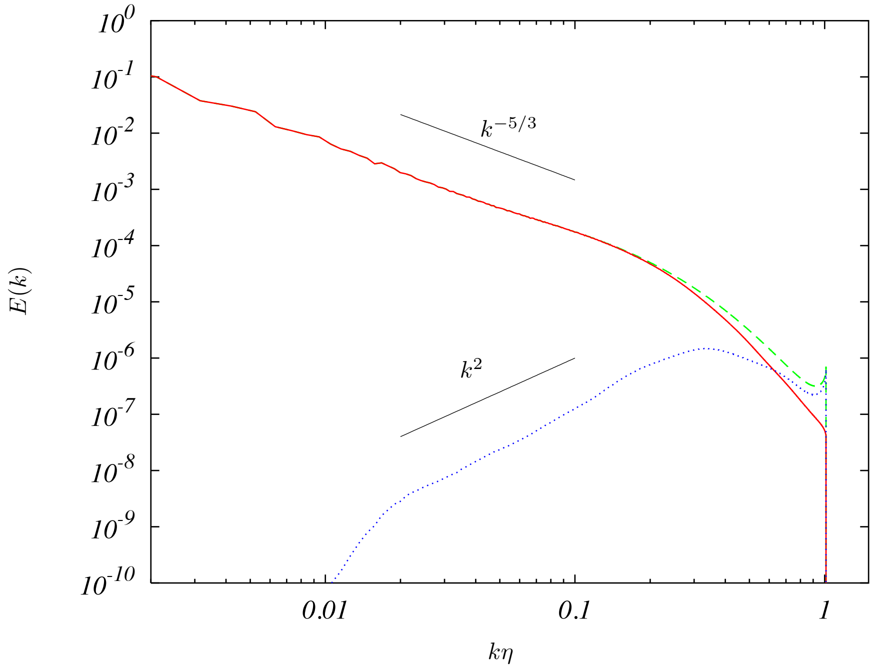
- Homogenous isotropic turbulent flow at $R_\lambda = 732$ and resolution $N = 2048^3$
- subcubes of size $N = 256^3$ are visualized

*Okamoto N, Yoshimatsu K, Schneider K, Farge M, Kaneda Y. 2007. Phys. Fluids 19:115109

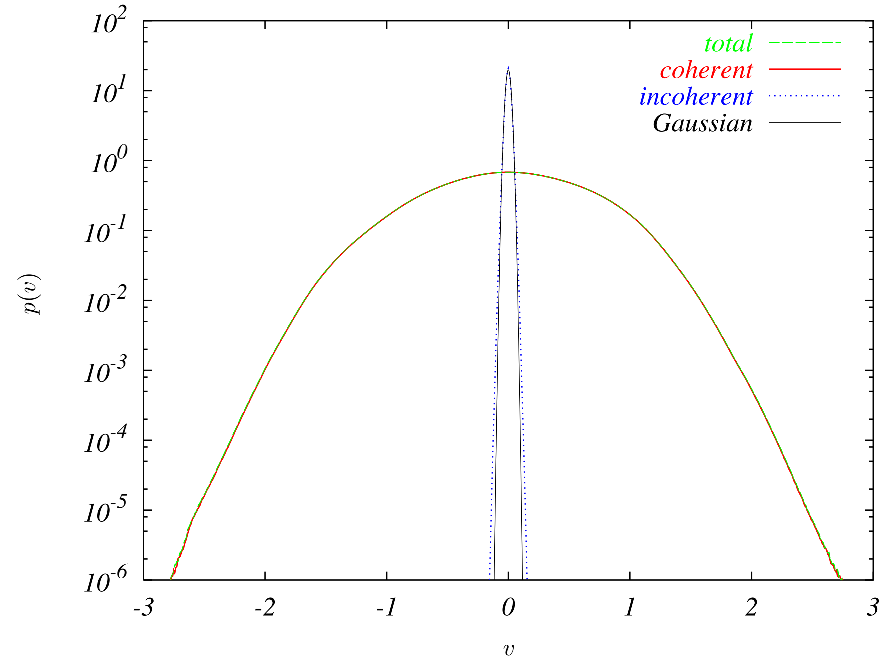
Coherent Vortex Extraction*

$$\omega = \omega_C + \omega_I$$

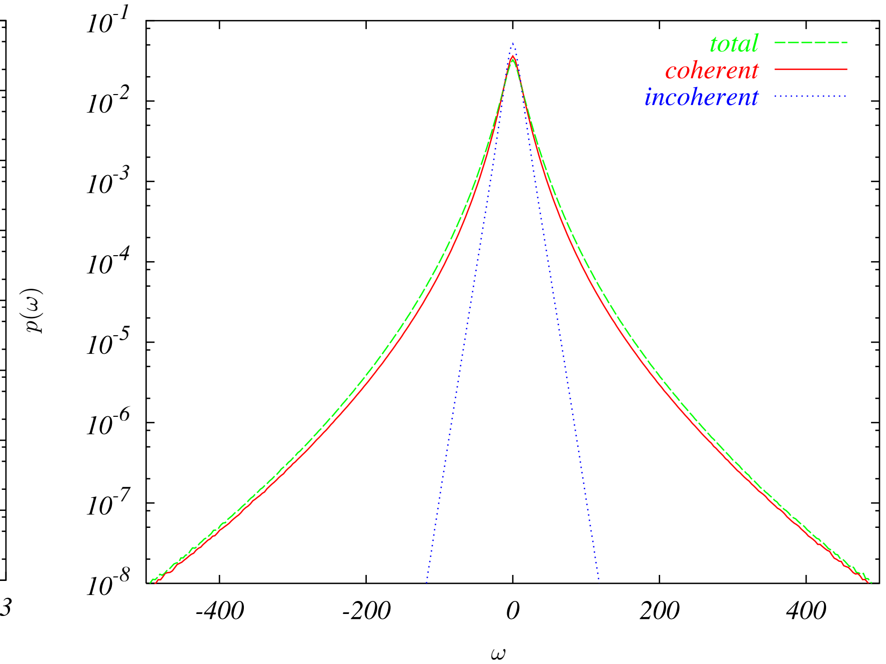
Energy spectra



PDF of velocity



PDF of vorticity



- Homogenous isotropic turbulent flow at R_λ = *732* and resolution *N* = 2048^3
- subcubes of size N = 256^3 are visualized

Okamoto N, Yoshimatsu K, Schneider K, Farge M, Kaneda Y. 2007. Phys. Fluids 19:115109

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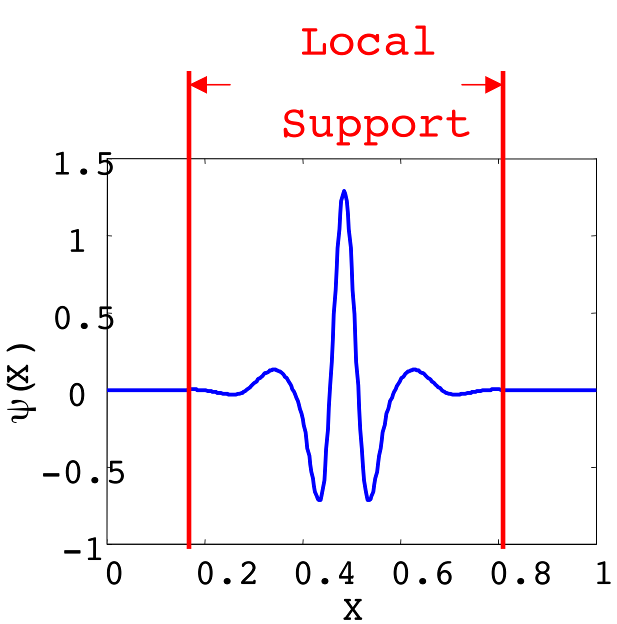
Wavelet thresholding filter:

$$\overline{u}_i^{>\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c_{\mathbf{l}}^0 \phi_{\mathbf{l}}^0(\mathbf{x}) + \sum_{j=0}^{+\infty} \sum_{\mu=1}^{2^n-1} \sum_{\substack{\mathbf{k} \in \mathcal{K}^j \\ |d_{\mathbf{k}}^j| \geq \epsilon \|\mathbf{u}\|}} d_{\mathbf{k}}^{\mu,j} \psi_{\mathbf{k}}^{\mu,j}(\mathbf{x})$$

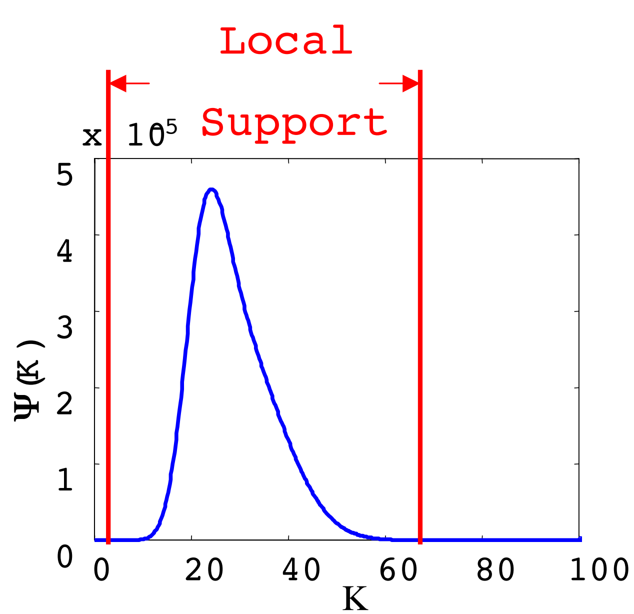
Wavelet thresholding filter:

$$\overline{u}_i^{>\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c^0_{\mathbf{l}}\,\phi^0_{\mathbf{l}}(\mathbf{x}) + \sum_{j=0}^{+\infty}\sum_{\mu=1}^{2^n-1}\sum_{\substack{\mathbf{k}\in\mathcal{K}^j \\ |d^j_{\mathbf{k}}|\ge\epsilon\|\mathbf{u}\|}} d^{\mu,j}_{\mathbf{k}}\,\psi^{\mu,j}_{\mathbf{k}}(\mathbf{x})$$

Physical Space



Wave Number Space



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$$\overline{u}_i^{>\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c_{\mathbf{l}}^0 \phi_{\mathbf{l}}^0(\mathbf{x}) + \sum_{j=0}^{+\infty} \sum_{\mu=1}^{2^n-1} \sum_{\substack{\mathbf{k}\in\mathcal{K}^j \\ |d_{\mathbf{k}}^j| \geq \epsilon \|\mathbf{u}\|}} d_{\mathbf{k}}^{\mu,j} \psi_{\mathbf{k}}^{\mu,j}(\mathbf{x})$$

$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}^{>\epsilon}(\mathbf{x},t) + \overline{\mathbf{u}}^{\leq\epsilon}(\mathbf{x},t)$$

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$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}^{>\epsilon}(\mathbf{x},t) + \overline{\mathbf{u}}^{\leq\epsilon}(\mathbf{x},t)$$

Choice of ϵ:

- WDNS - $\epsilon \ll 1$
- CVS* - $\epsilon \approx \epsilon_{\mathrm{opt}}$
- SCALES† - $\epsilon > \epsilon_{\mathrm{opt}}$

*Coherent Vortex SImulation (CVS): Farge M, Schneider K, Kevlahan N. Phys. Fluids 11:2187–201, 1999.
†Stochastic Coherent Adaptive Large Eddy Simulations (SCALES): Goldstein, D.E. and Vasilyev, O.V., Phys. Fluids 16: 2497-2513, 2004.

Wavelet thresholding filter:

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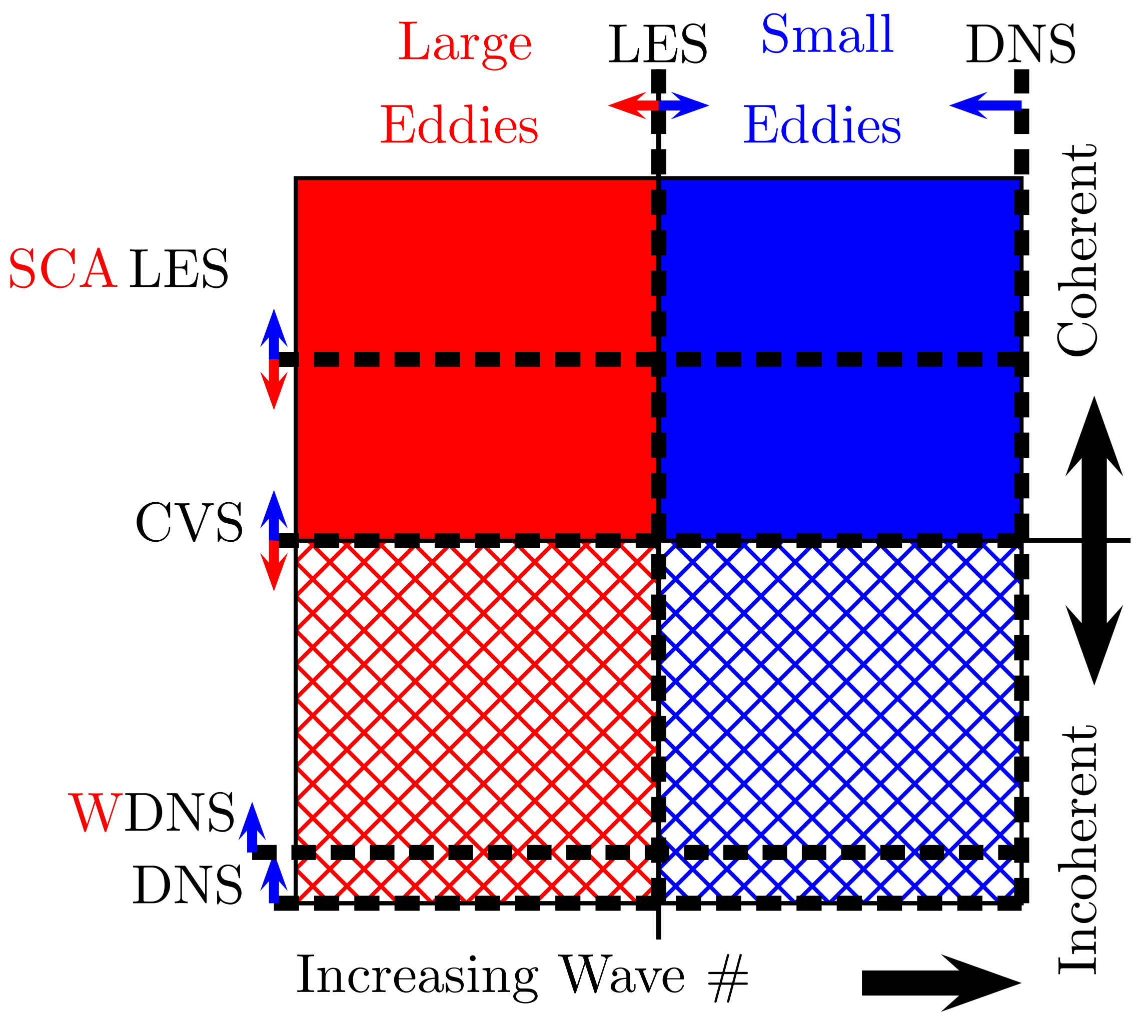

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Simulate the evolution of the most energetic coherent vortices (track them), while modeling the effect of the subgrid scales.

$$\frac{\partial\overline{u}_i^{>\epsilon}}{\partial t} + \frac{\partial\overline{u}_i^{>\epsilon}\overline{u}_j^{>\epsilon}}{\partial x_j} = -\frac{\partial\overline{p}^{>\epsilon}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2\overline{u}_i^{>\epsilon}}{\partial x_j\partial x_j} + \frac{\partial\tau_{ij}}{\partial x_j}$$

- Use wavelet thresholding filter to separate the numerically significant flow structures from the insignificant ones.

$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}^{>\epsilon}(\mathbf{x},t) + \overline{\mathbf{u}}^{\leq\epsilon}(\mathbf{x},t)$$

Wavelet threshold is set to sufficiently small value ($\epsilon \ll 1$), so the ignored scales are insignificant and **no model** is necessary.

$$\frac{\partial \overline{u}_i^{>\epsilon}}{\partial t} + \frac{\partial \overline{u}_i^{>\epsilon}\overline{u}_j^{>\epsilon}}{\partial x_j} = -\frac{\partial \overline{p}^{>\epsilon}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i^{>\epsilon}}{\partial x_j \partial x_j} + \cancel{\frac{\partial \tau_{ij}}{\partial x_j}}$$

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Coherent Vortex Simulation*

- Use an “ideal” wavelet thresholding filter to separate the energetic coherent vortices from the “incoherent Gaussian" subgrid scales at each time step.

$$\omega(x,t) = \omega_{\geq}(x,t) + \omega_{<}(x,t)$$

Simulate the evolution of the coherent vortices, (track them), while modeling the effect of the “incoherent Gaussian” subgrid scales.

$$\frac{\partial \omega_{i\geq}}{\partial t} + u_{j\geq}\frac{\partial \omega_{i\geq}}{\partial x_j} = \omega_{j\geq}\frac{\partial u_{i\geq}}{\partial x_j} + \frac{1}{Re}\frac{\partial^2 \omega_{i\geq}}{\partial x_j \partial x_j} + M_i$$

*Farge M, Schneider K, Kevlahan N. 1999. Phys. Fluids 11:2187–201

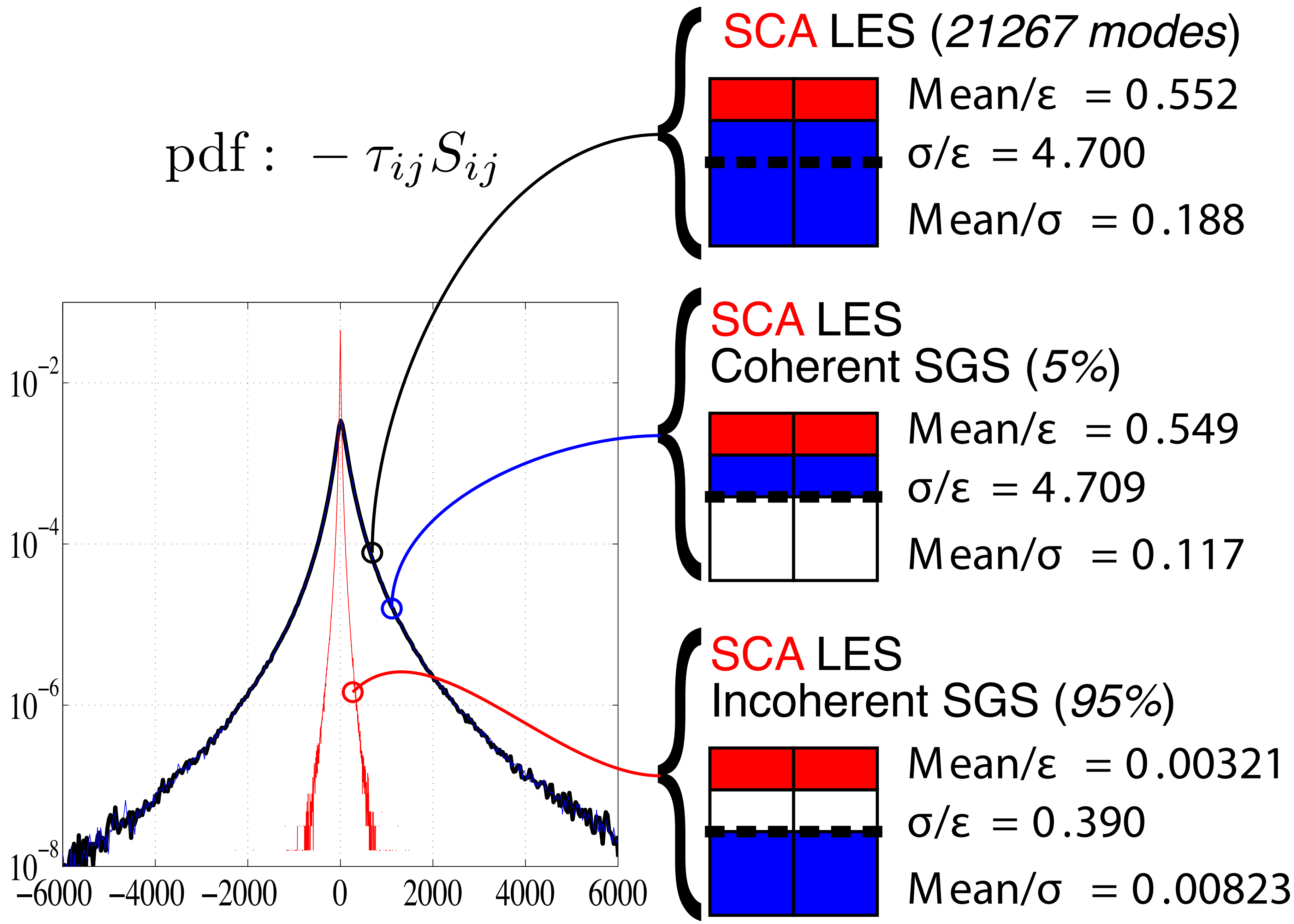
- Use wavelet thresholding filter to separate the most energetic coherent vortices from the subgrid scales.

$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}^{>\epsilon}(\mathbf{x},t) + \overline{\mathbf{u}}^{\leq\epsilon}(\mathbf{x},t)$$

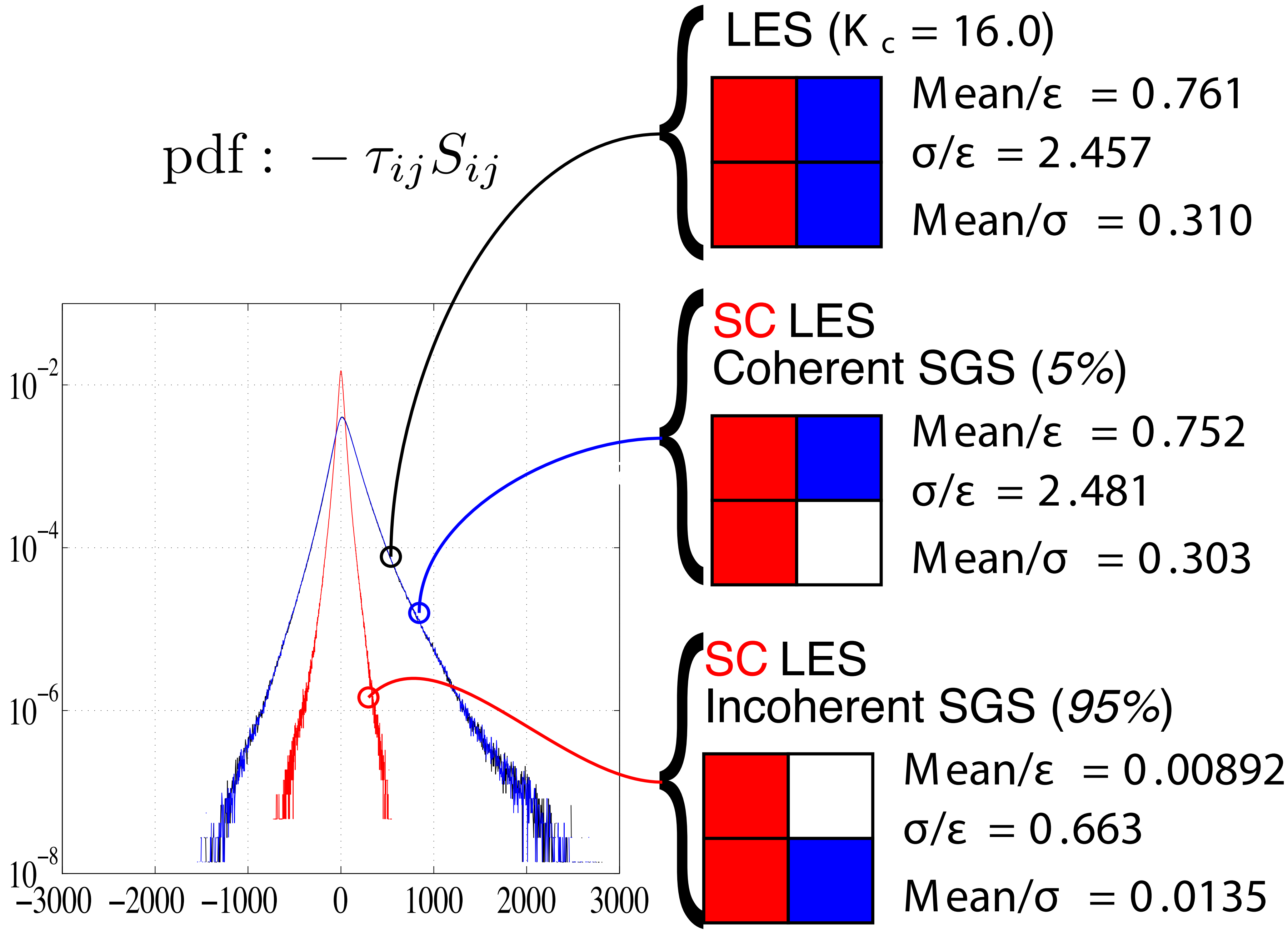
Simulate the evolution of the most energetic coherent vortices (track them), while modeling the effect of the subgrid scales.

$$\frac{\partial \overline{u}_i^{>\epsilon}}{\partial t} + \frac{\partial \overline{u}_i^{>\epsilon}\overline{u}_j^{>\epsilon}}{\partial x_j} = -\frac{\partial \overline{p}^{>\epsilon}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i^{>\epsilon}}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j}$$

SCALES Dissipation



$\varepsilon = \langle 2\nu S_{ij} S_{ij} \rangle$ (mean viscous dissipation)



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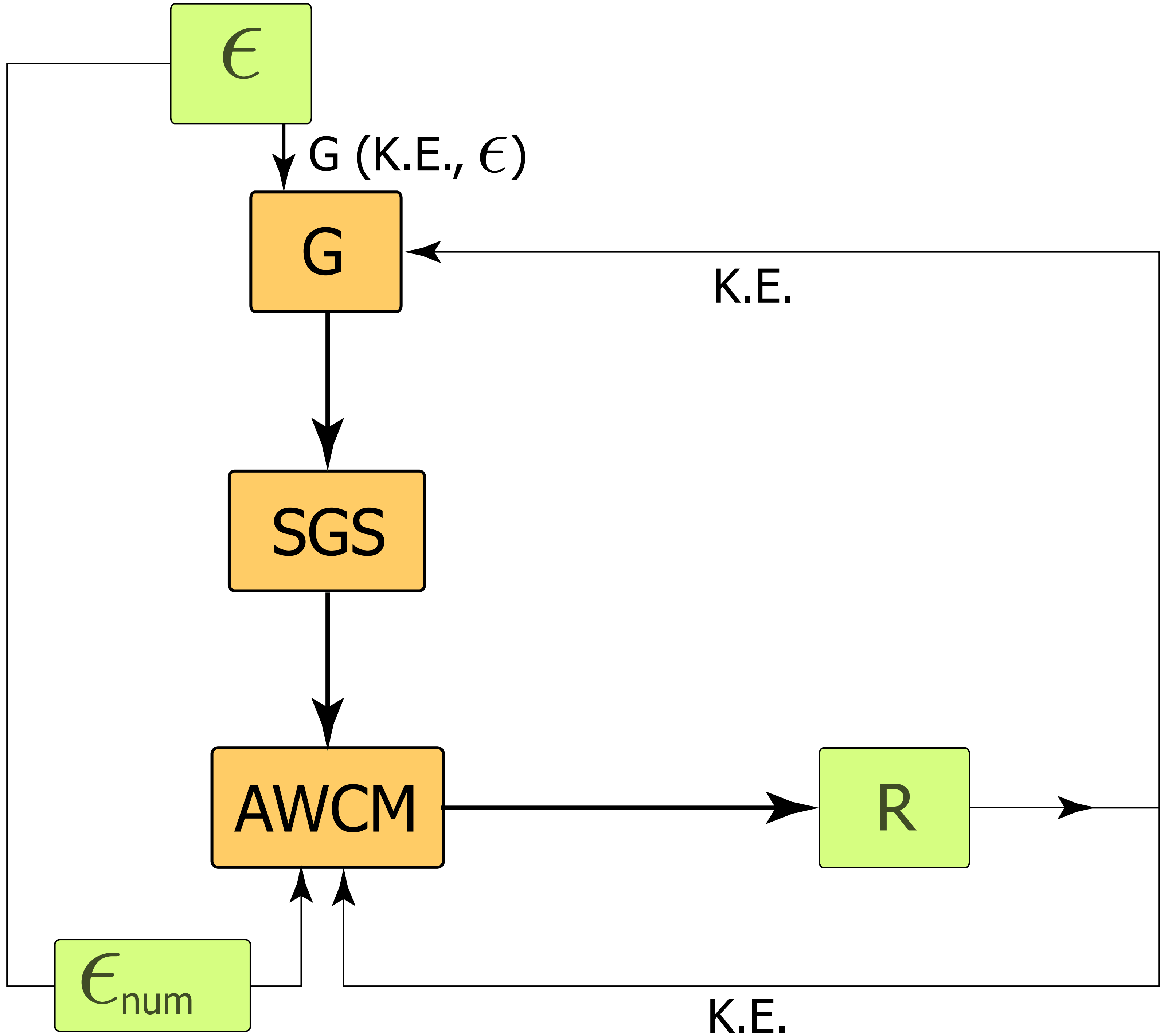
Dependency Diagram – SCALES



Kinetic Energy Based: $$\mathcal{F} = \frac{k_{\mathrm{sgs}}}{k_{\mathrm{res}}+k_{\mathrm{sgs}}}$$

SGS dissipation Based: $$\mathcal{F} = \frac{\Pi}{\varepsilon_{\mathrm{res}}+\Pi}$$

Kinetic Energy Based: $\mathcal{F} = \frac{k_{\mathrm{sgs}}}{k_{\mathrm{res}} + k_{\mathrm{sgs}}}$

SGS dissipation Based: $\mathcal{F} = \frac{\Pi}{\varepsilon_{\mathrm{res}} + \Pi}$

Fidelity of the simulation is a function of Turbulence Resolution

Objective - control the level of fidelity

Kinetic Energy Based: $\mathcal{F} = \frac{k_{\mathrm{sgs}}}{k_{\mathrm{res}} + k_{\mathrm{sgs}}}$

SGS dissipation Based: $\mathcal{F} = \frac{\Pi}{\varepsilon_{\mathrm{res}} + \Pi}$

Kinetic Energy Based: $\mathcal{F} = \frac{k_{\text{sgs}}}{k_{\text{res}} + k_{\text{sgs}}}$

SGS dissipation Based: $\mathcal{F} = \frac{\Pi}{\varepsilon_{\text{res}} + \Pi}$

Homogeneous Turbulence:

LES with $\mathcal{F}_{KE}$ fixed complexity $\sim Re^{0} = 1$

LES with $\mathcal{F}_{D}$ fixed complexity $\sim Re^{9/4}$

Spatial Variable Thresholding

Scales Dependency

$\epsilon\downarrow \implies$ Buildup scales $\varepsilon_{\mathrm{res}}\uparrow$

$\epsilon\uparrow \implies$ Losing some scales $\Pi\uparrow$

Goal: $\epsilon\downarrow$ wherever $\Pi > \Pi_{\mathrm{Goal}}$

In General:

Wavelet-Threshold-Filtered Velocity
Depends on:
1) Threshold Level
2) Velocity Scale

$$\overline{u}_i^{>\epsilon} \sim \epsilon \ \& \ \|u_i\|$$

Idea:

Threshold is determined on-the-fly
By Tracking areas of Locally Significant:
1) SGS Dissipation or
2) Any other Physical quantity

Fully Adaptive Wavelet Thresholding Filter

$$\epsilon = \epsilon(\Pi)$$

Lagrangian “Variable Thresholding” SCALES

$$\partial_t \epsilon + \overline{u}_j^{>\epsilon} \partial_{x_j} \epsilon = -\mathrm{forcing}_{\mathrm{term}} + \nu_\epsilon \partial^2_{x_j x_j} \epsilon$$

$$\mathrm{forcing}_{\mathrm{term}} = \epsilon^{\mathrm{old}} \left(\mathbf{x} - \overline{\mathbf{u}}^{>\epsilon} \Delta t, t\right) \tfrac{1}{\tau_\epsilon} \left(\mathcal{F} - \mathcal{G}\right)$$

$$\mathcal{F} = \tfrac{\Pi}{\varepsilon_{\mathrm{res}} + \Pi}$$

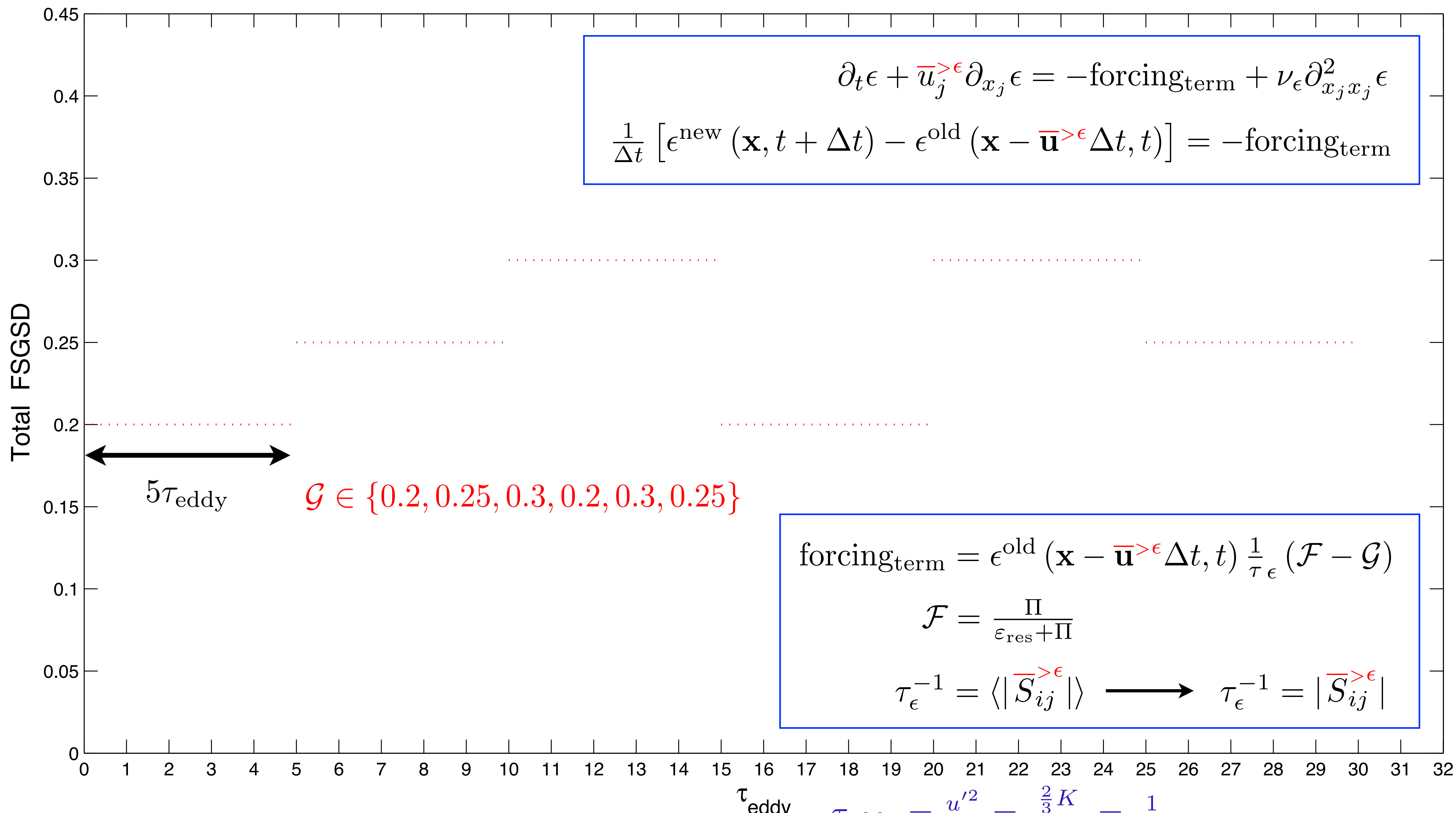
$$\tau_\epsilon = \left| \overline{S}_{ij}^{>\epsilon} \right|^{-1}$$

$$\nu_\epsilon \left(\mathbf{x}, t\right) = C_{\nu_\epsilon} \Delta^2 \left(\mathbf{x}, t\right) \left| \overline{S}_{ij}^{>\epsilon} \right|$$

numerical diffusion time-scale: τ_ϵ $>$ convective time-scale: $\frac{\Delta^2}{\nu_\epsilon}$

$$C_{\nu_\epsilon} < 1$$

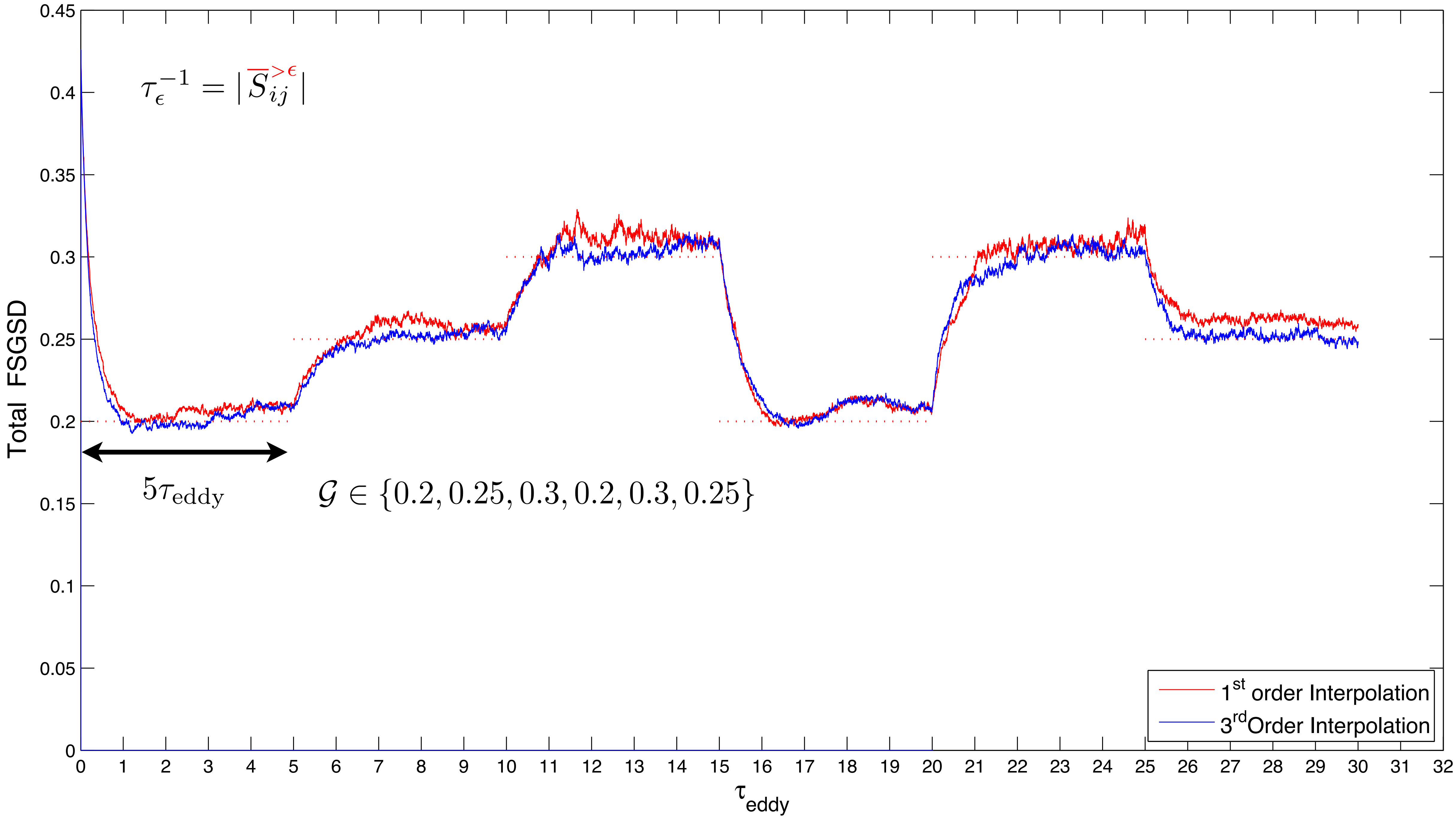
$$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\text{res}} \rangle + \langle \Pi \rangle}$$



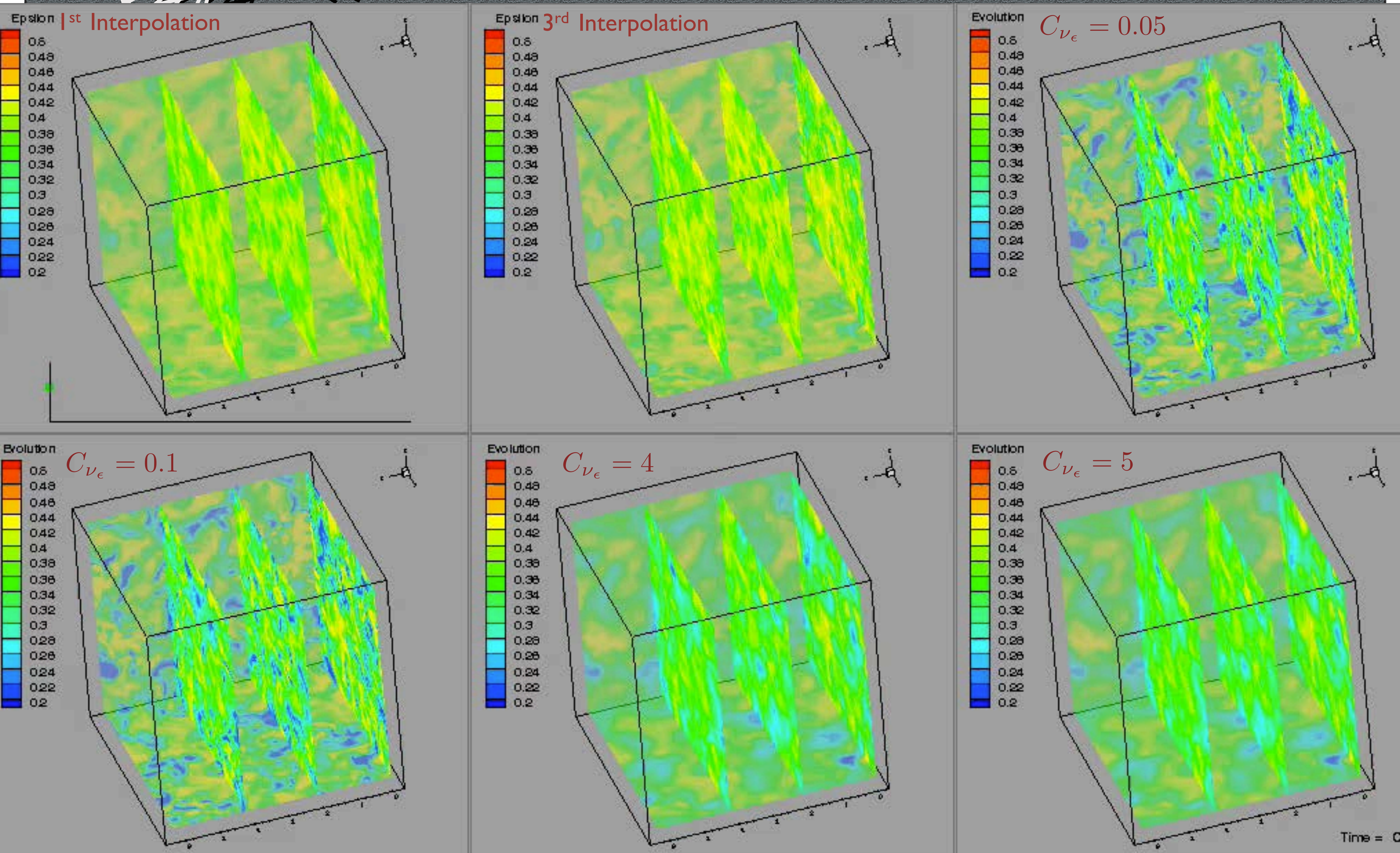
$$\tau_{\text{eddy}} = \frac{u'^2}{\langle \varepsilon \rangle} = \frac{\frac{2}{3}K}{2QK} = \frac{1}{3Q}$$

$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle}$

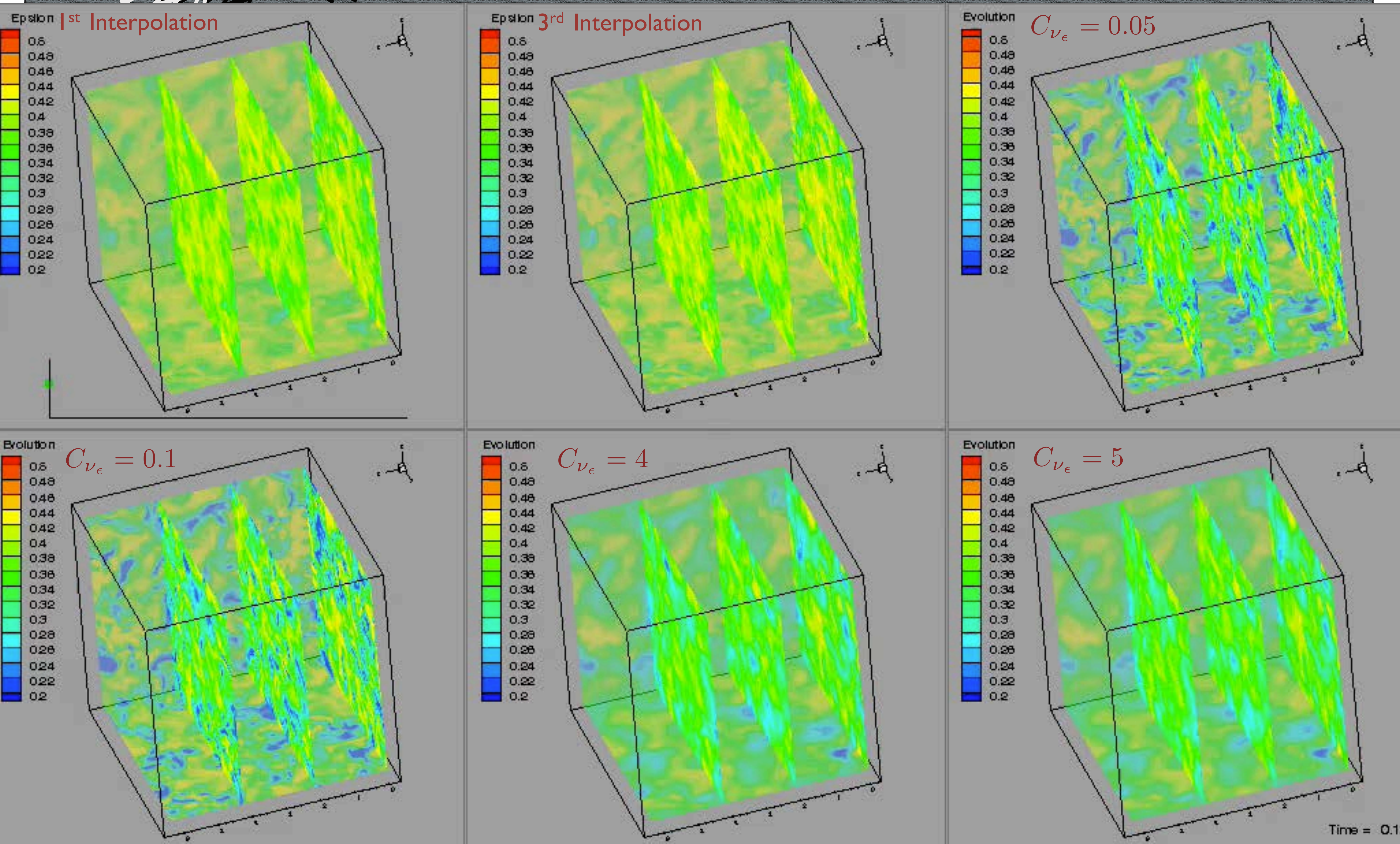
1st & 3rd Order Interpolation Approach



Hybrid CVS / SCALES – Threshold Animation



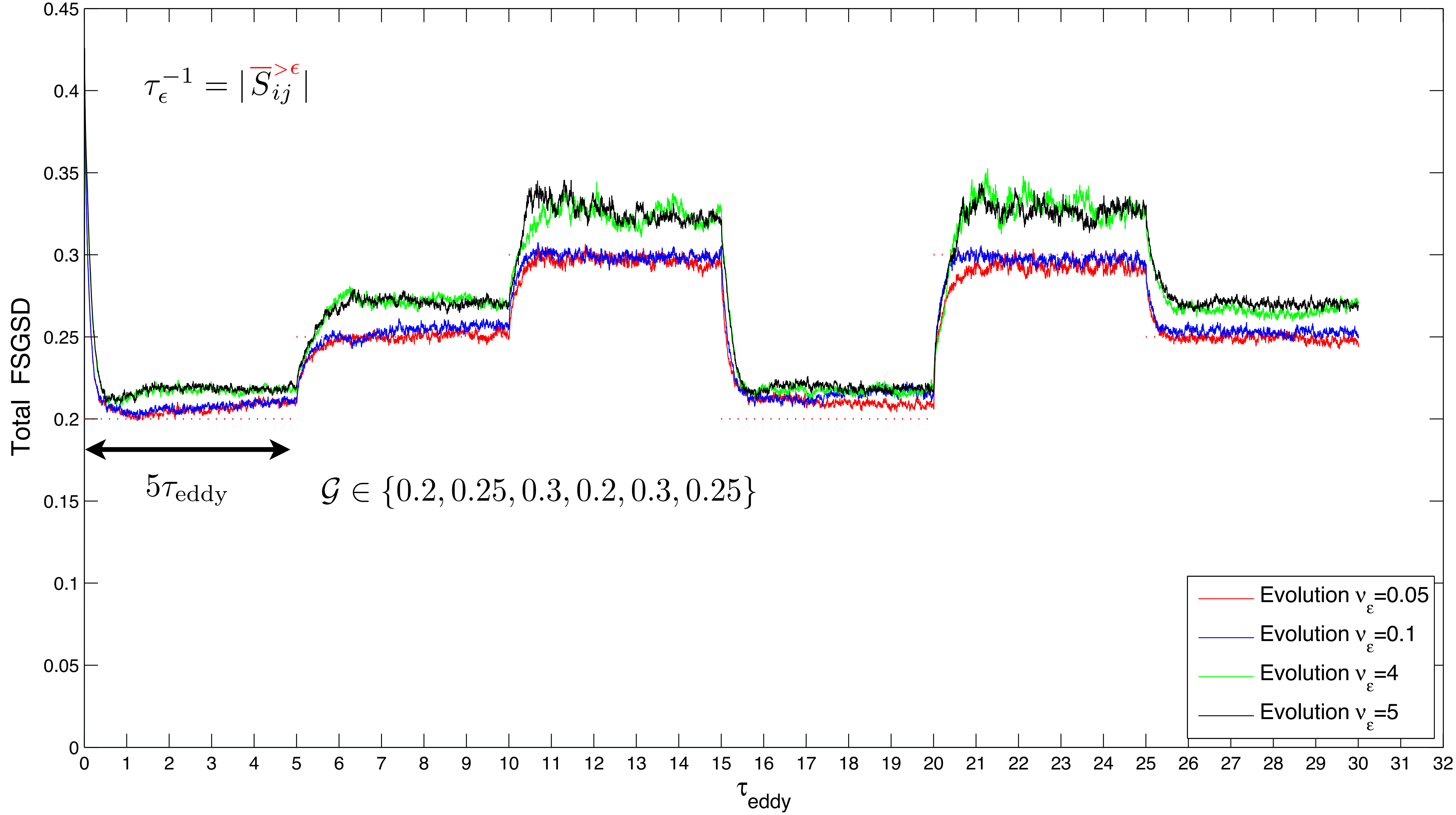
Hybrid CVS / SCALES – Threshold Animation

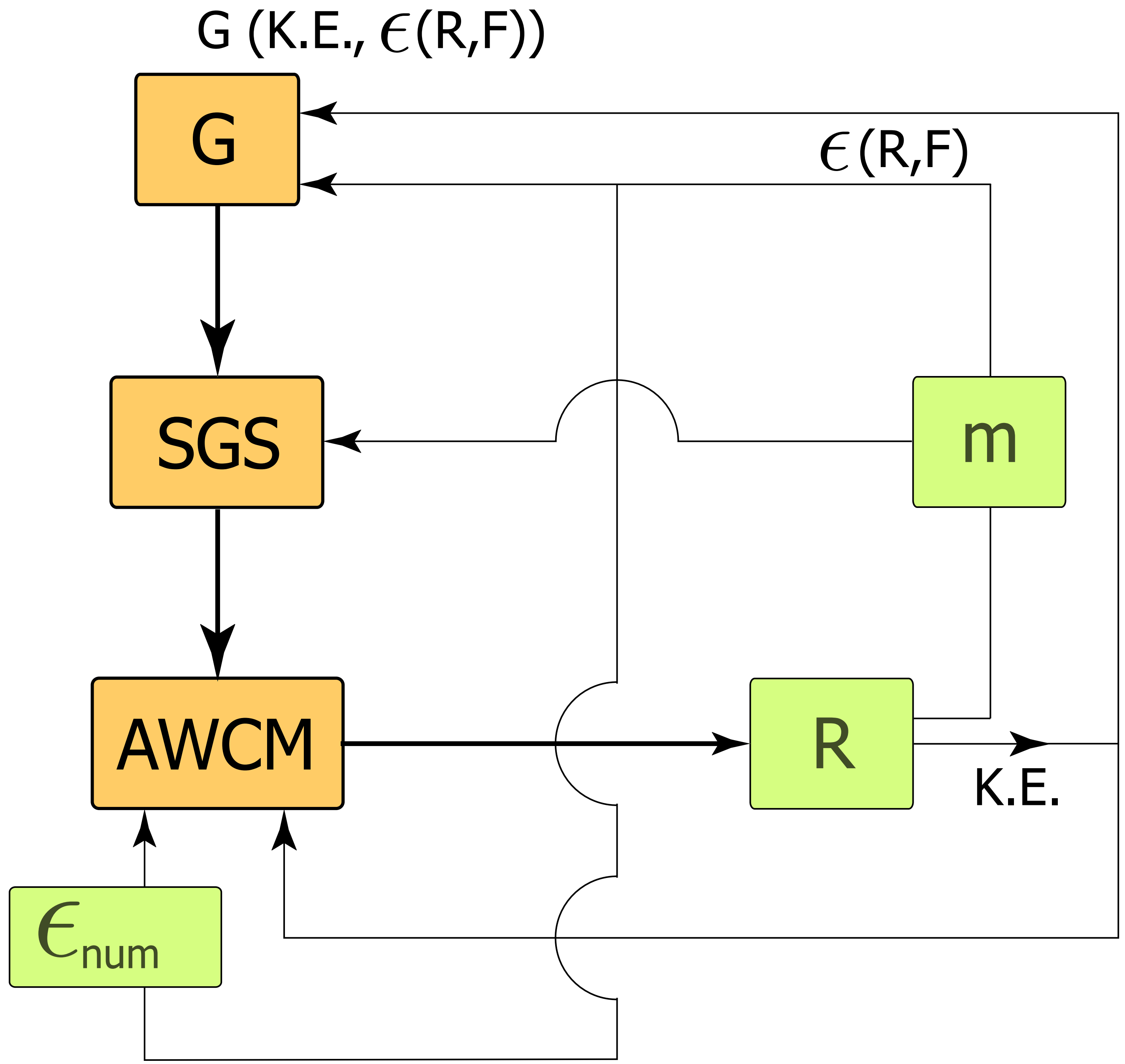


$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle}$

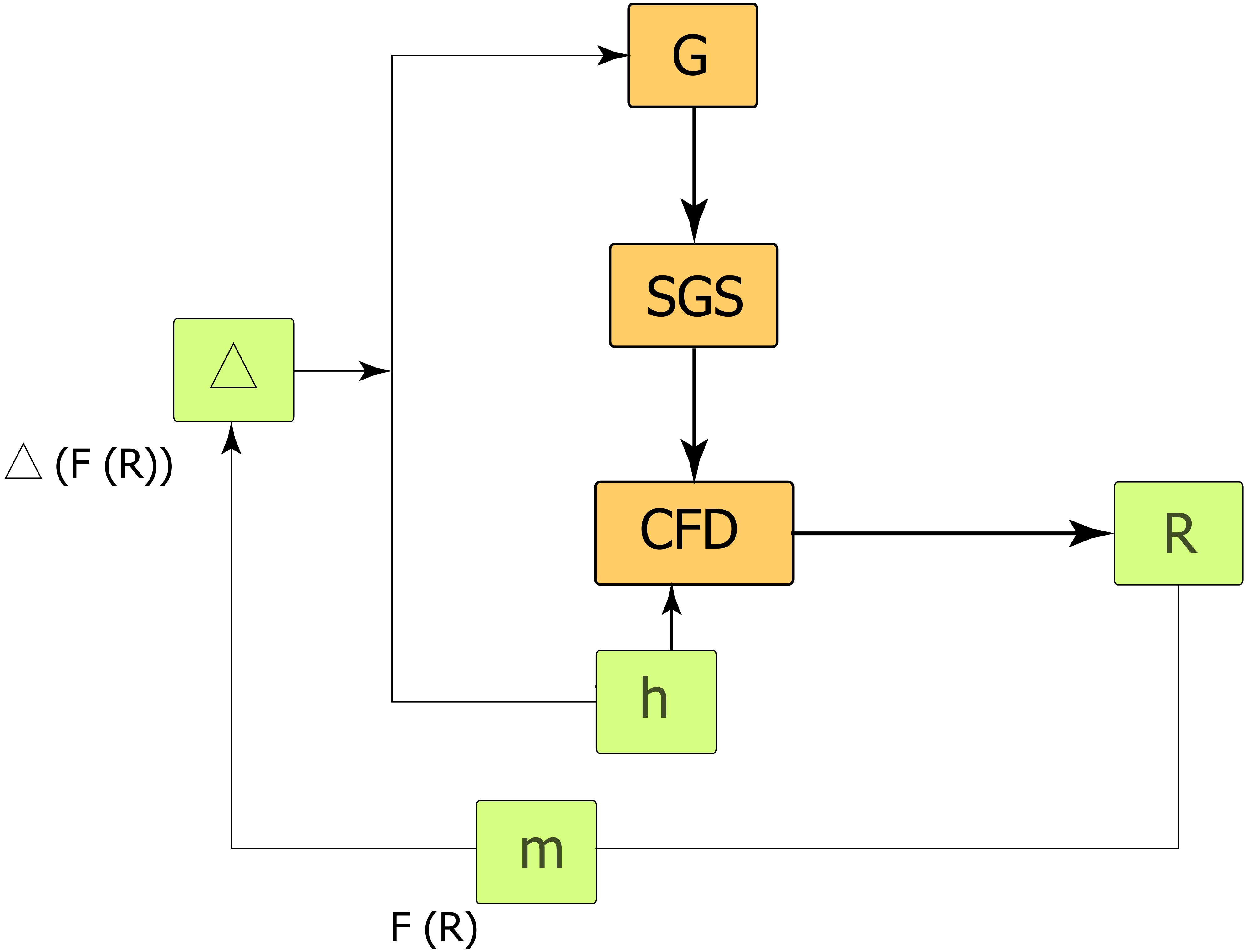
$\nu_\epsilon \in \{0.05, 0.1, 4, 5\}$

Solving Evolution Equation Directly




G (K.E., ϵ(R,F))
G
ϵ(R,F)
SGS
m
AWCM
R
K.E.
ϵ_{num}

model refinement is not limited to SCALES – m-LES



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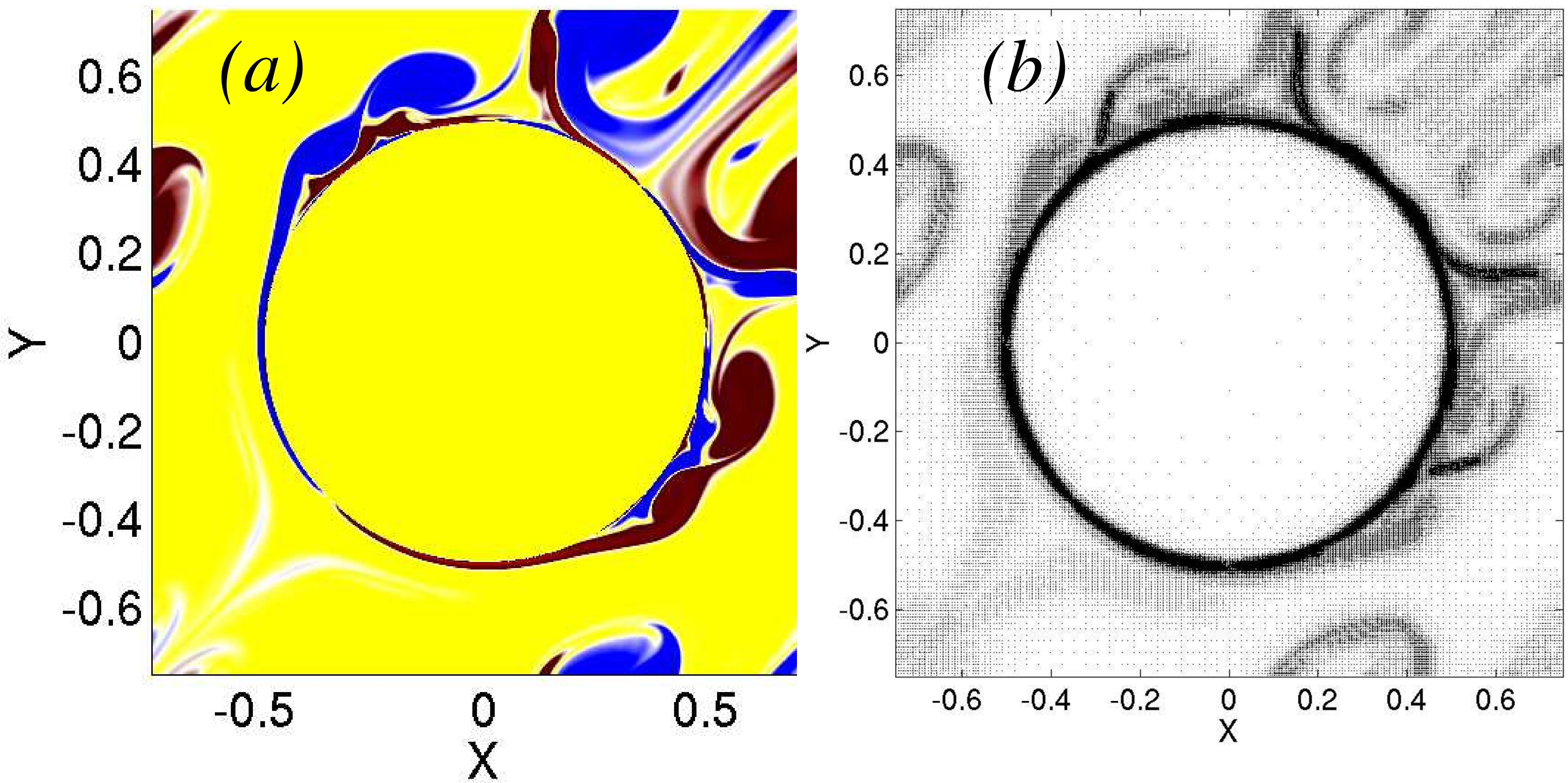
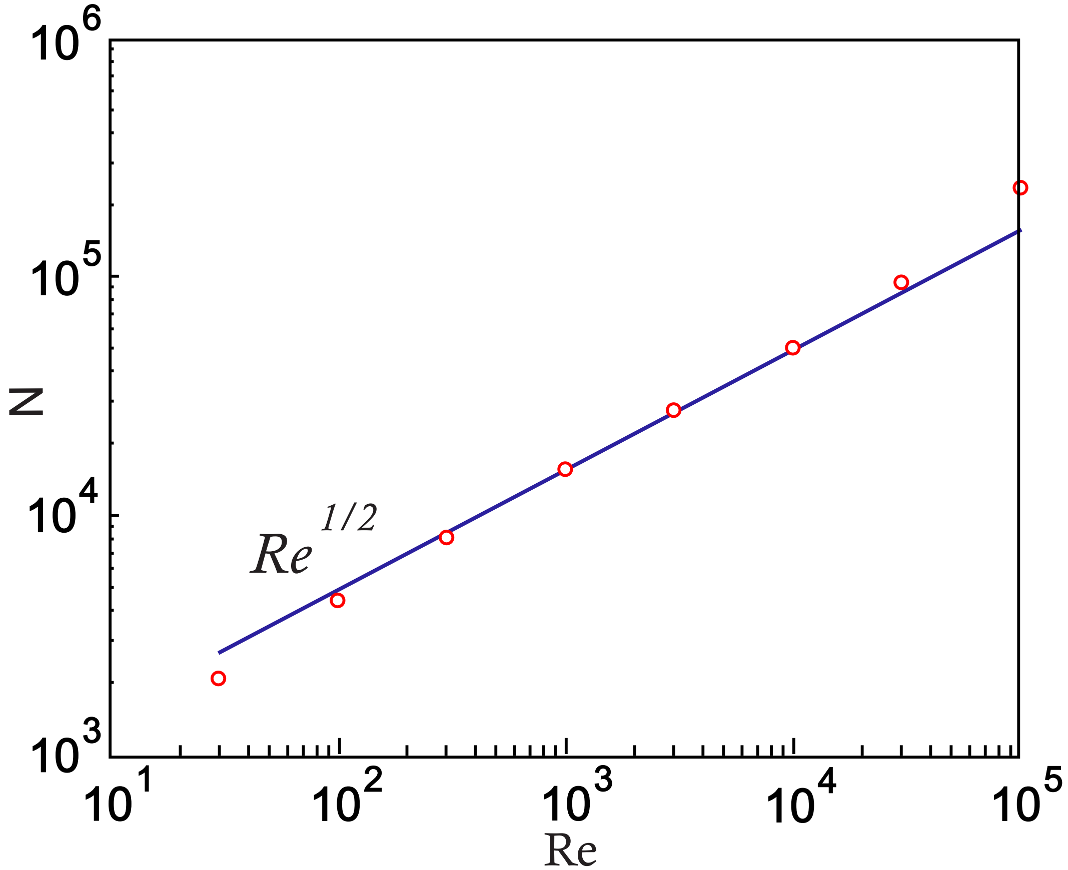
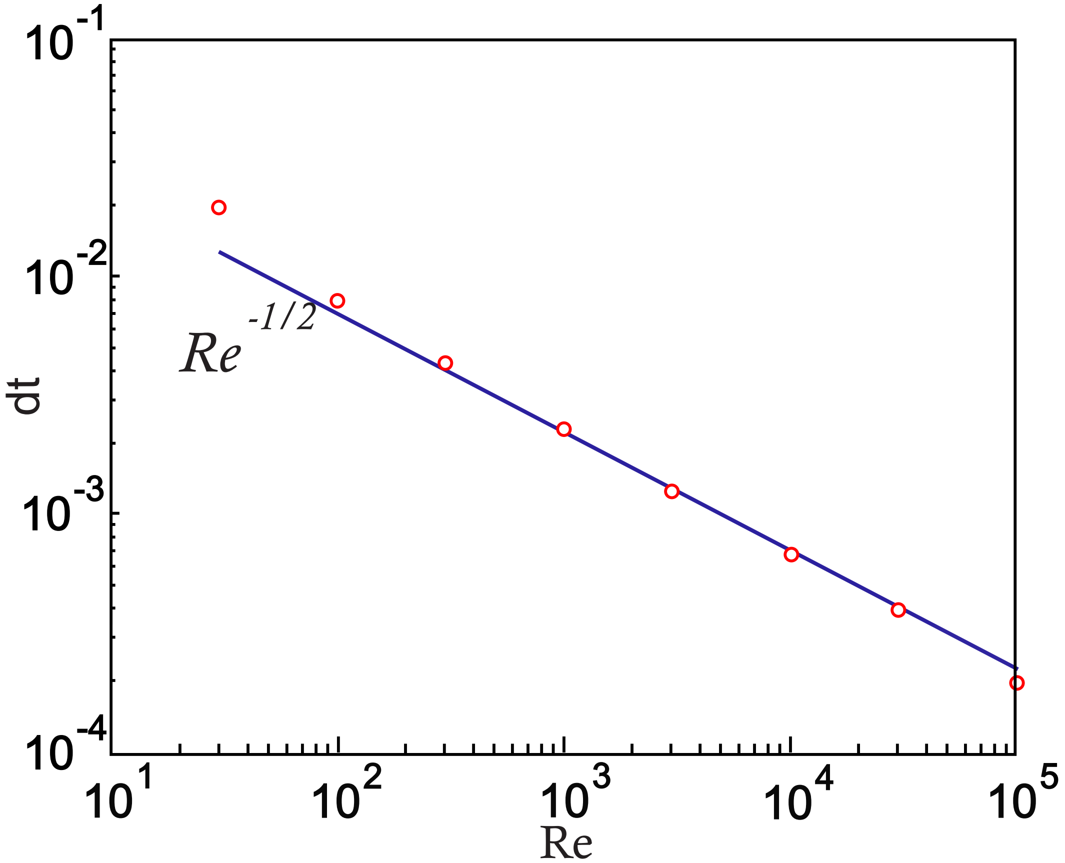
Computational Complexity
or
Reynolds Number Scaling

- Threshold parameter ϵ is significantly small



Computational Complexity*

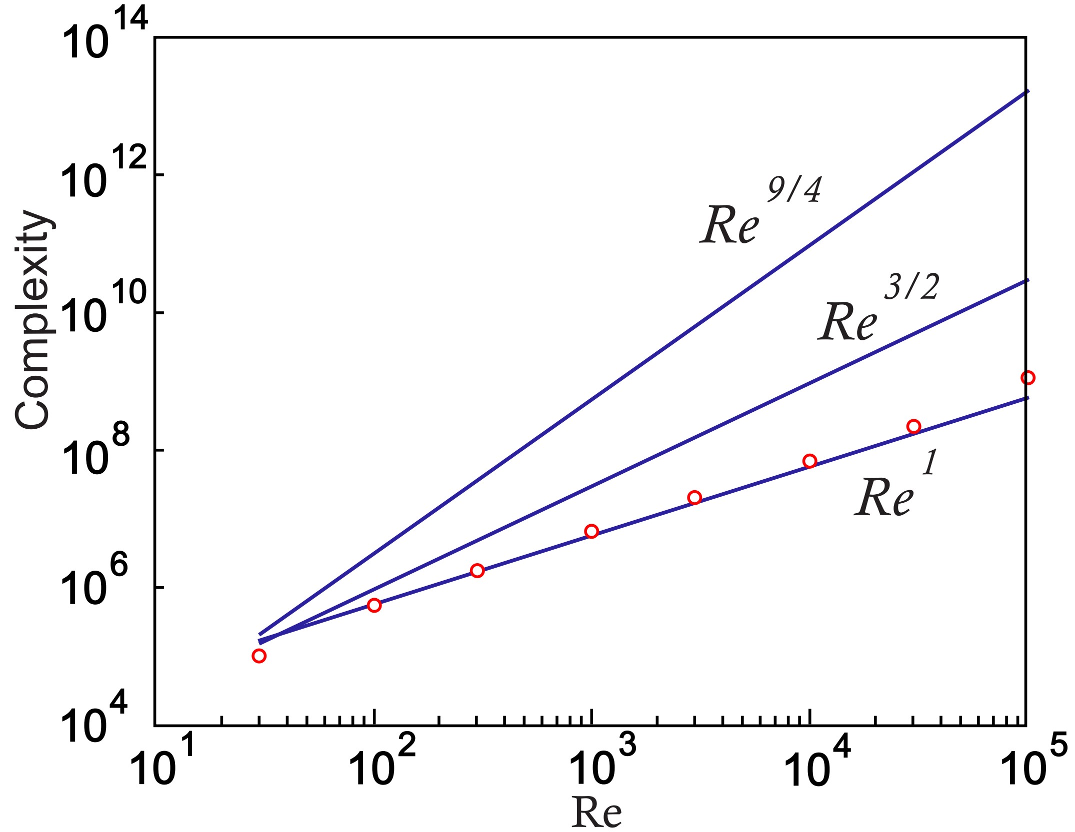




- Studied: $3 \times 10^{1} < Re < 10^{5}$
- Number of grid points ($\mathcal{N}$) scales like $Re^{1/2}$
- Δt scales like $\mathrm{Re}^{-1/2}$

*Kevlahan NKR, Vasilyev OV. 2005. SIAM J. Sci. Comput. 26:1894–915

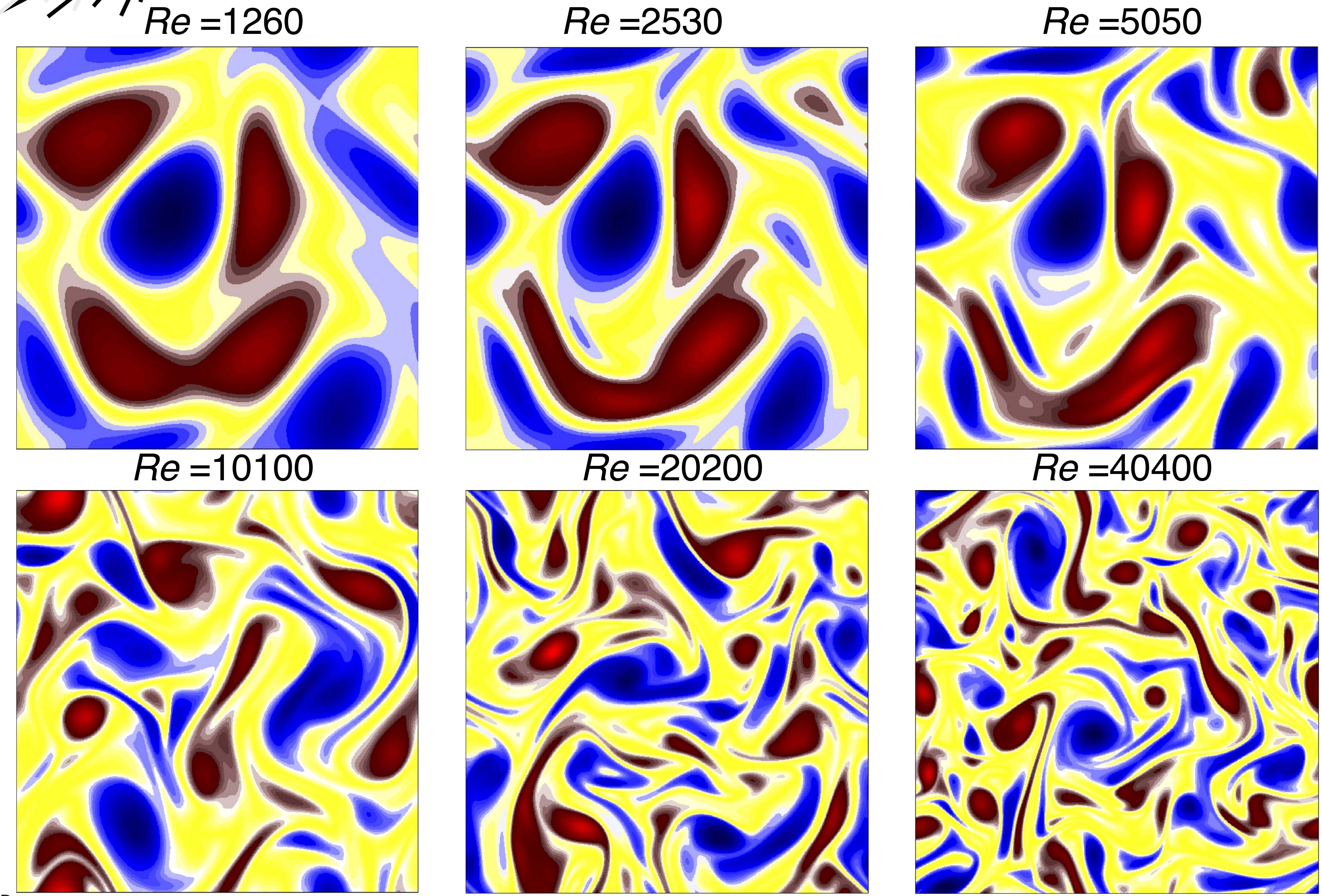
Computational Complexity



- Computational Complexity ($\mathcal{N}/\Delta t$) scales like *Re*.
- Improvement on standard scaling estimate of $Re^{9/4}$.
  (for 2D turbulence based on Kolmogorov scale)

**Kevlahan NKR, Vasilyev OV. 2005. SIAM J. Sci. Comput. 26:1894–915*

Space–time Modes in 2D Turbulence*



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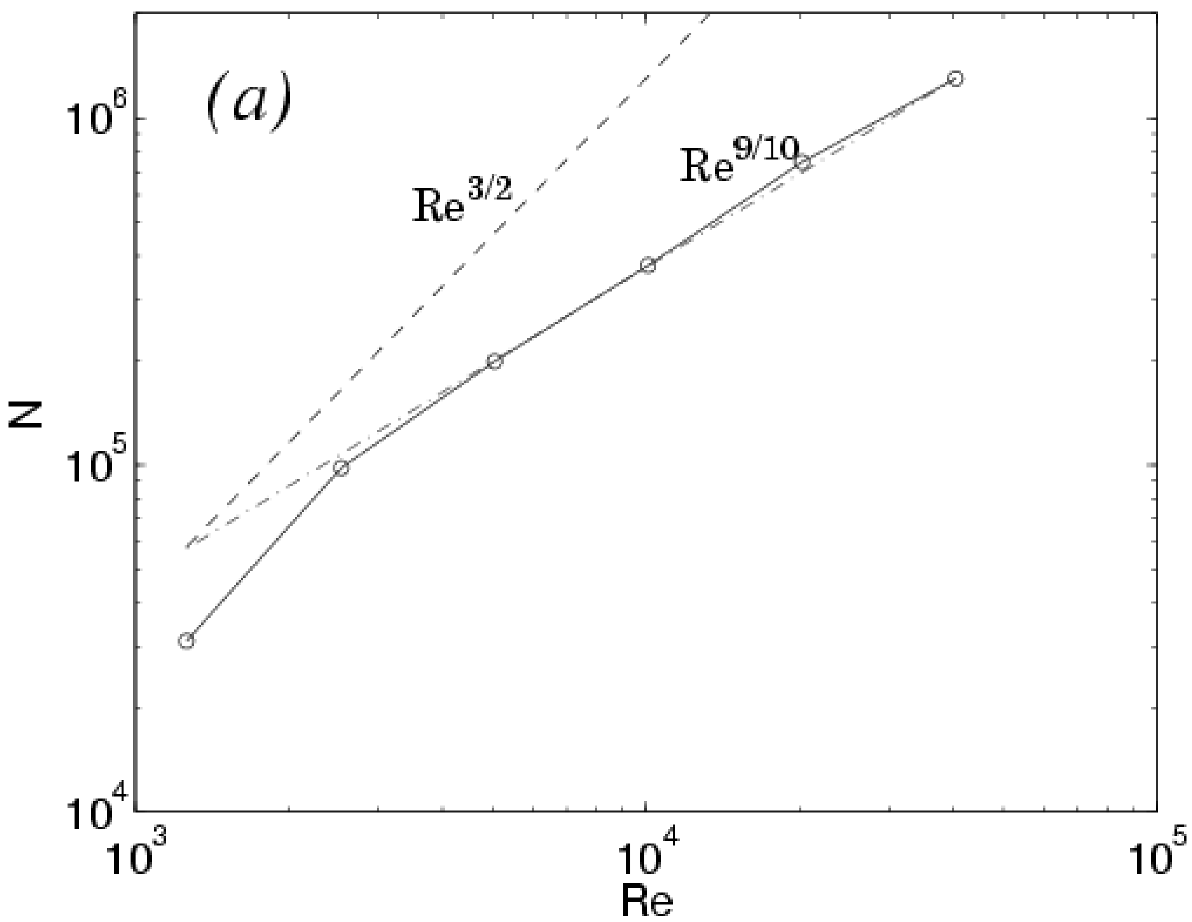
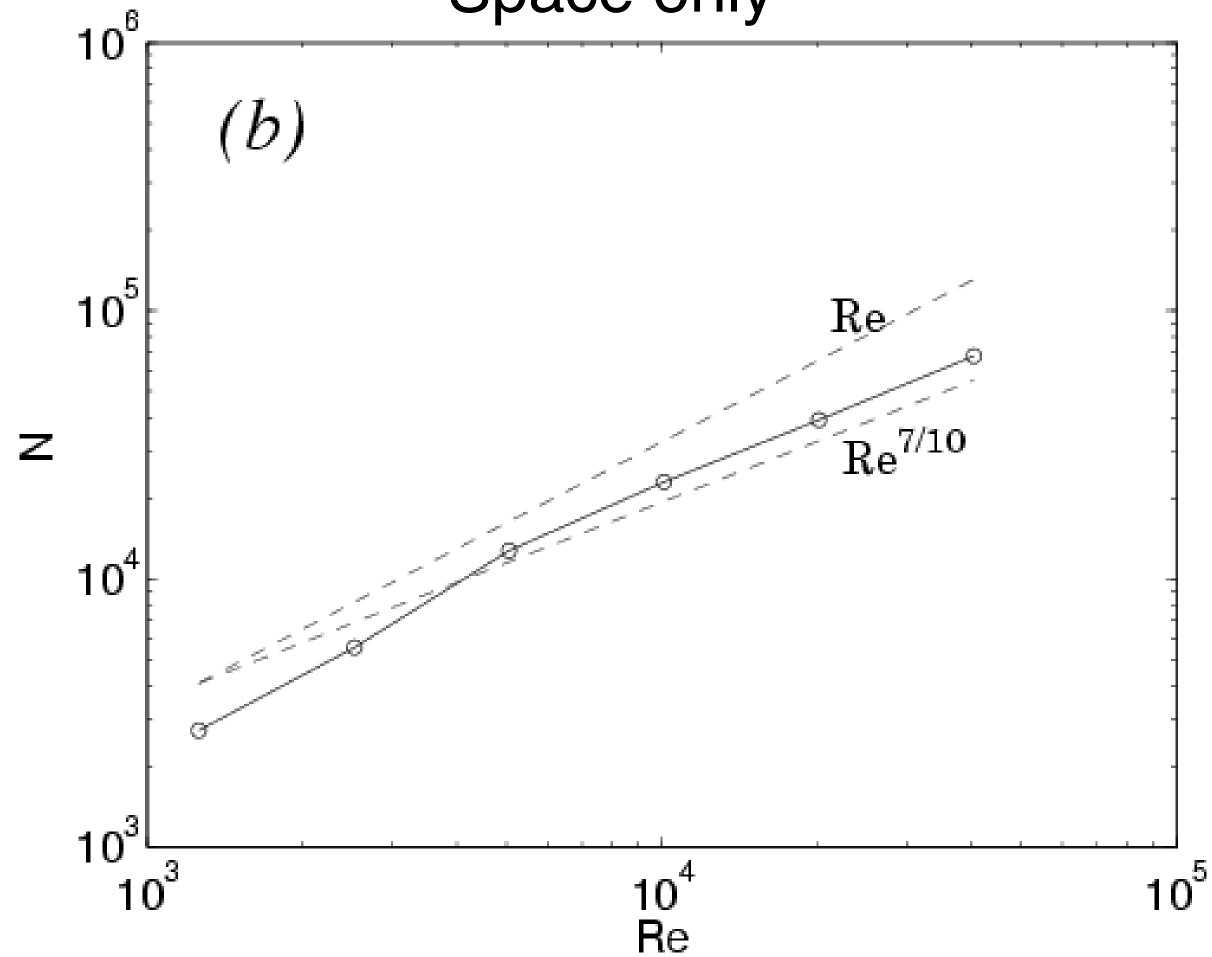
Space–time Modes in 2D Turbulence*





- 2D decaying turbulence 1 260 ≤ *Re* ≤ 40 400.
- The non-intermittent computational estimate: $\mathcal{N} \sim Re^{3/2}$
- Mathematical upper bound: $\mathcal{N} \sim Re^{2}$

Kevlahan NKR, Alam JM, Vasilyev OV. 2007. J. Fluid Mech. 570:217–26

Are similar trends observed for 3-D turbulence?

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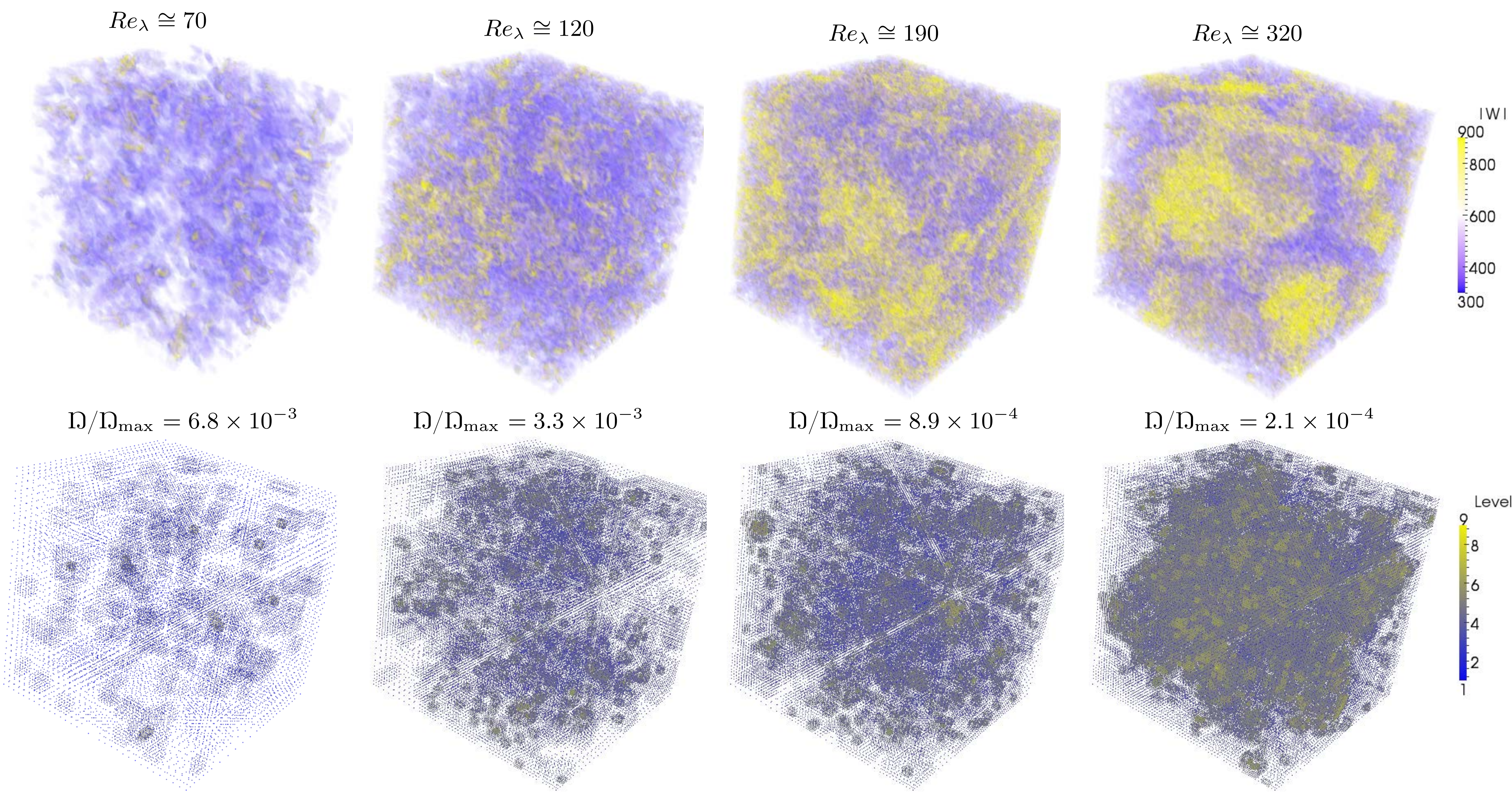
Time-Averaged Energy Spectra – CVS and SCALES

Linear Forcing Coefficient : $Q = 6.\bar{6}$

Taylor micro-scale Reynolds number : $Re_\lambda \cong 70, 120, 190, 320$ $\nu = 0.09, 0.035, 0.015, 0.006$

Adaptive Grid corresponds to $256^3, 512^3, 1024^3, 2048^3$ (at highest level of resolution) $J_{\max} = 6, 7, 8, 9$ $\epsilon = 0.2, 0.43$



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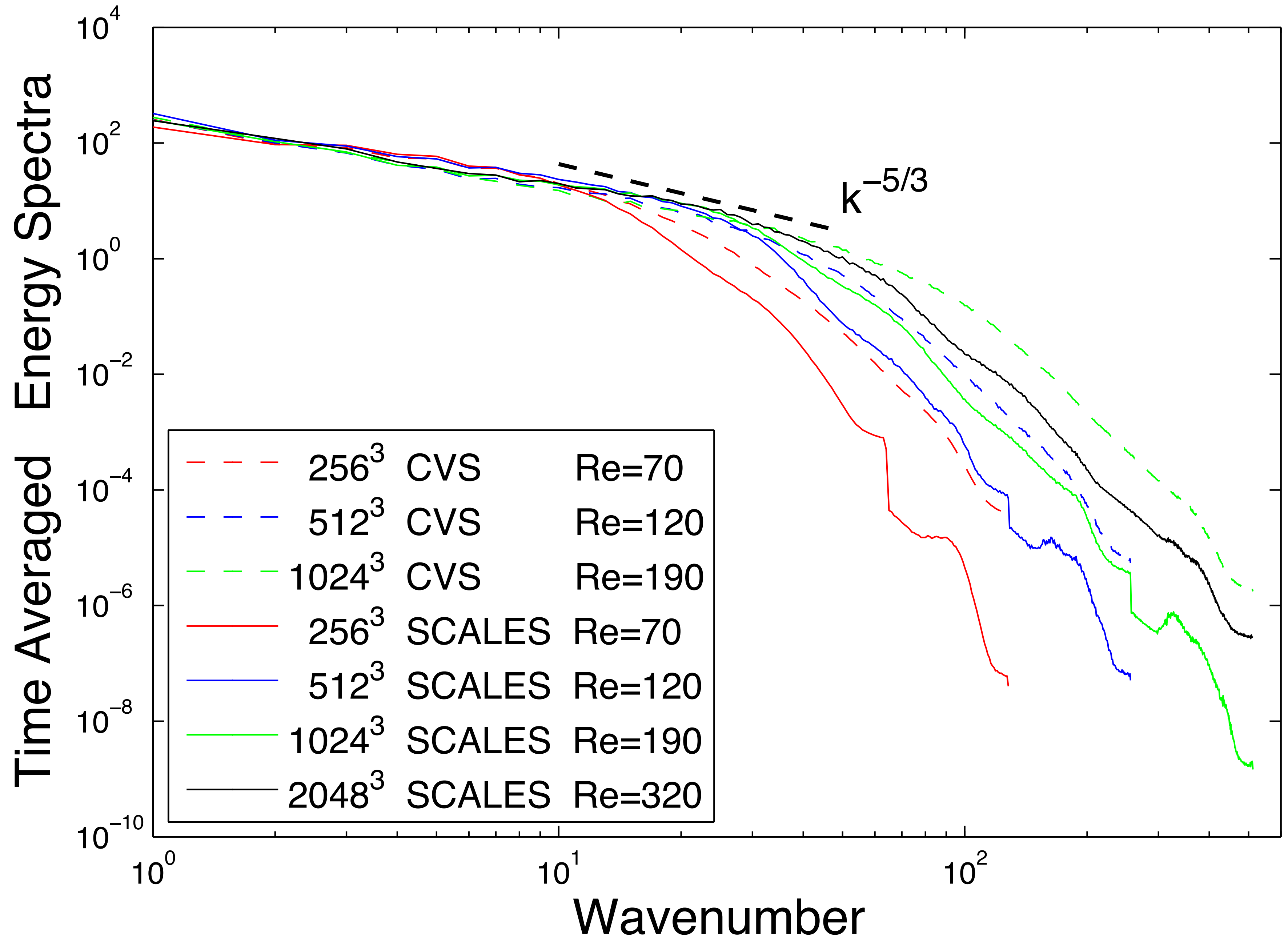
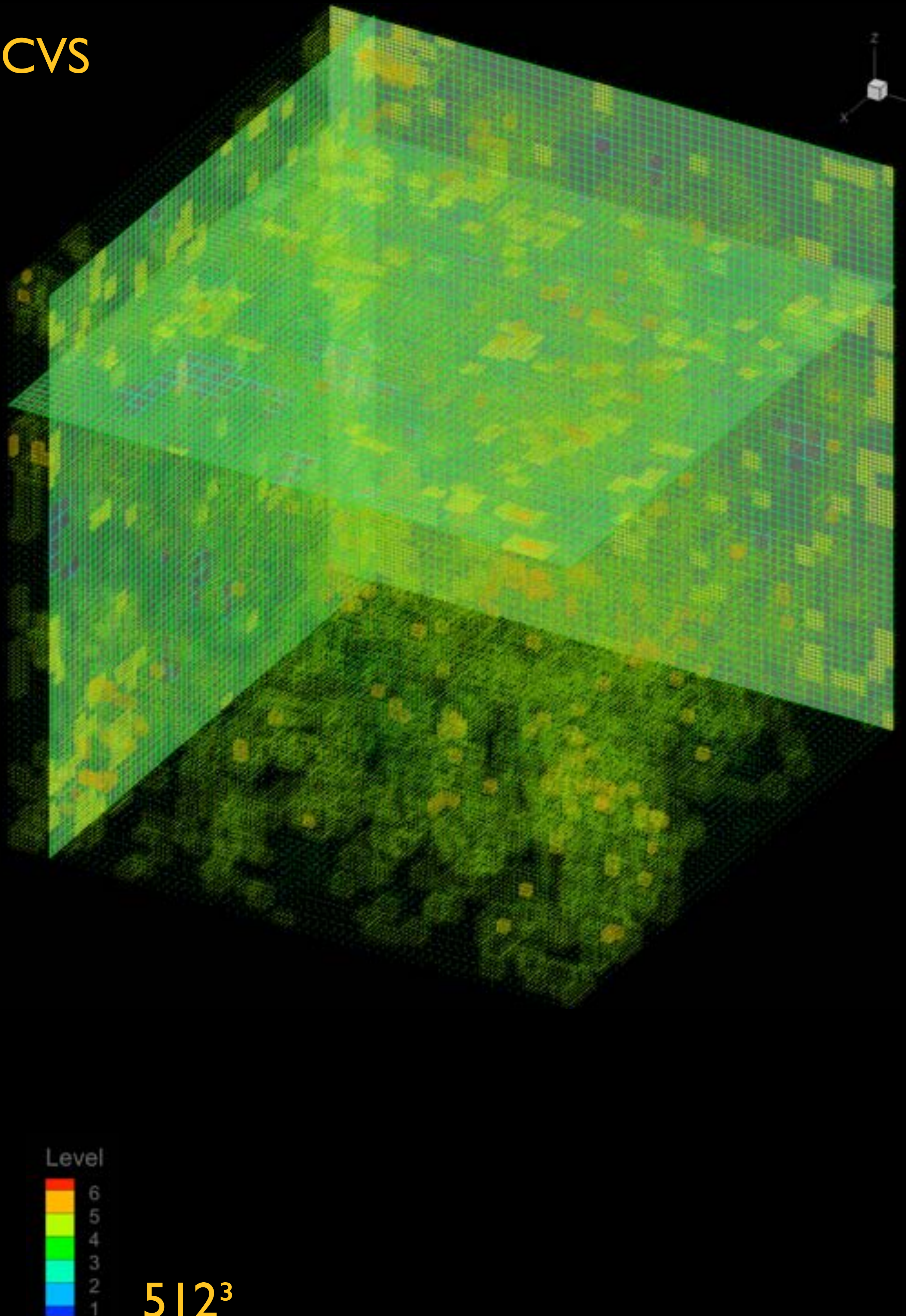
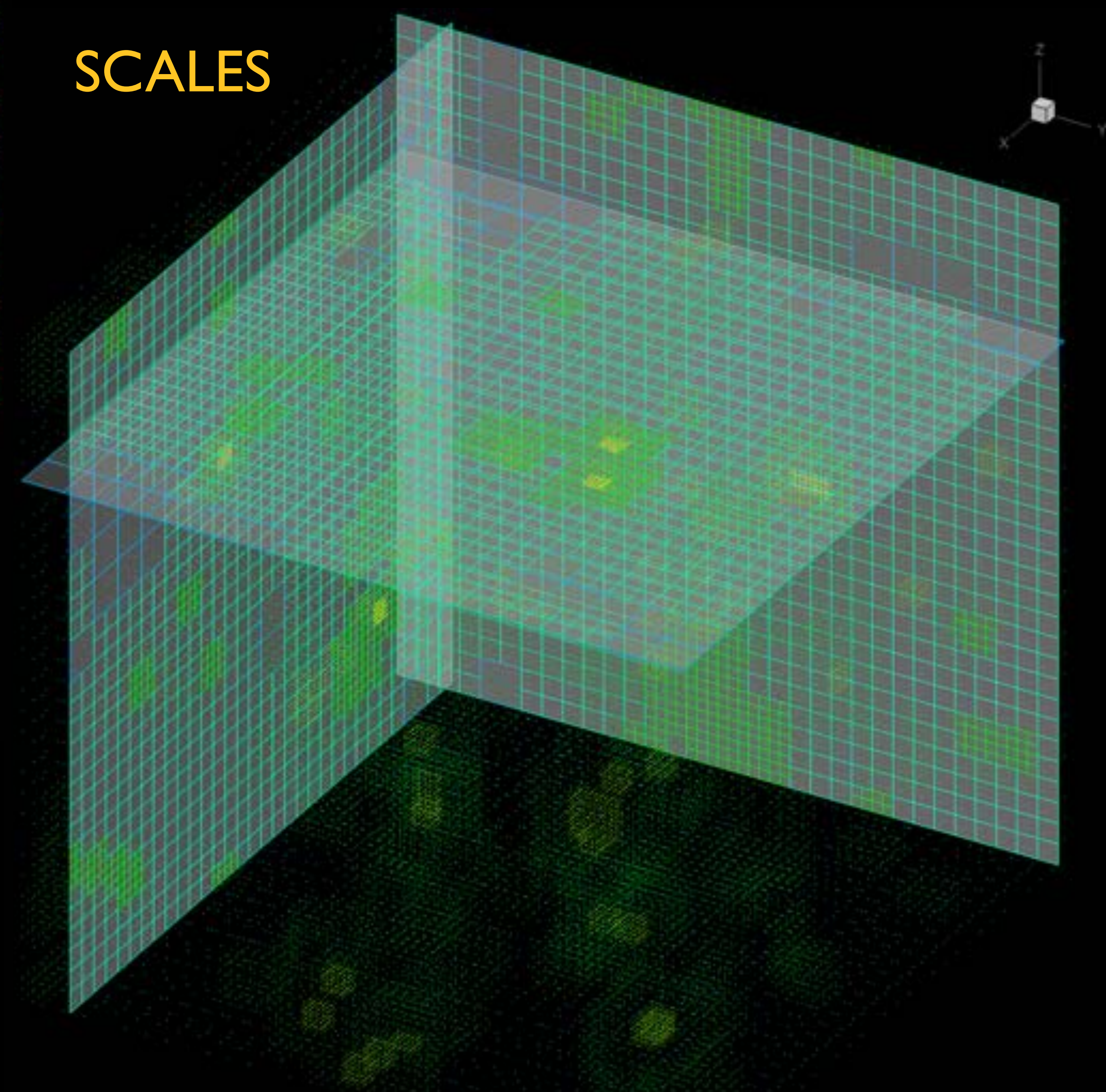


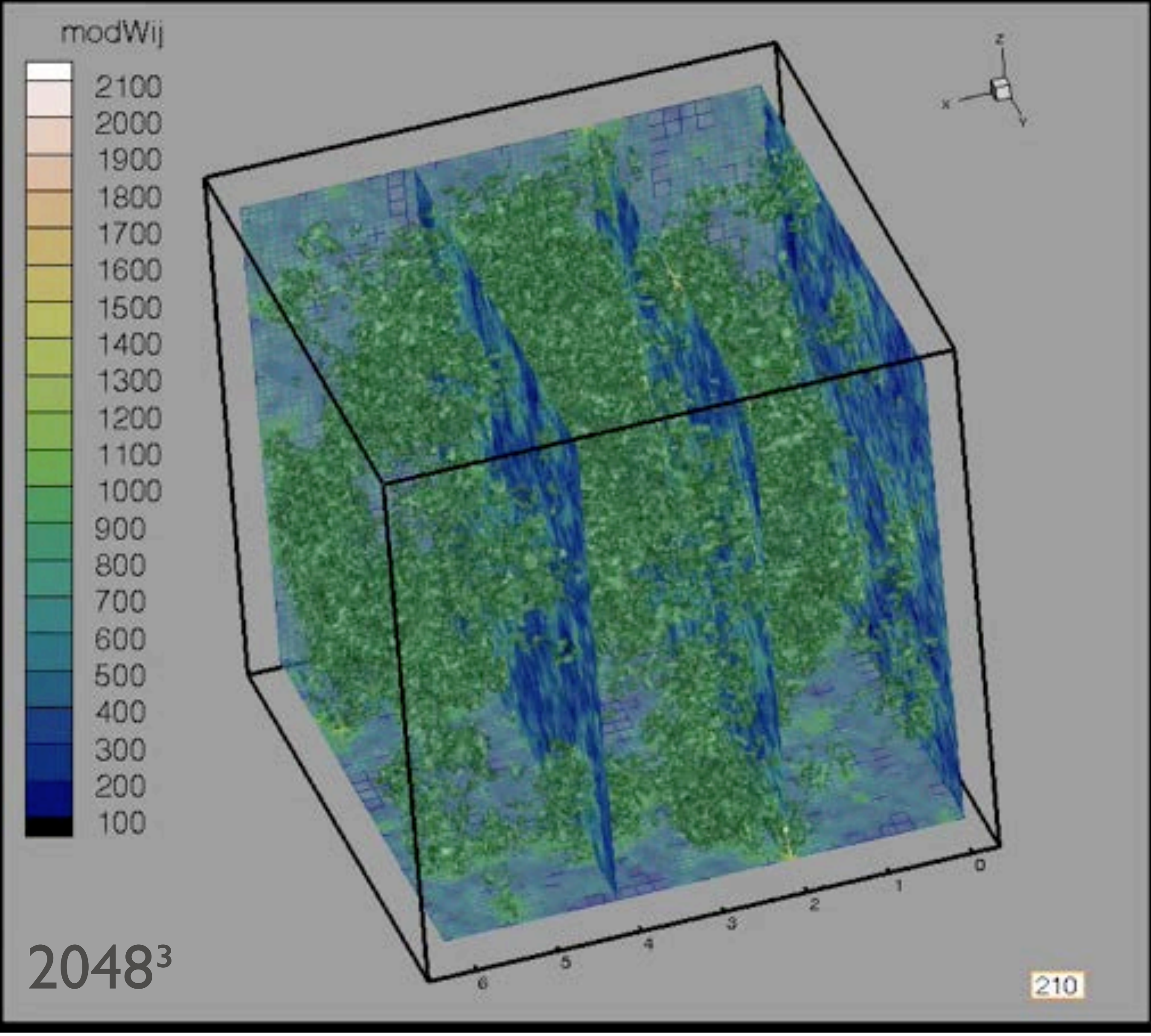
CVS



SCALES



512^3


modWij
2100
2000
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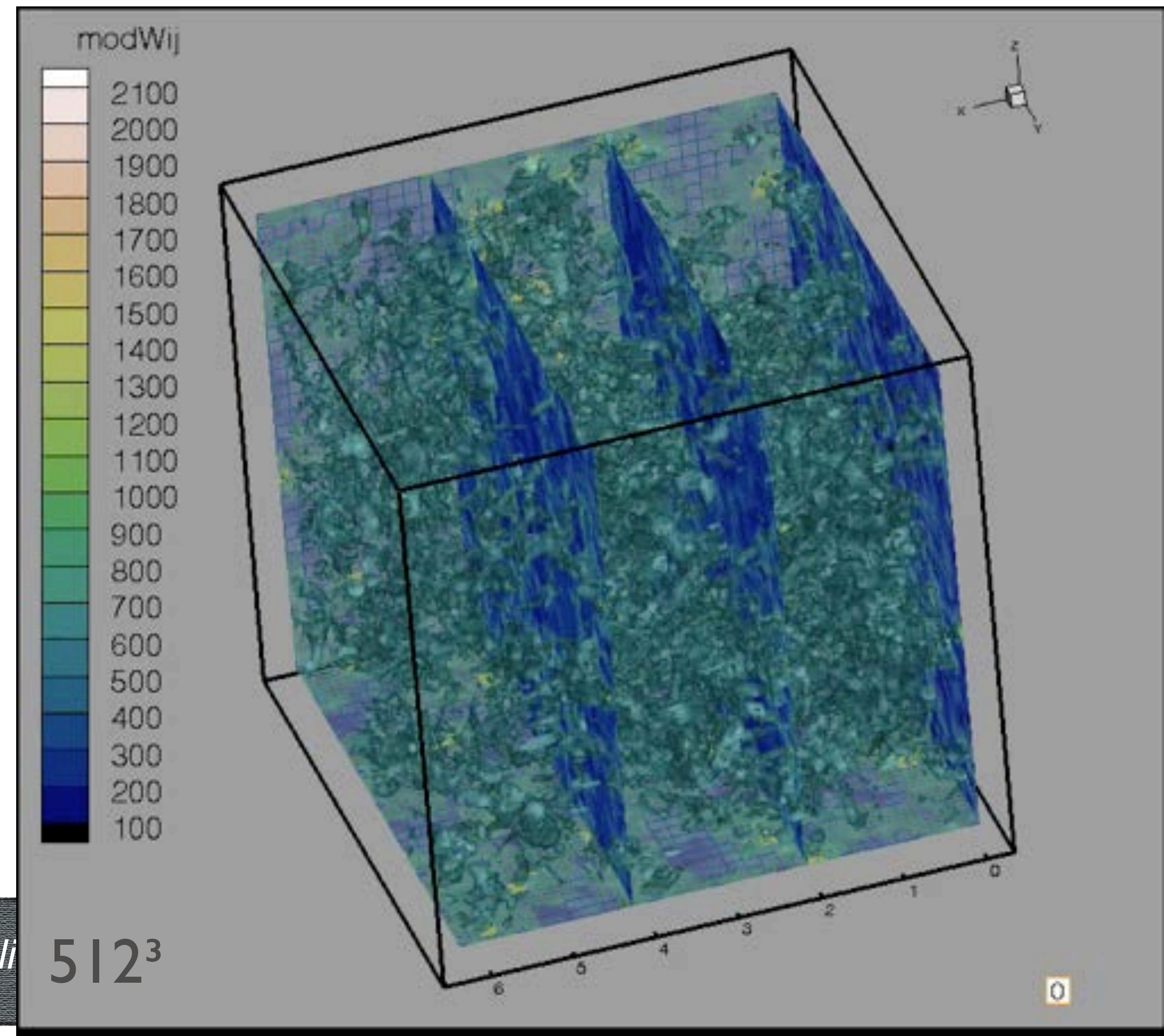
200
100
2048^3
210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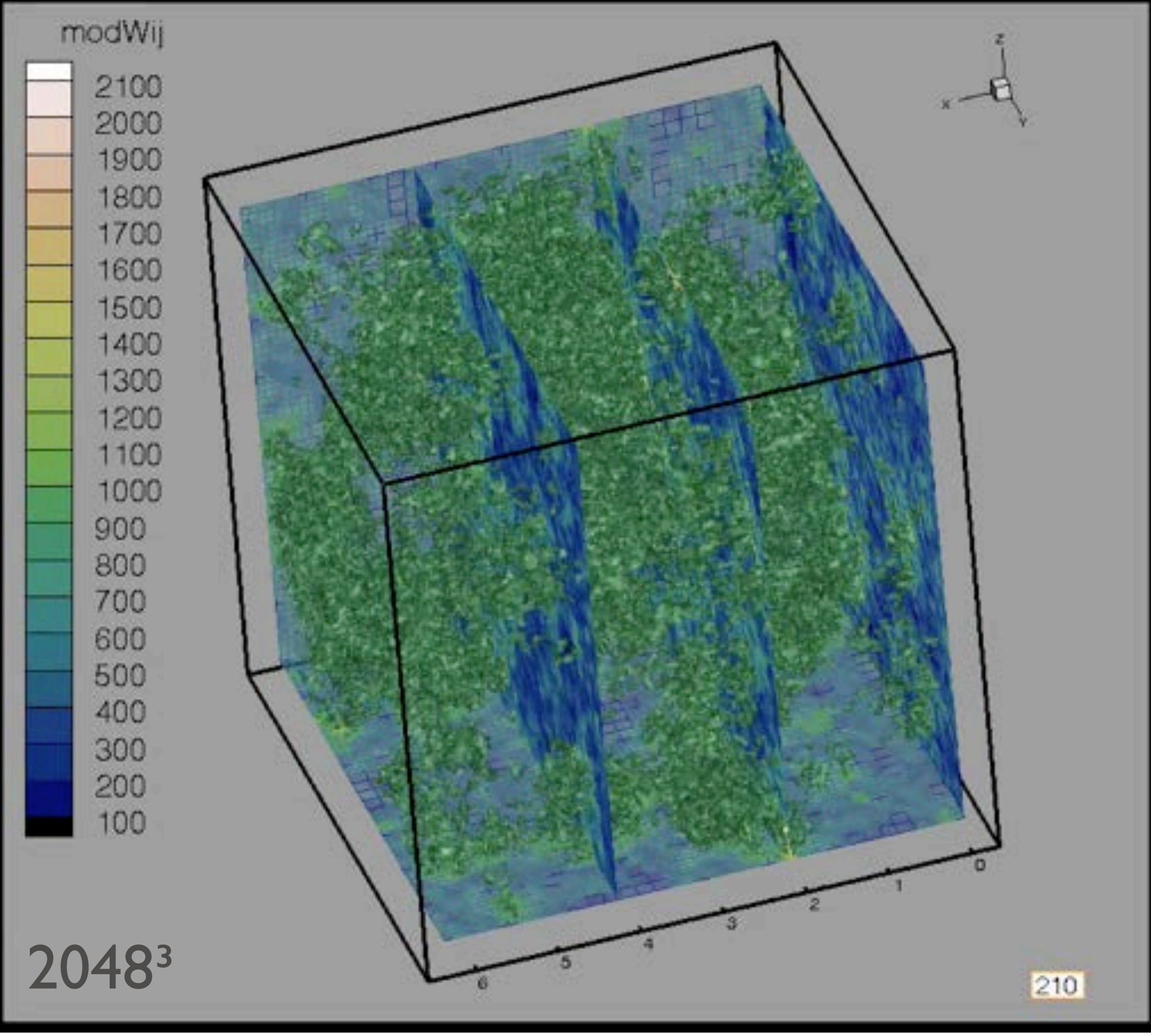
512^3
0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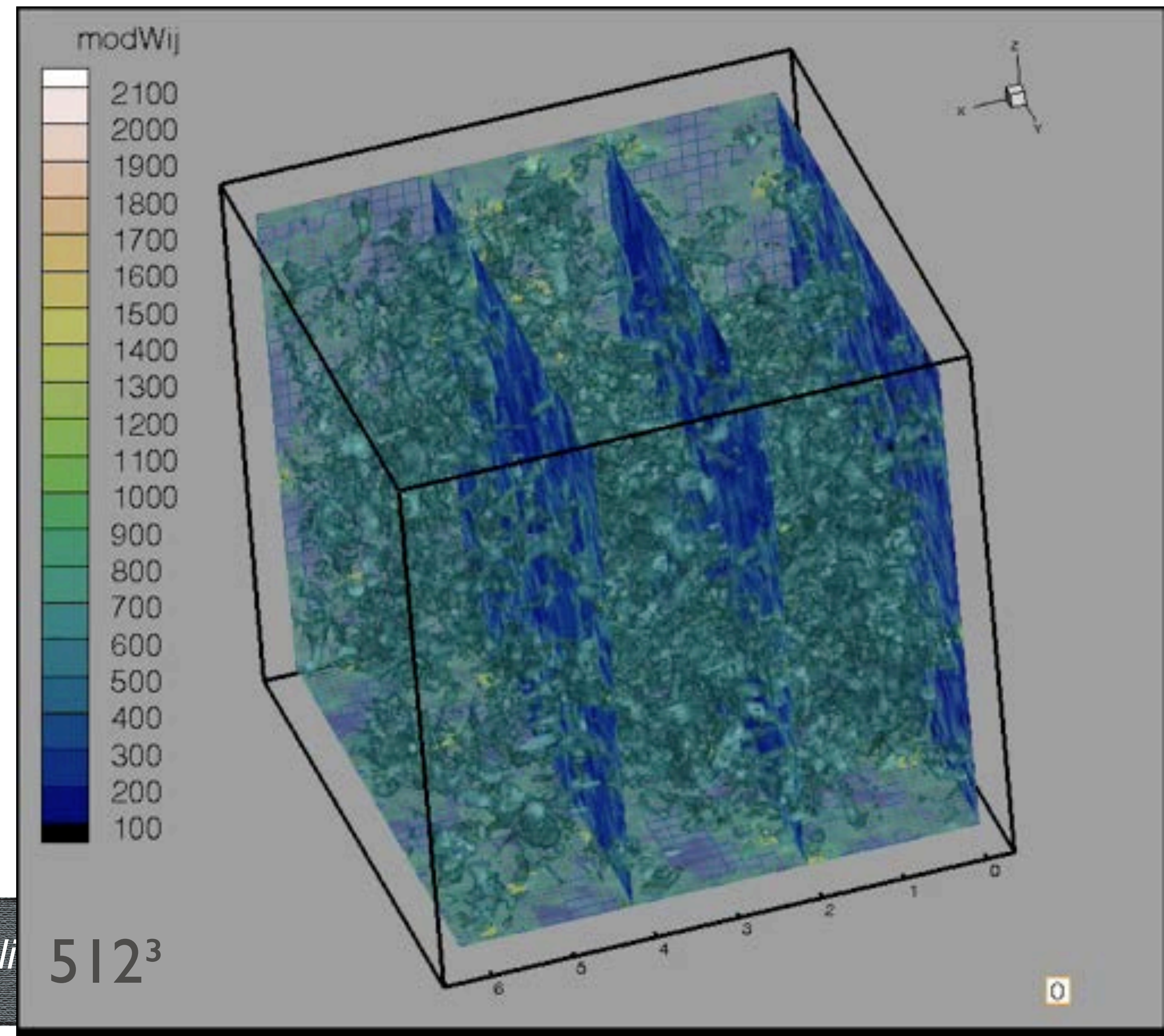
2
3
4
5
6
2048^3
210


modWij
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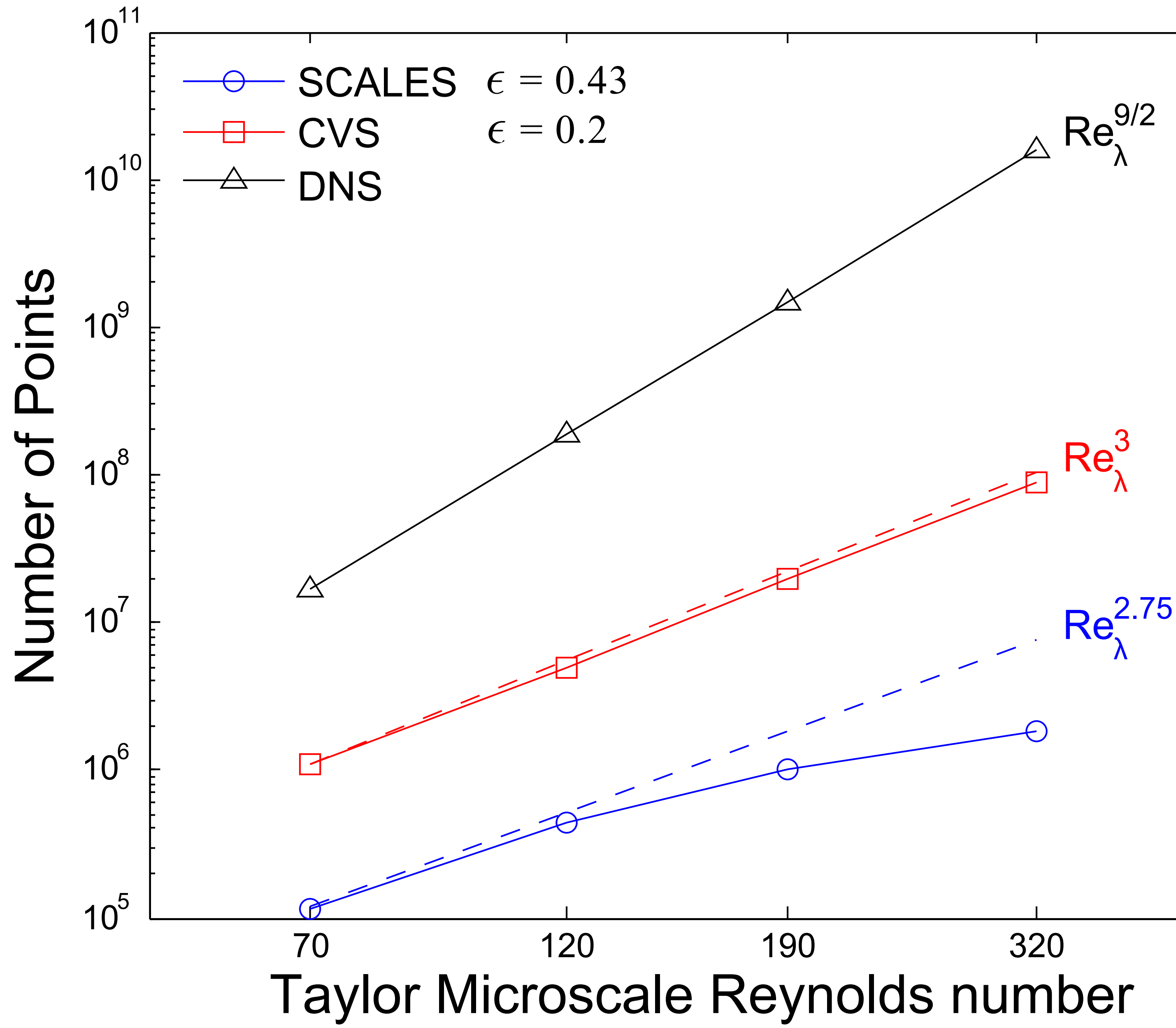
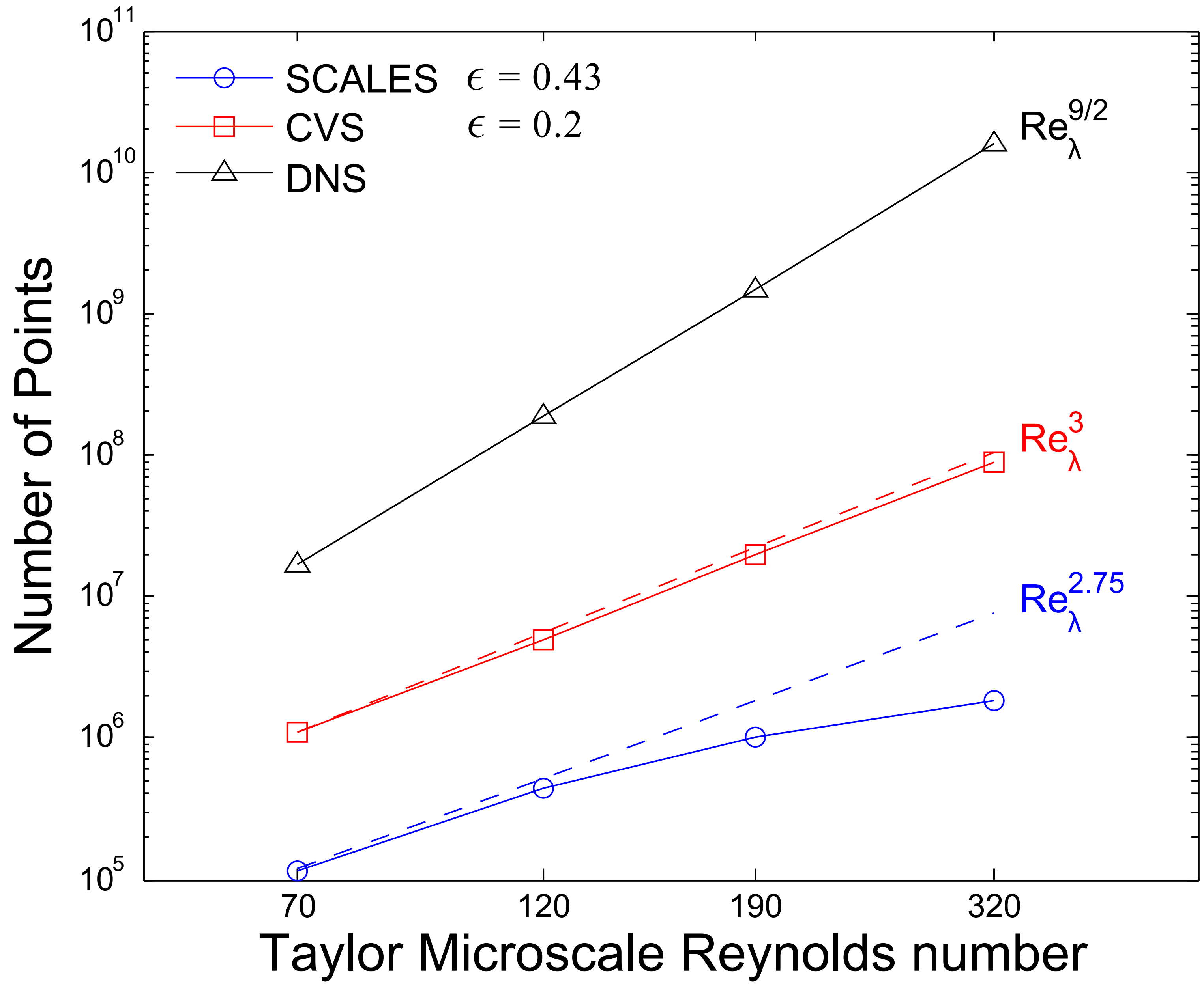
100
0
1
2
3
4
5
6
512^3
0

Computational Complexity –


SCALES $\epsilon = 0.43$
CVS $\epsilon = 0.2$
DNS
$Re_\lambda^{9/2}$
Re_λ^{3}
$Re_\lambda^{2.75}$
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
70
120
190
320
Number of Points
Taylor Microscale Reynolds number



$Re^{3D_F/(D_F+1)}$

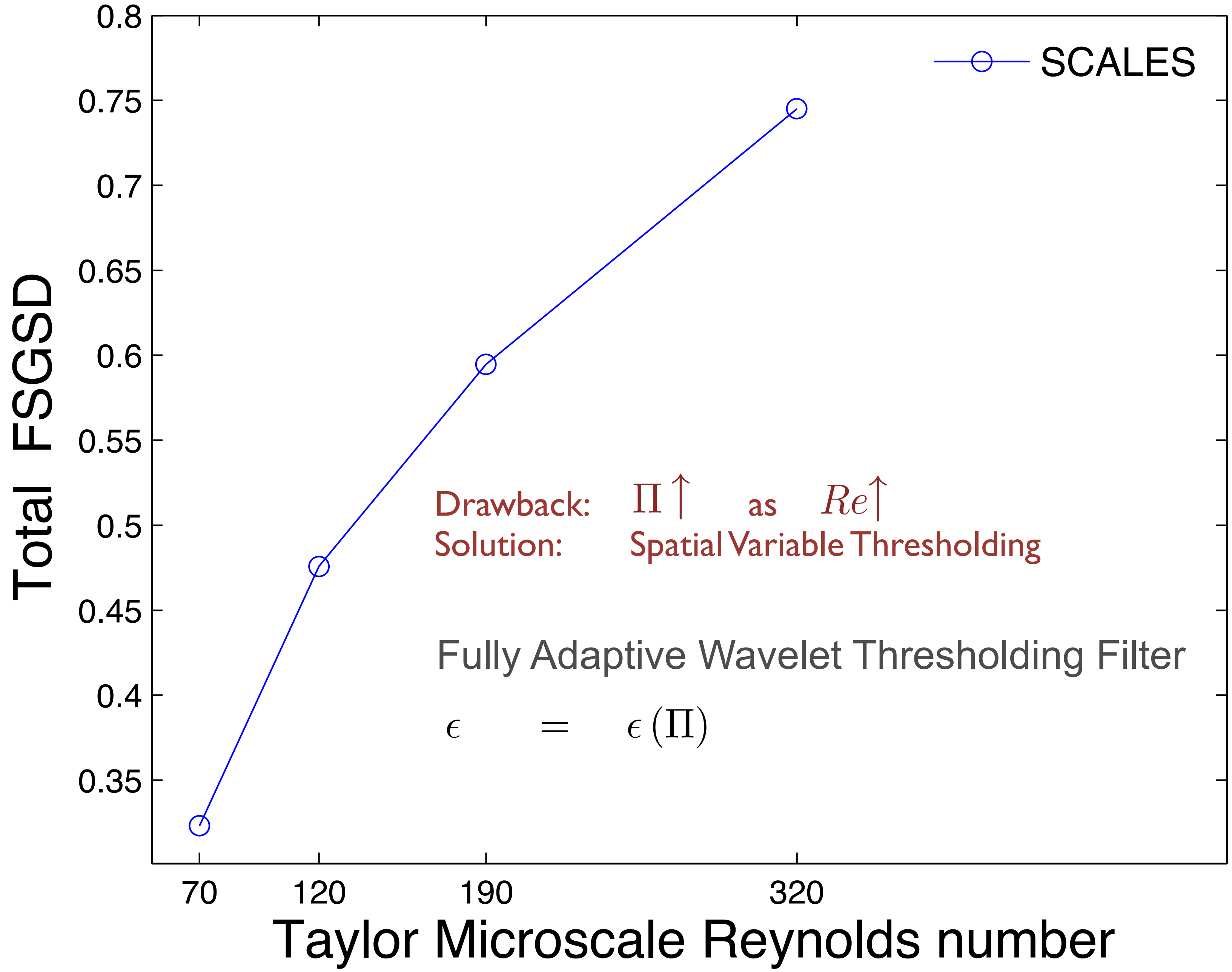
$D_F \leq 3$

$D_{F_{\text{CVS}}} \lesssim 1$

$D_{F_{\text{SCALES}}} \lesssim \frac{11}{13} = 0.\overline{846153}$

$D_{F_{\text{SCALES}}} < 1$

$$\overline{\langle \mathcal{F} \rangle} = \overline{\left(\frac{\langle \Pi \rangle}{\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle} \right)}$$



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Linear Forcing Coefficient : $Q = 6.\bar{6}$

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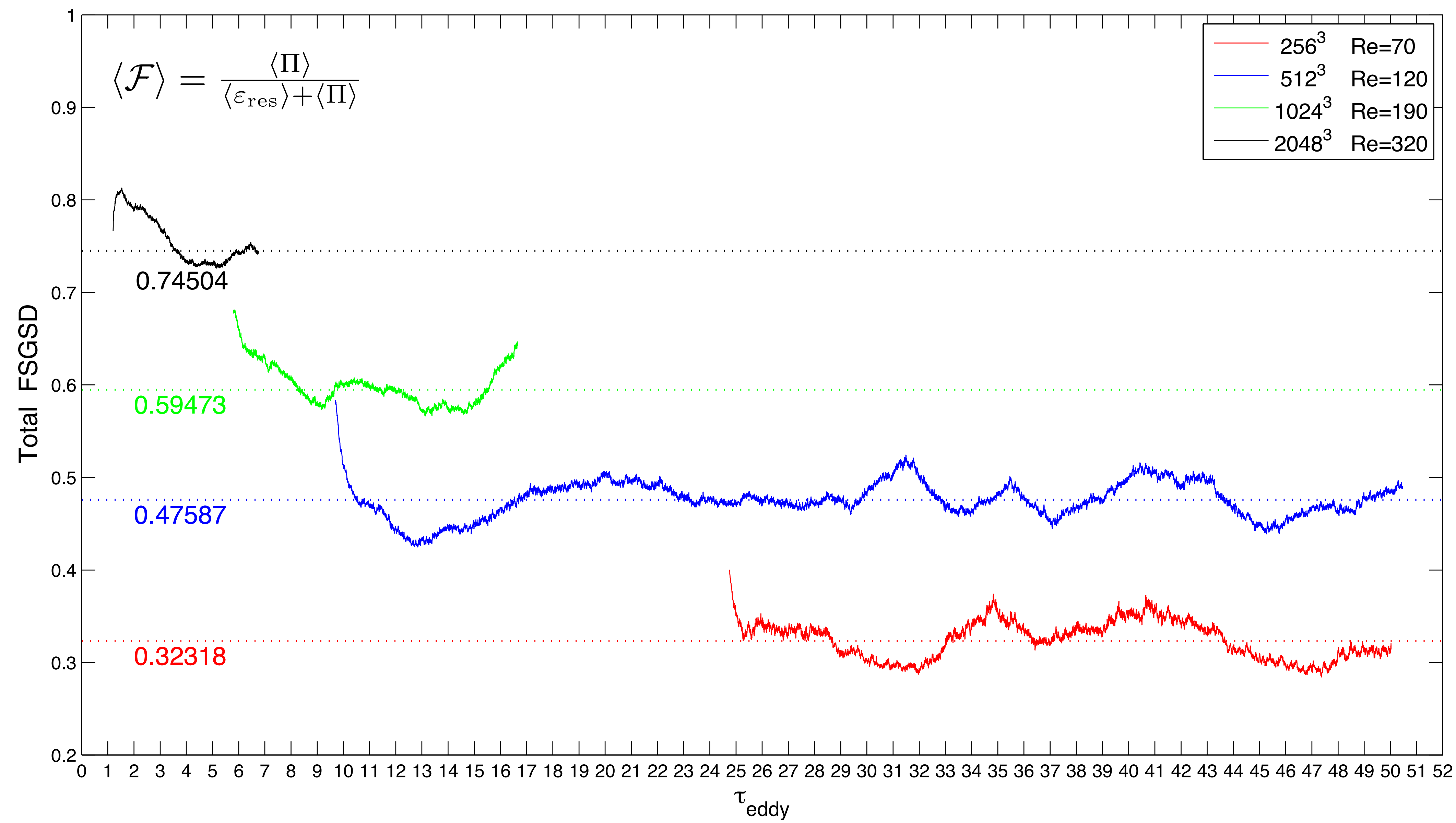
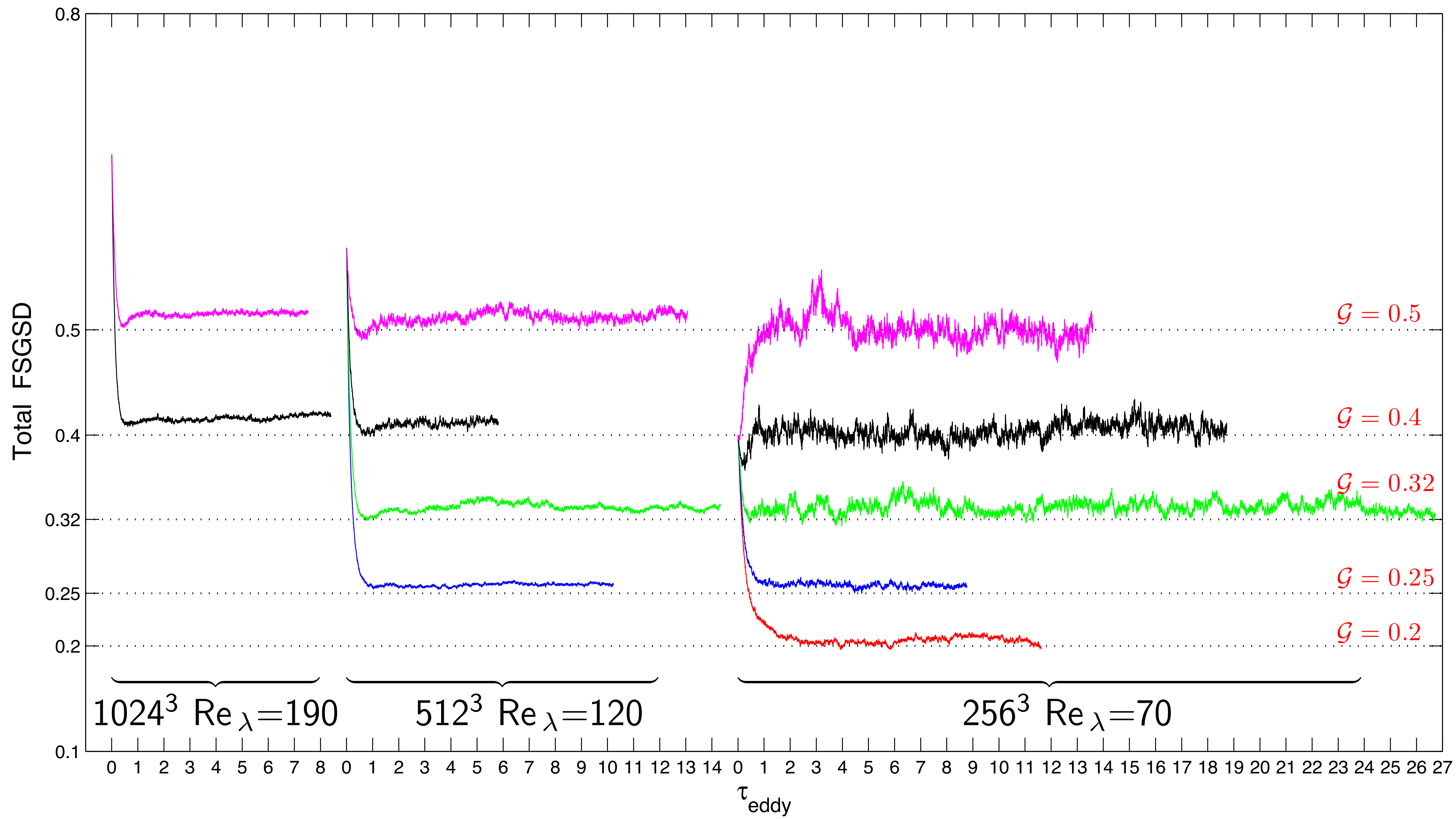
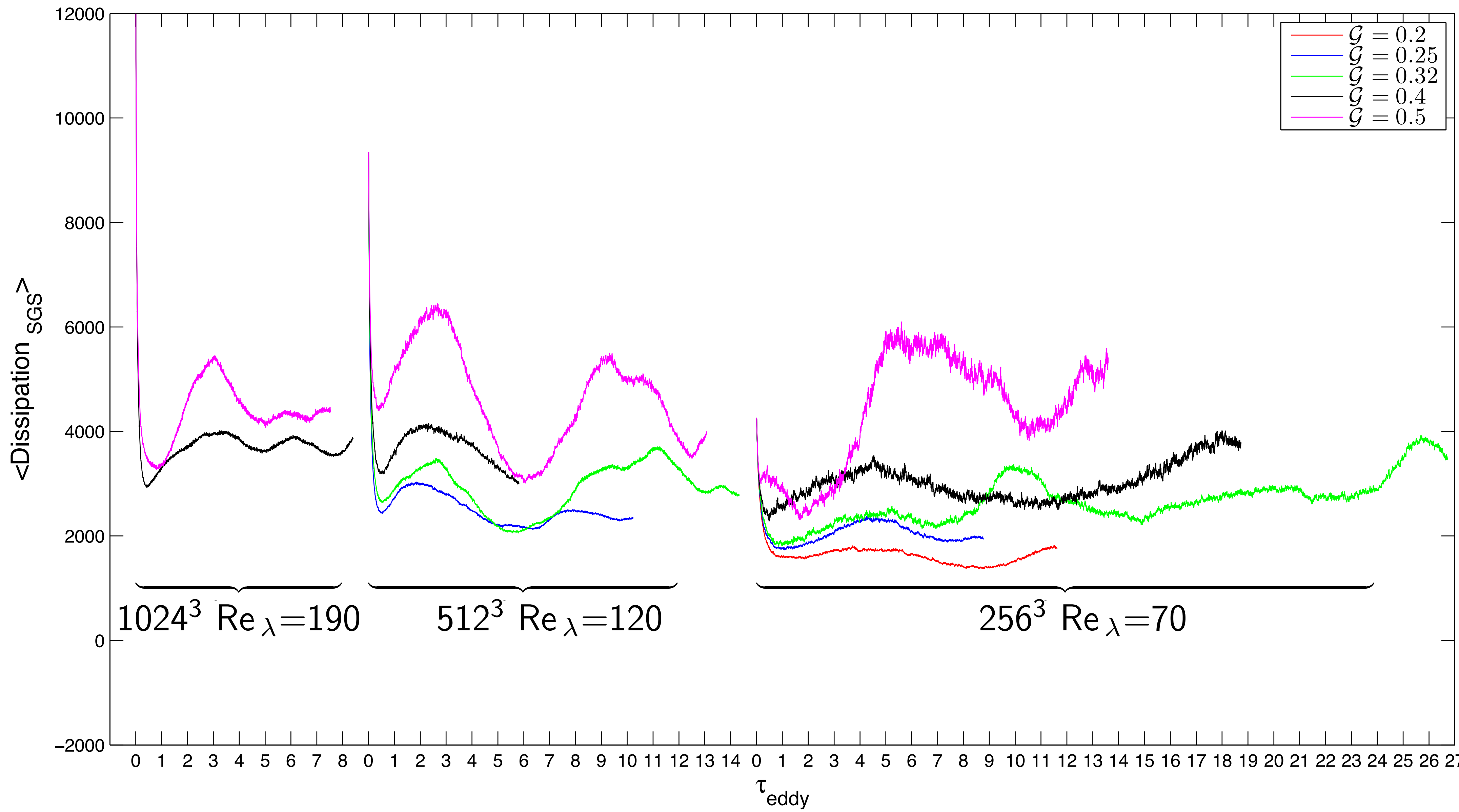


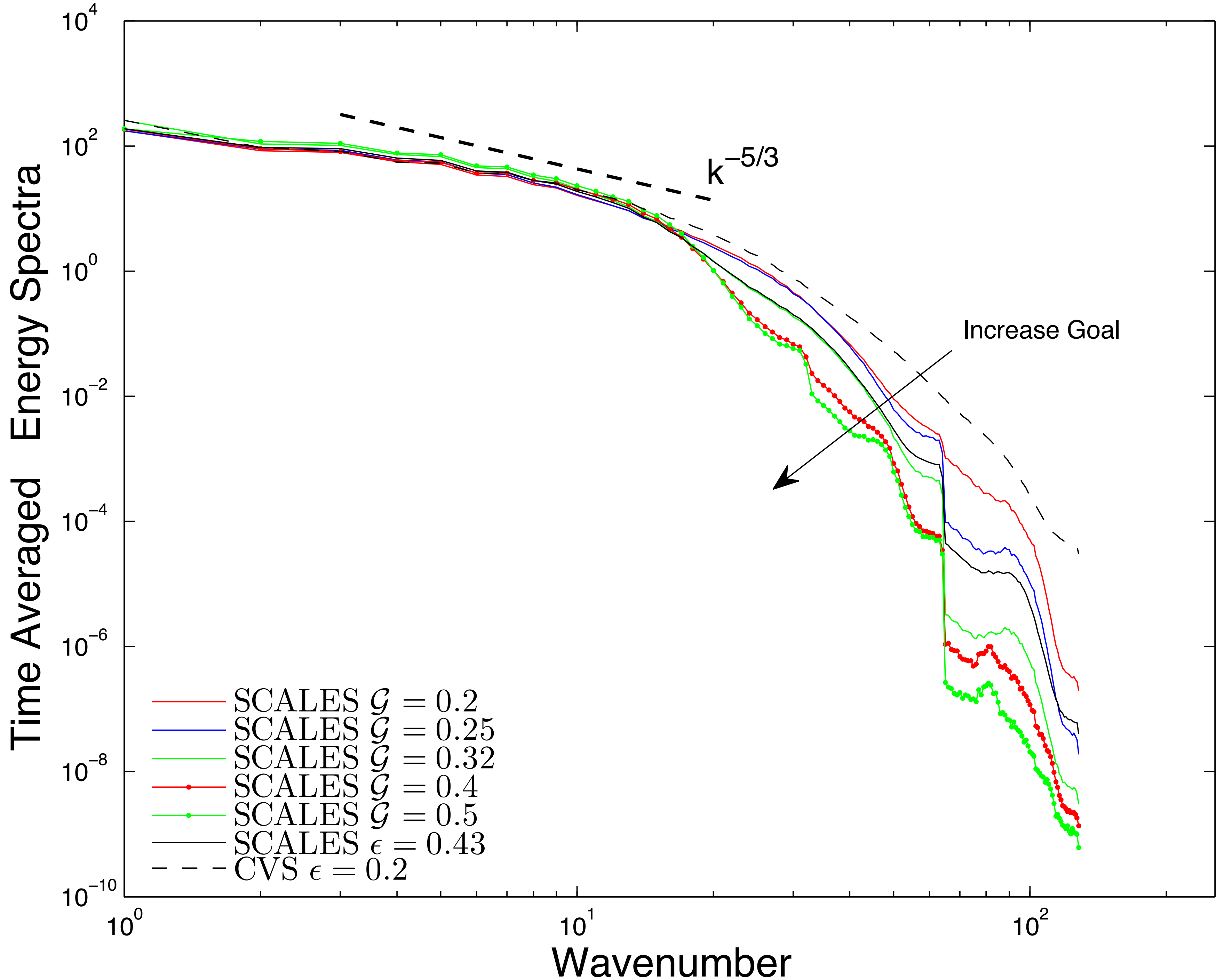
$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle}$



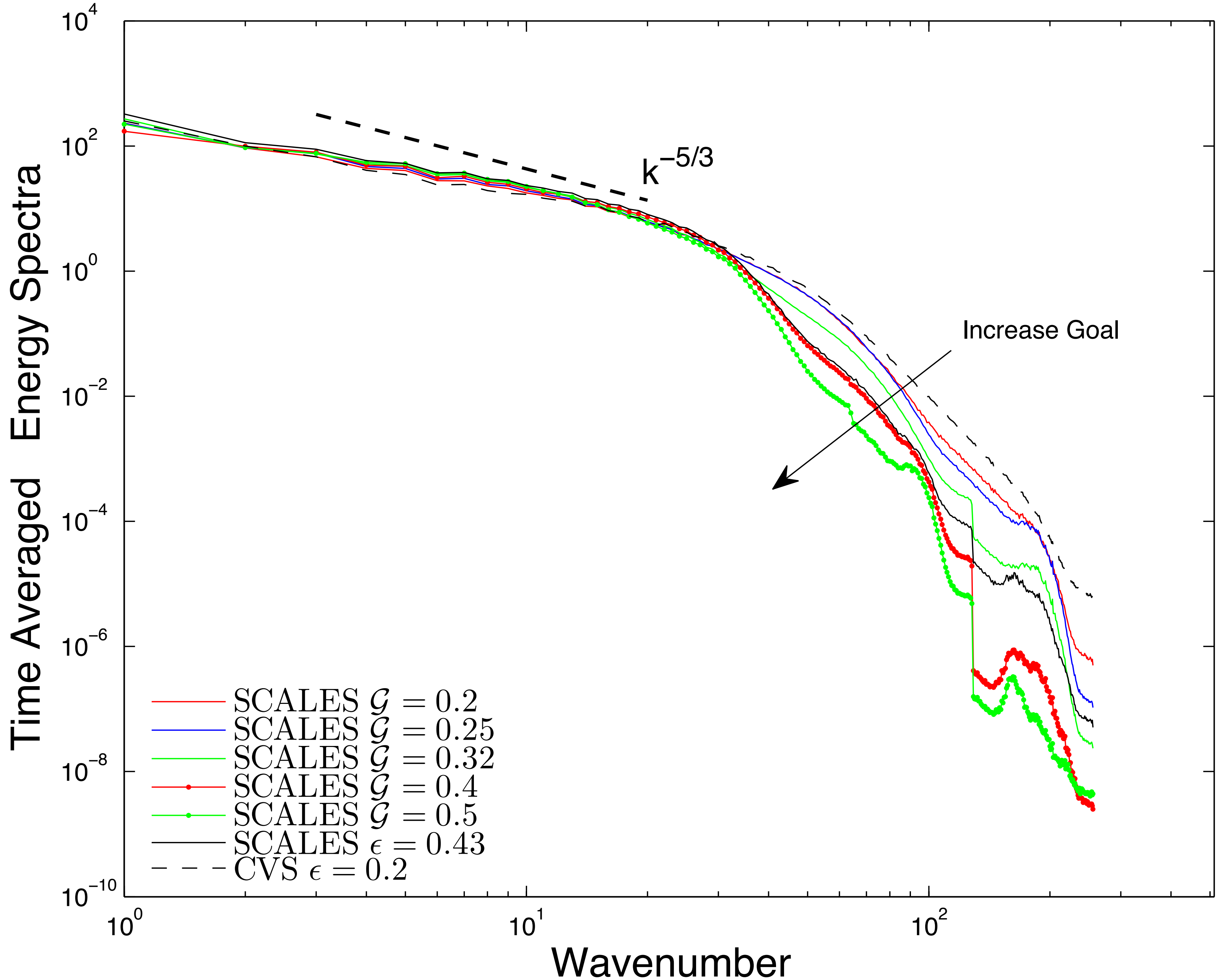
$$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle}$$



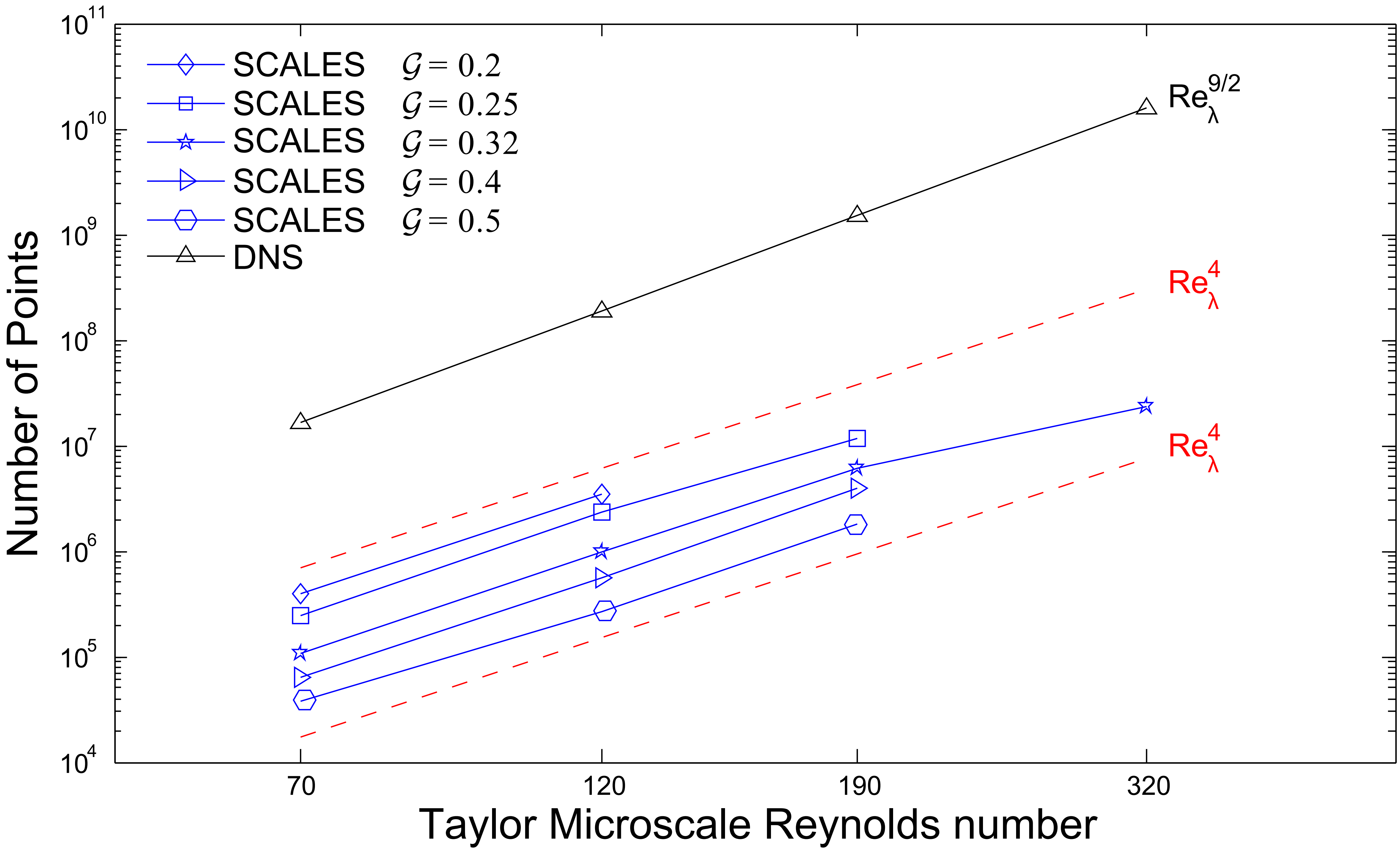
Energy-Spectra of constant-ϵ CVS, constant-ϵ SCALES, constant-$\mathcal{F}$ SCALES



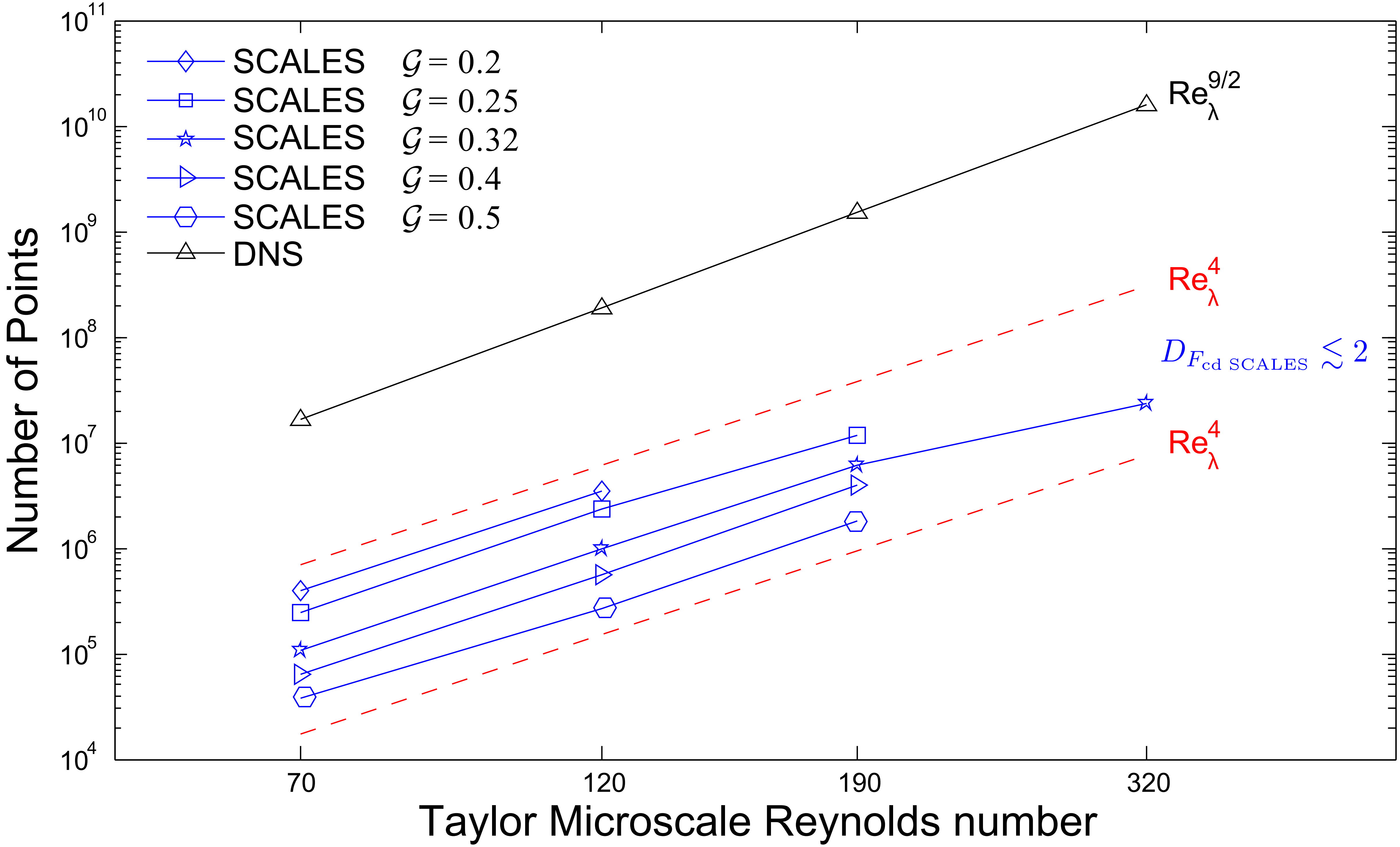
Energy-Spectra of constant-ϵ CVS, constant-ϵ SCALES, constant-$\mathcal{F}$ SCALES

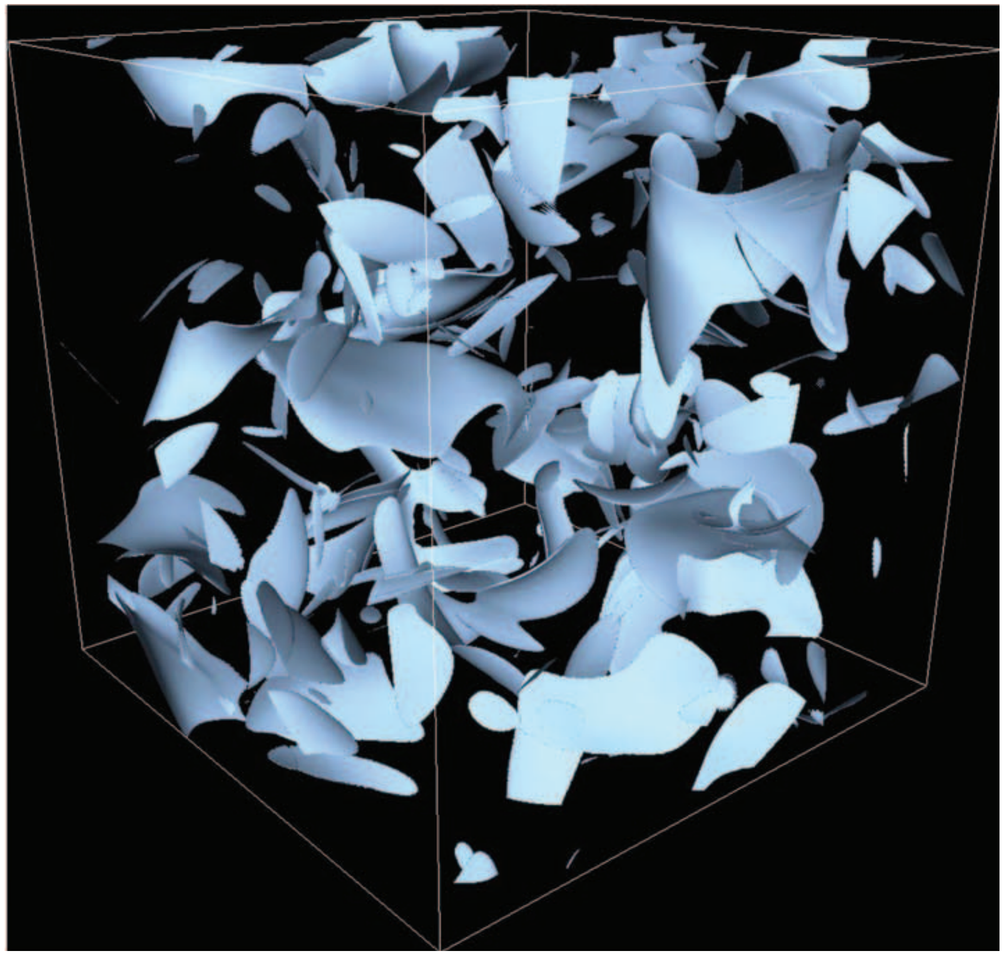


Computational Complexity – Different $\mathcal{G}$



SCALES $\mathcal{G} = 0.2$
SCALES $\mathcal{G} = 0.25$
SCALES $\mathcal{G} = 0.32$
SCALES $\mathcal{G} = 0.4$
SCALES $\mathcal{G} = 0.5$
DNS
$Re_\lambda^{9/2}$
Re_λ^4
Re_λ^4
Number of Points
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}
70
120
190
320
Taylor Microscale Reynolds number


SCALES $\mathcal{G} = 0.2$
SCALES $\mathcal{G} = 0.25$
SCALES $\mathcal{G} = 0.32$
SCALES $\mathcal{G} = 0.4$
SCALES $\mathcal{G} = 0.5$
DNS
$Re_\lambda^{9/2}$
Re_λ^4
$D_{F_{\mathrm{cd\ SCALES}}} \lesssim 2$
Re_λ^4
Number of Points
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
70
120
190
320
Taylor Microscale Reynolds number



Constant-Dissipation SCALES (cd-SCALES)

$\Longrightarrow$

maintains dissipation ($\mathcal{F}$) at a fixed level ($\mathcal{G}$)

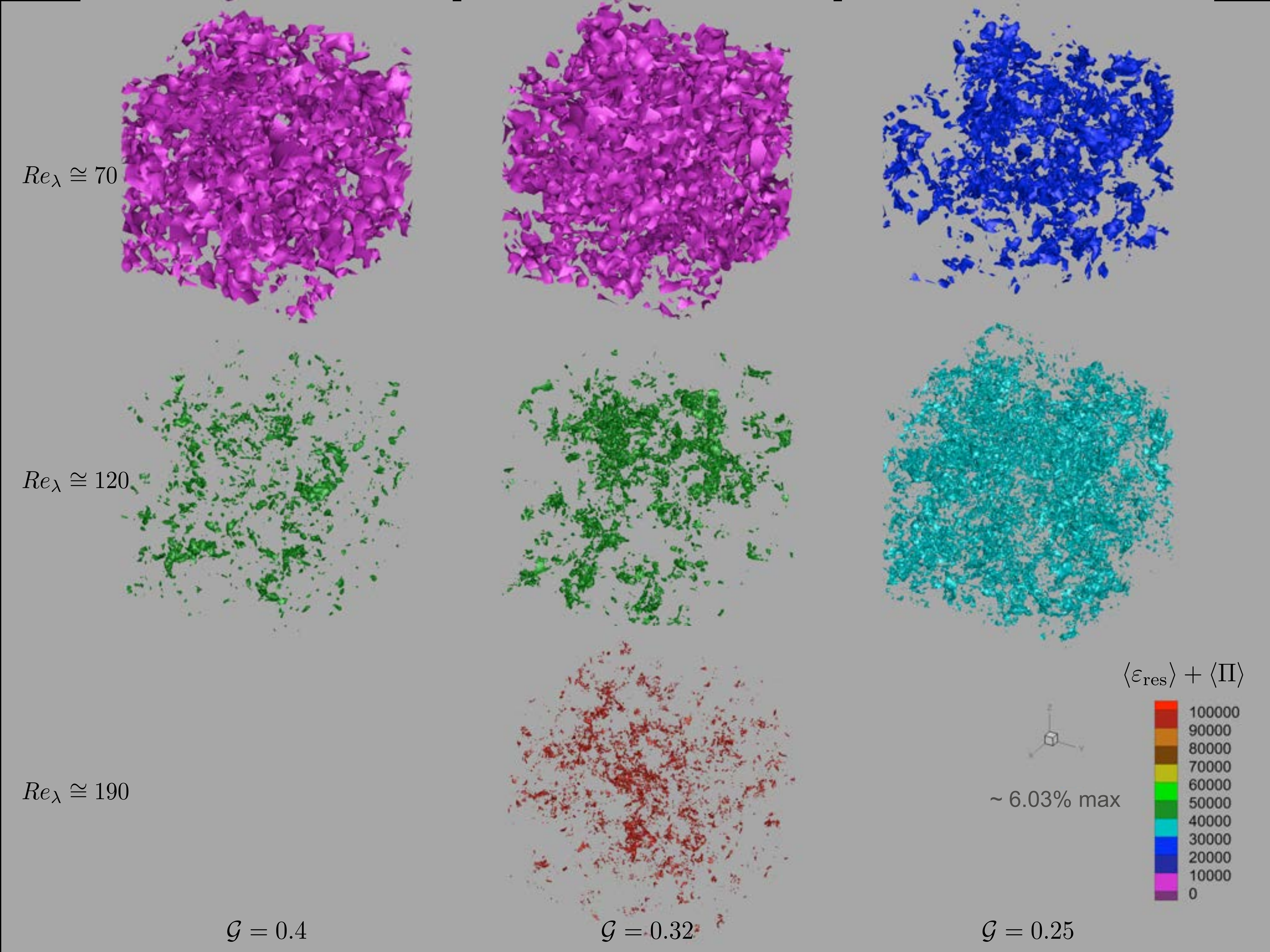
,i.e., captures iso-surfaces of dissipation,

which are known to be sheet-like structure*,

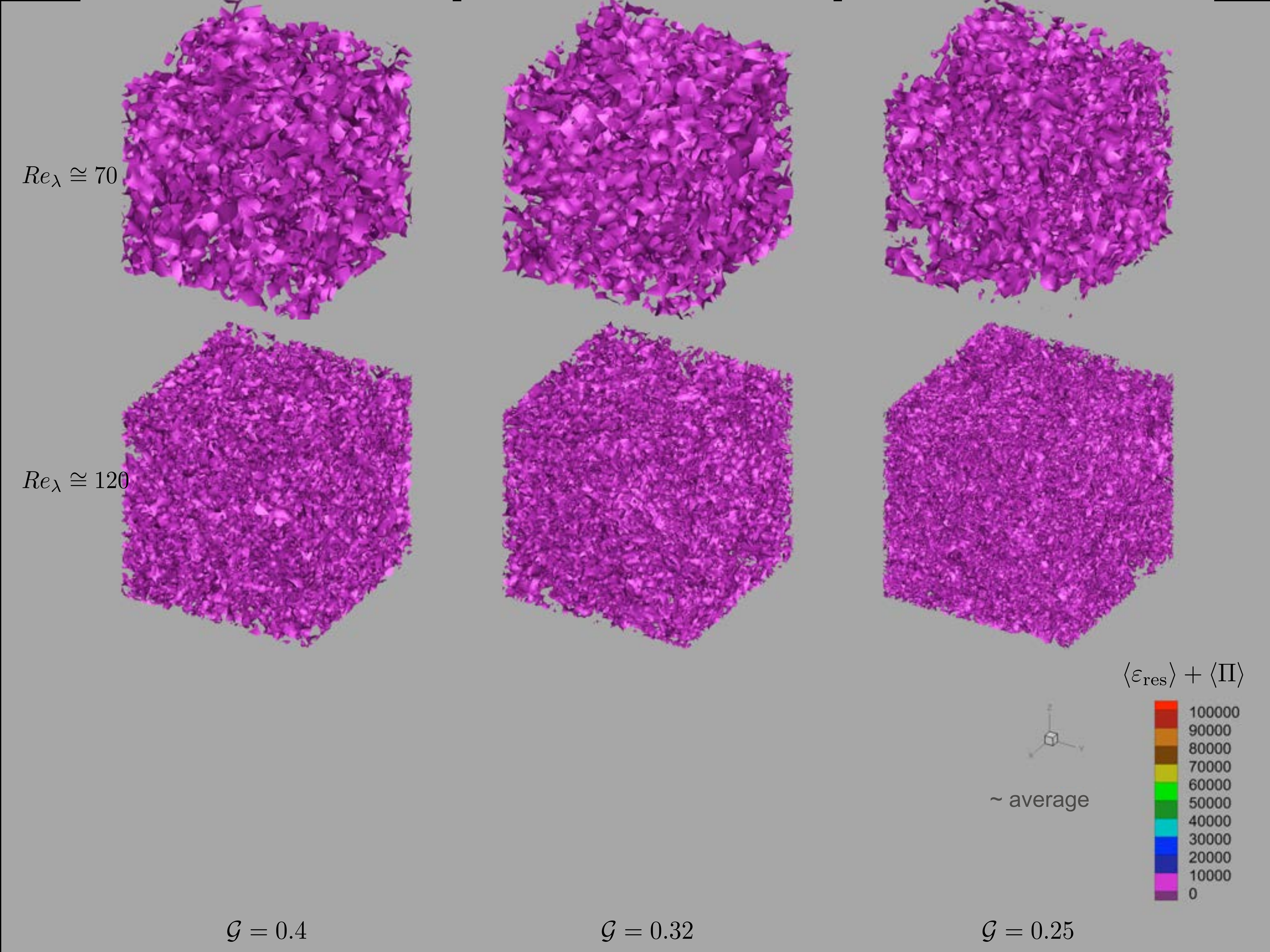
$\Longrightarrow$

cd-SCALES fills the space with $D_F \cong 2$.

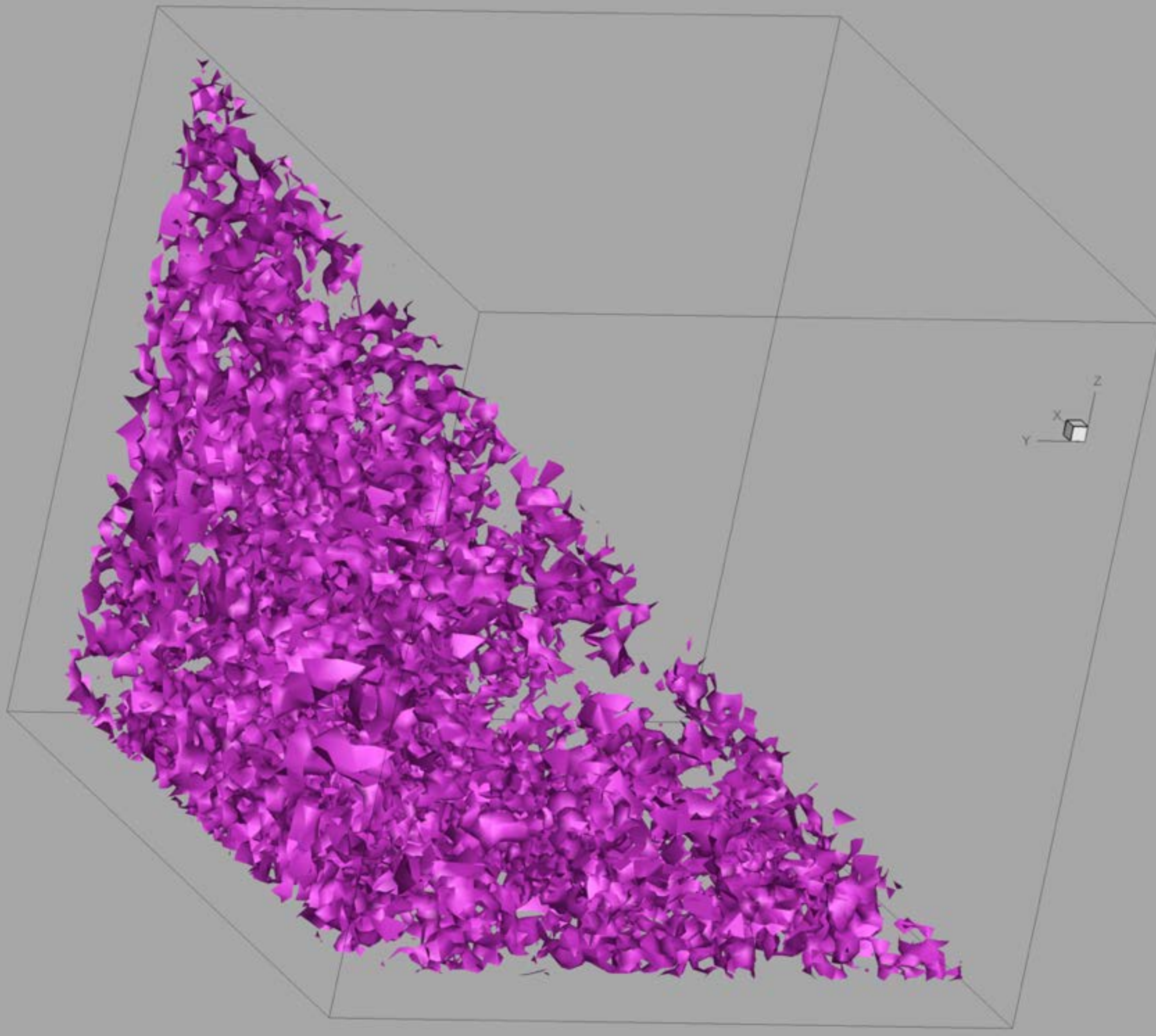
*J. Schumacher, H. Zilken, B. Eckhardt, and K.R. Sreenivasan, *Scalar dissipation fronts in high-Schmidt number mixing,* *Chaos* 15, 041105 (2005). doi



$Re_\lambda \cong 70$
$Re_\lambda \cong 120$
$Re_\lambda \cong 190$
$\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle$
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
~ 6.03% max
$\mathcal{G} = 0.4$
$\mathcal{G} = 0.32$
$\mathcal{G} = 0.25$



$Re_\lambda \cong 70$
$Re_\lambda \cong 120$
$\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle$
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
~ average
$\mathcal{G} = 0.4$
$\mathcal{G} = 0.32$
$\mathcal{G} = 0.25$



Z
X
Y

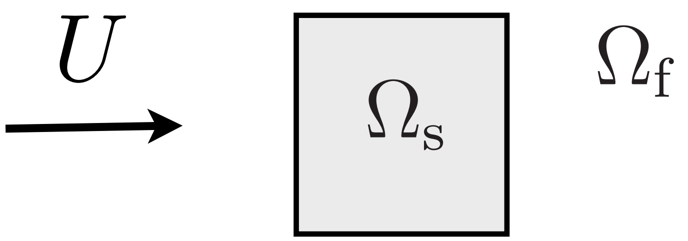
extending the wavelet-based eddy capturing approach (De Stefano & Vasilyev, JFM 2012)

typical multiscale problem (mixing layer, recirculating flow, separated boundary layer, ...)

unsteady 3-D flow at low Re (even for 2-D bodies)

wavelet multi-resolution analysis naturally able to identify/track dynamically dominant flow structures

obstacle simply modeled through volume-penalization



$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0 \qquad \tilde{u}_i \quad \text{penalized perturbation velocity} \qquad u_i + U_i = 0 \quad \text{on} \quad \partial\Omega_{\mathrm{s}}$$

$$\frac{\partial \tilde{u}_i}{\partial t} + (\tilde{u}_j + U_j)\frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{P}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} - \frac{\chi_{\mathrm{s}}}{\eta}(\tilde{u}_i + U_i)$$

penalty error scales with $\eta^{1/2}$

$$F_i(t) = \frac{\rho}{\eta}\int_{\Omega_s} (\tilde{u}_i + U_i)\, d\mathbf{x}$$

isolated stationary cylinder with square cross-section

supercritical Reynolds-number $Re = UL/\nu = 2 \times 10^3$

computational domain: $24L \times 16L \times 4L$

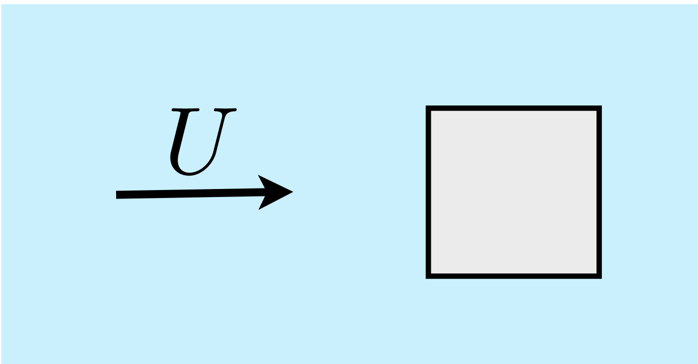
penalty constant: $\eta U/L = 5 \times 10^{-4}$

thresholding level: $\epsilon = 0.05$

levels of resolution: $j_{\max} = 8$

finest resolution: $\Delta x/L = \Delta y/L = 1/128$ $\Delta z/L = 1/64$

SGS/resolved dissipation about 10%

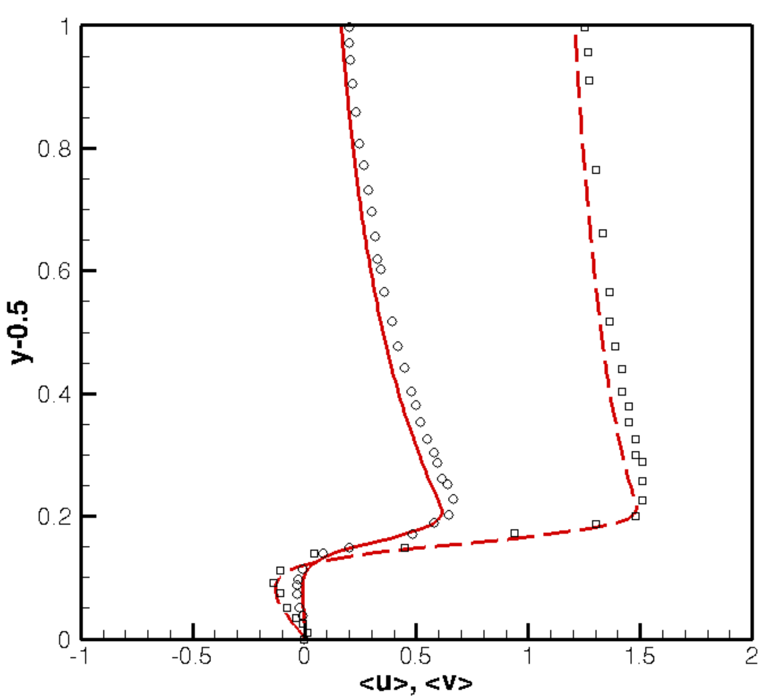


Mean = Time- and spanwise-averaged

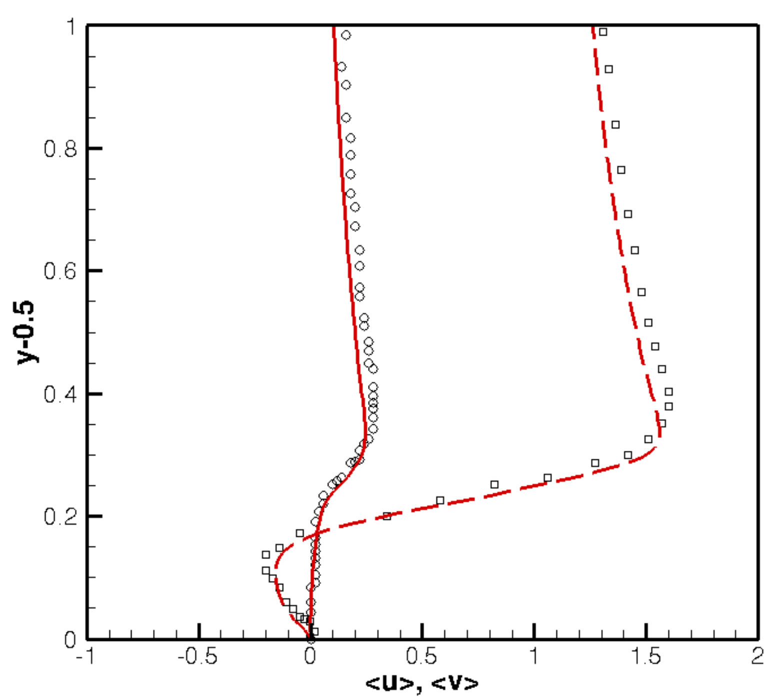
Streamwise and transverse components

Reference: non-adaptive numerical solution (Brun et al., FTC 2008)

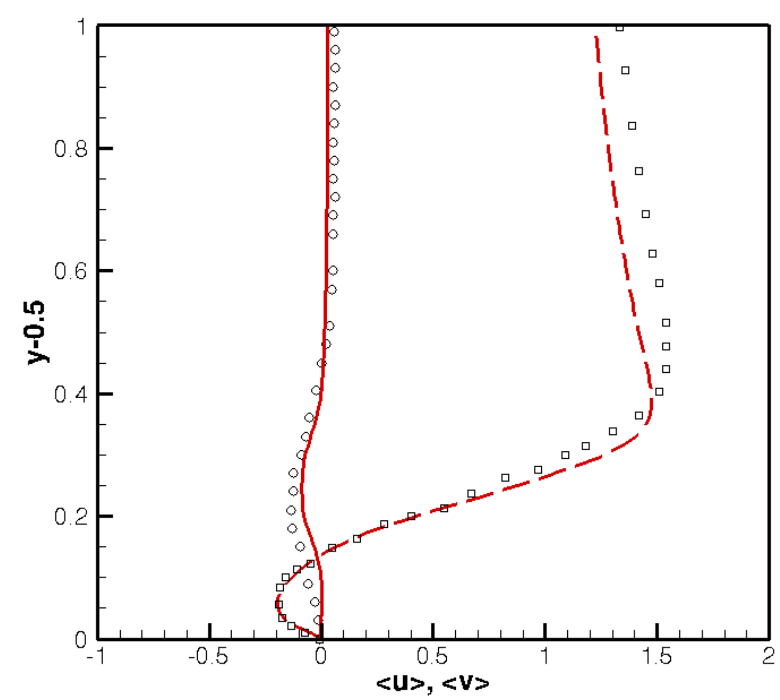
x = -0.3L



x = 0

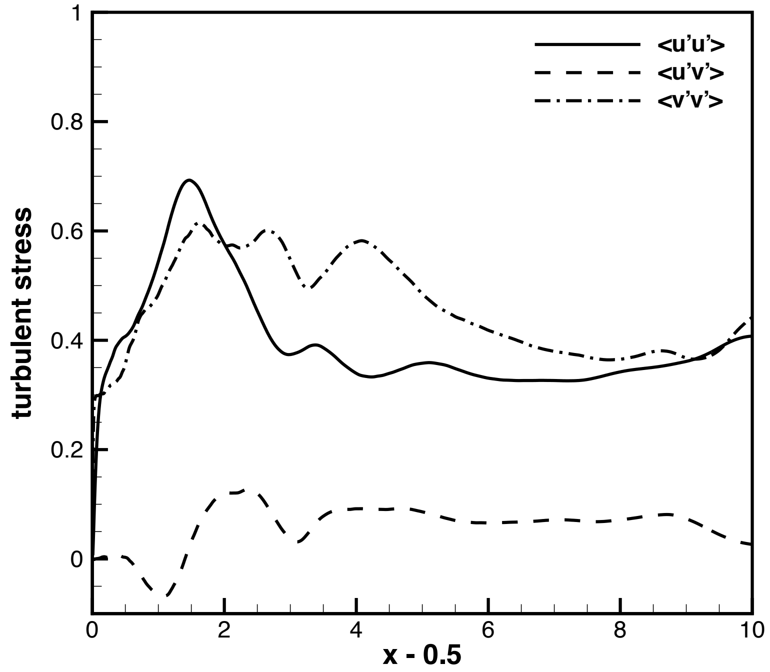


x = 0.3L



Mean turbulent stresses

y = 0

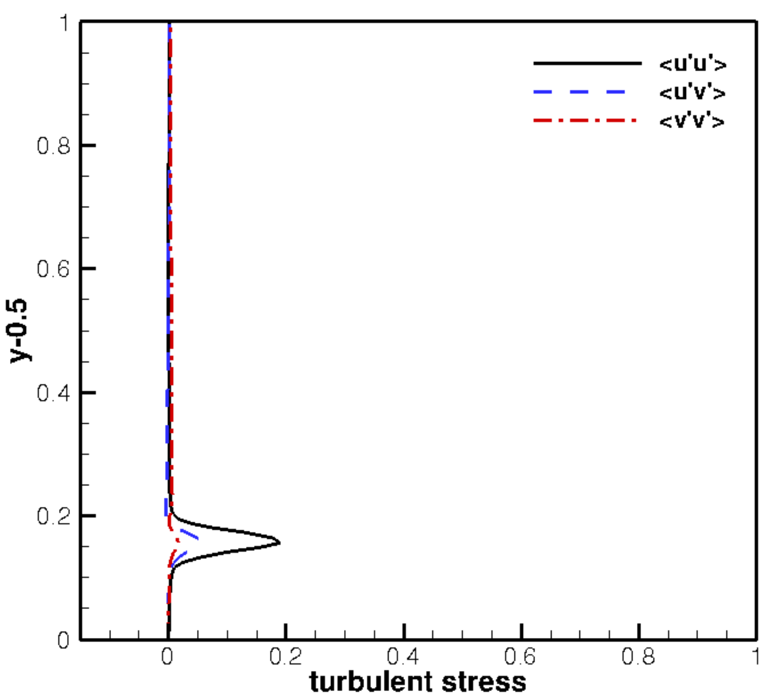


Time- and spanwise-averaged stresses

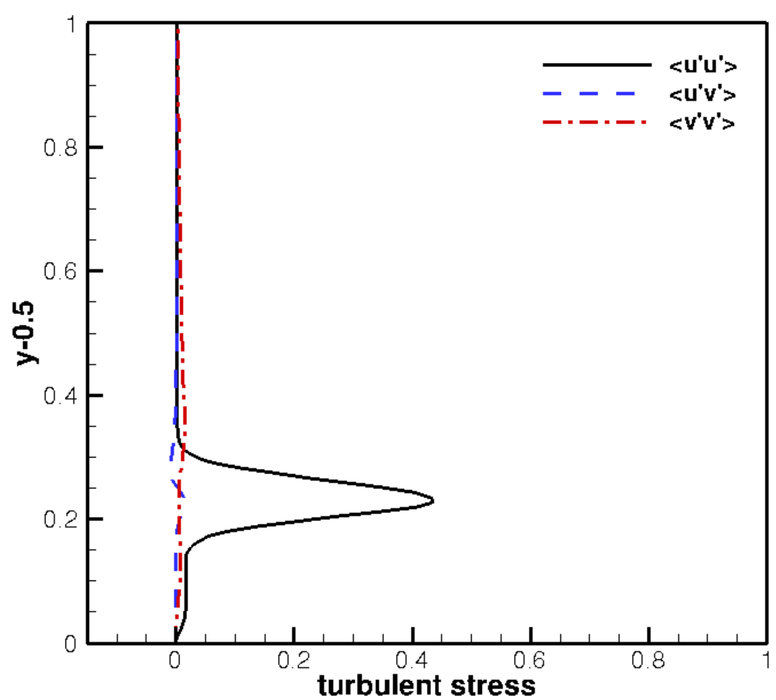
Experimental peak along the centerline captured (Lin et al., JFM 1995)

Separating shear layer becomes unstable and transition to turbulence occurs

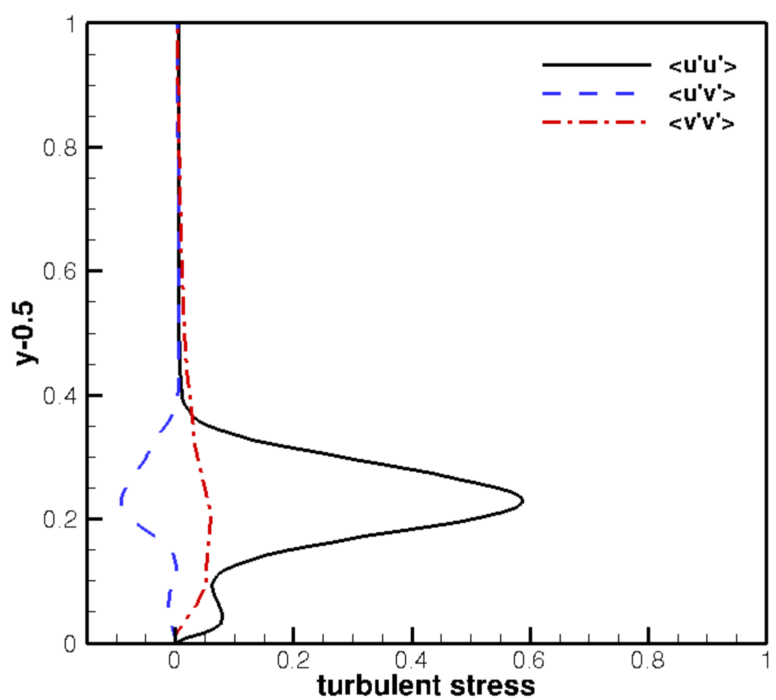
x = -0.3L



x = 0



x = 0.3L



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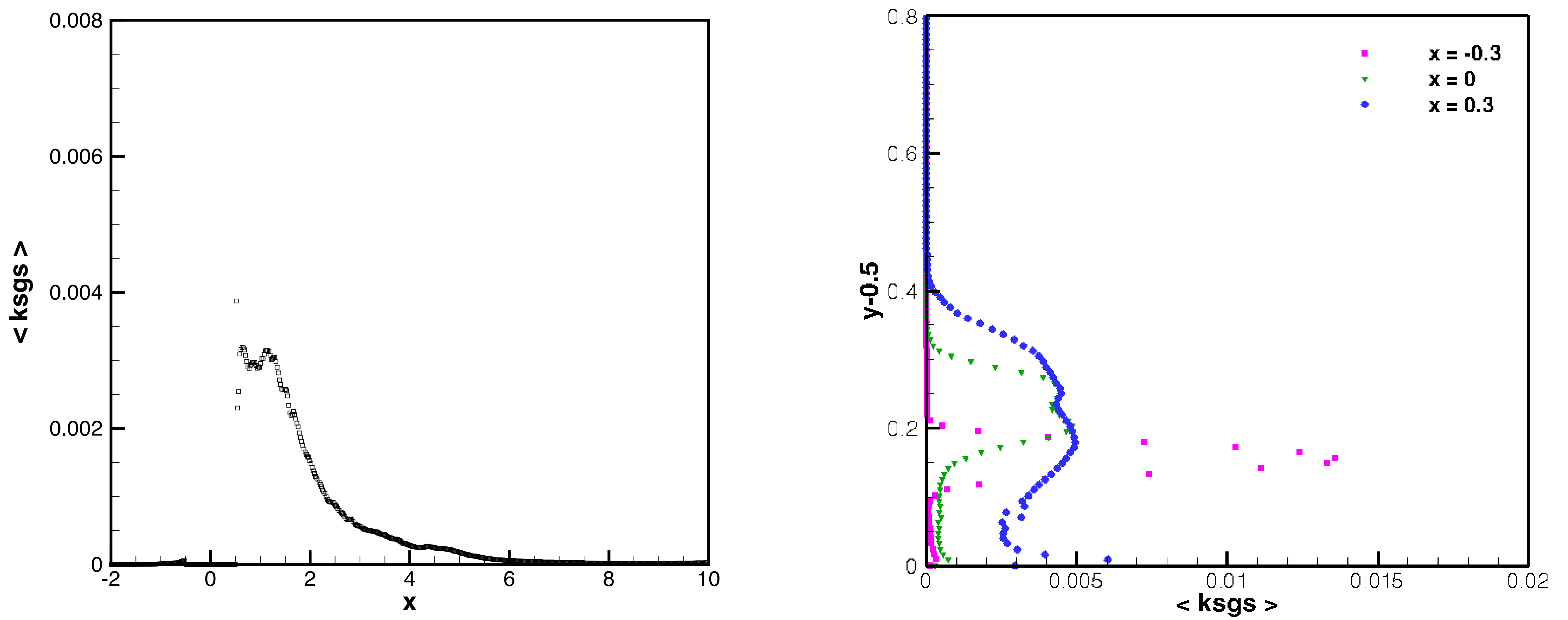
Mean SGS energy profiles

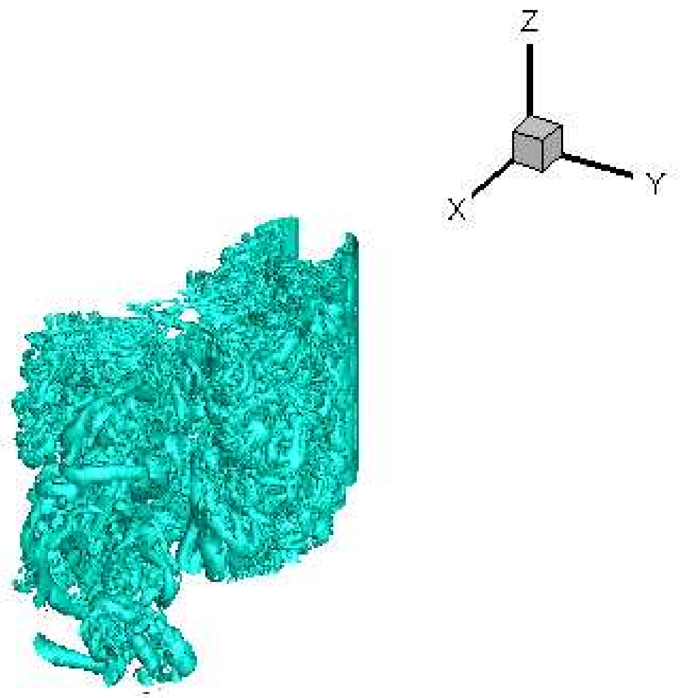
y = 0



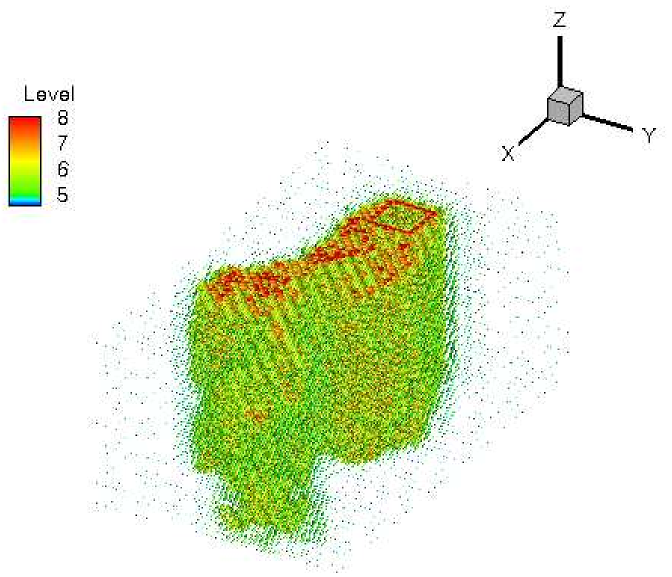
spanwise- and time-averaged modeled SGS energy

Verified global results: $C_D = 2.4$ $St = 0.13$ *(Okajima, JFM 1982)*

Instantaneous vorticity field



U



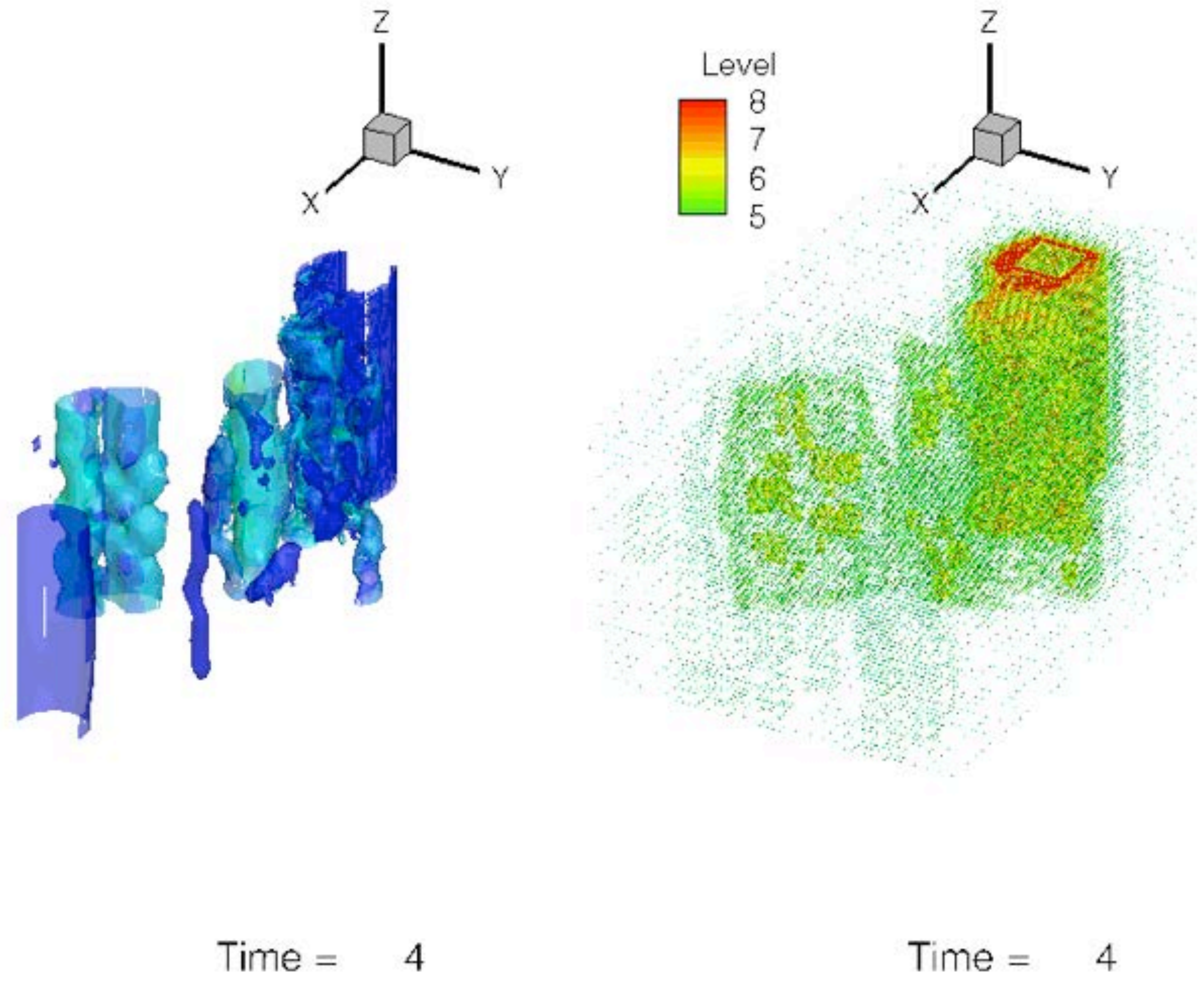
Main vortical structures identified by the Q-criterion (Q=0.25)

Scatter plot of the retained collocation points at highest levels

Spatial distribution of wavelets follows the physics *vortical structures*

&

retained collocation points scatter plot



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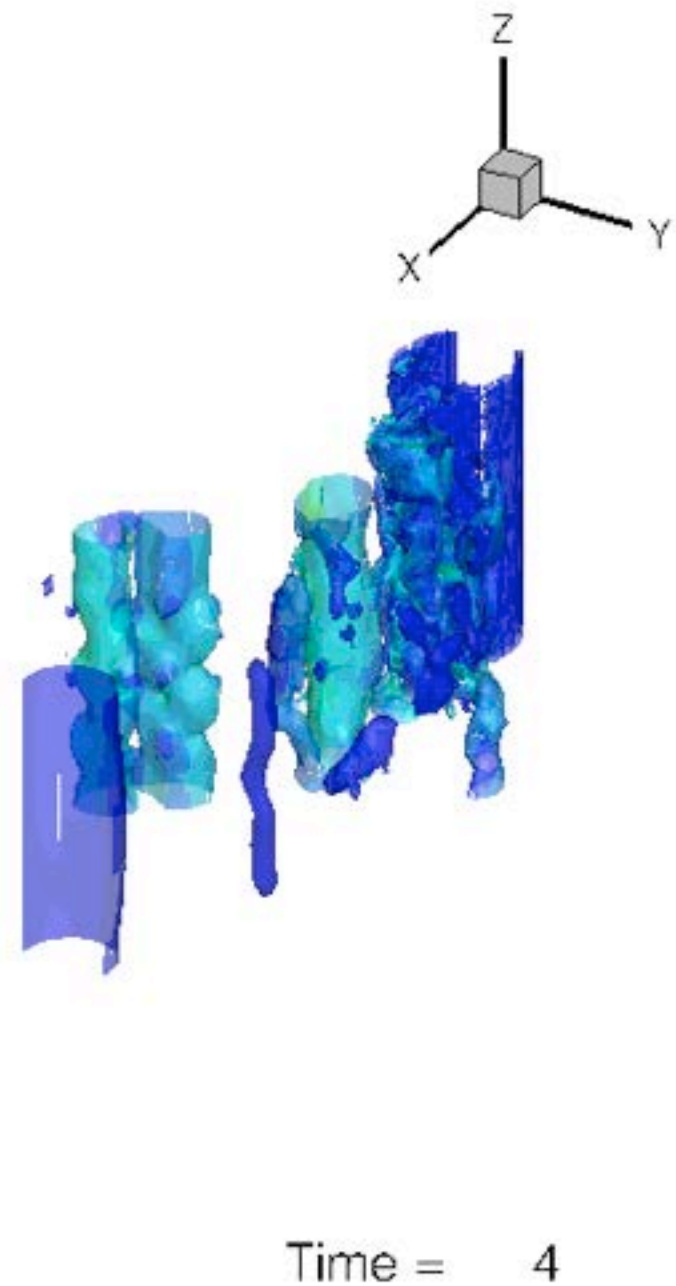
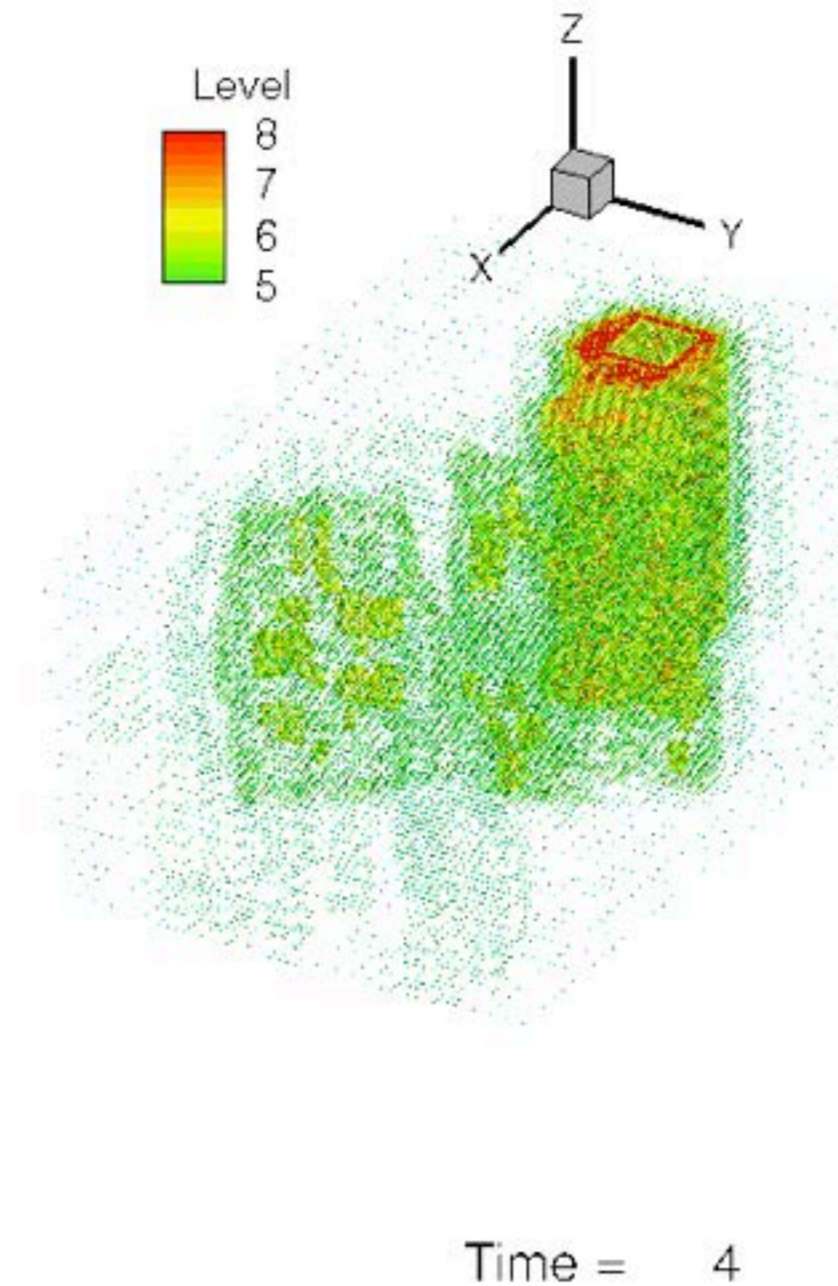
Time-dependent coupling

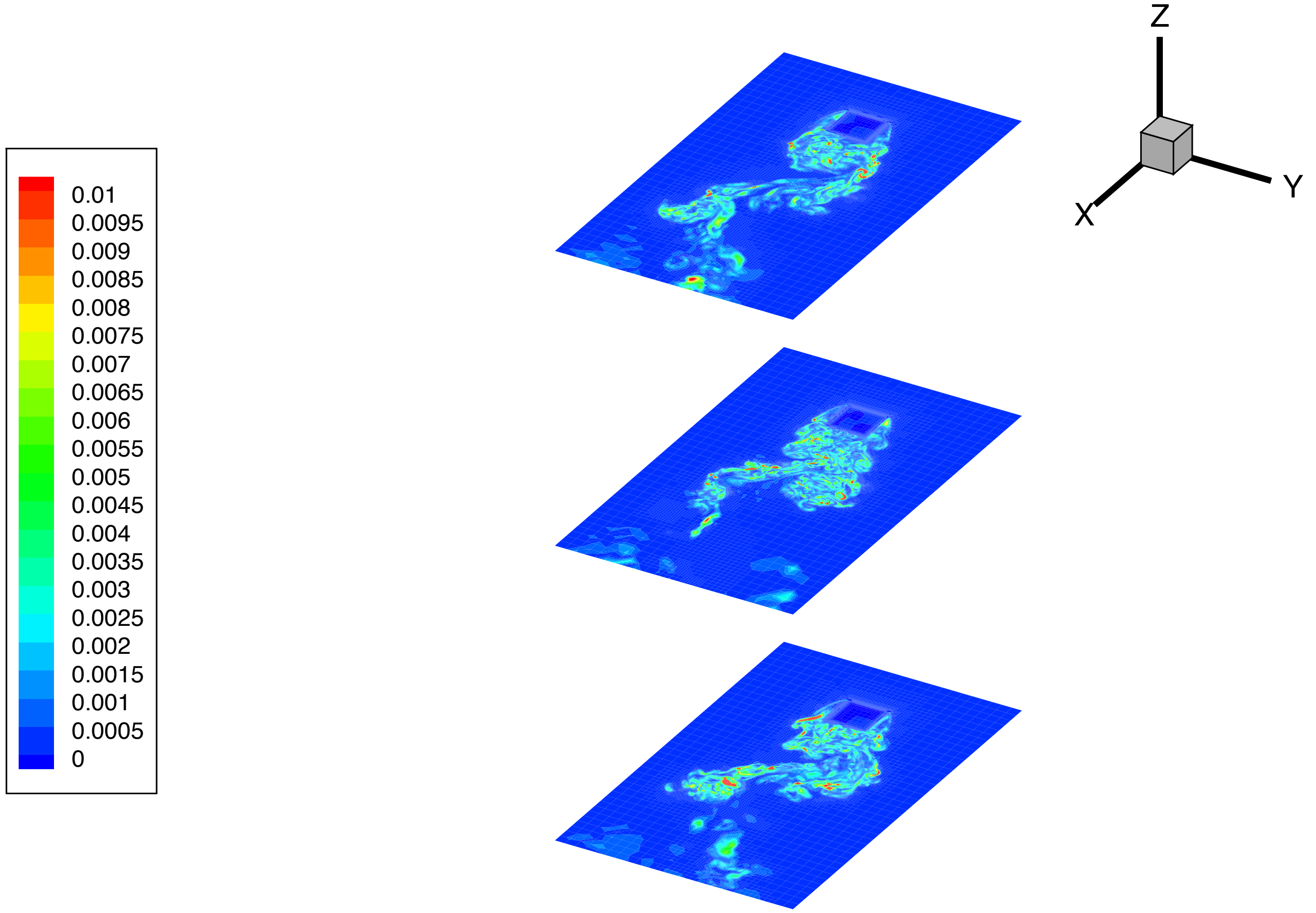
vortical structures

&

retained collocation points scatter plot



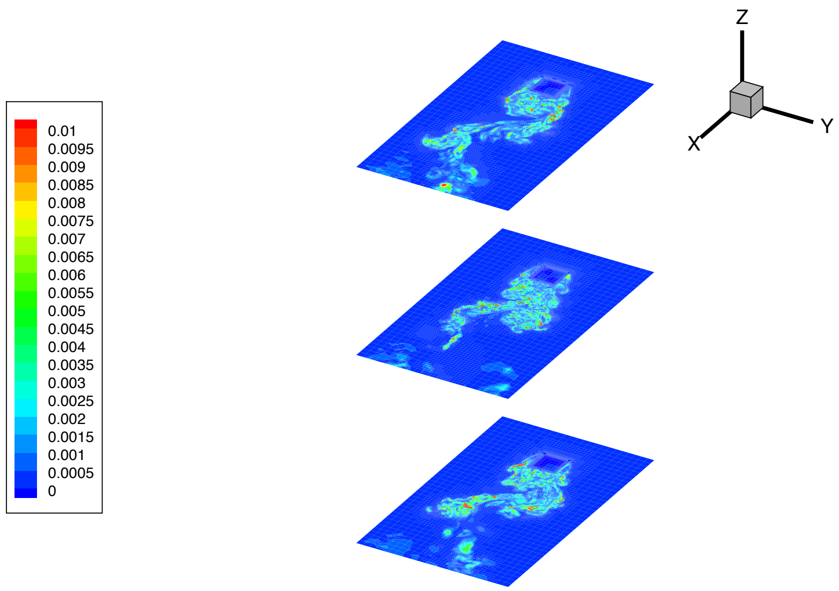


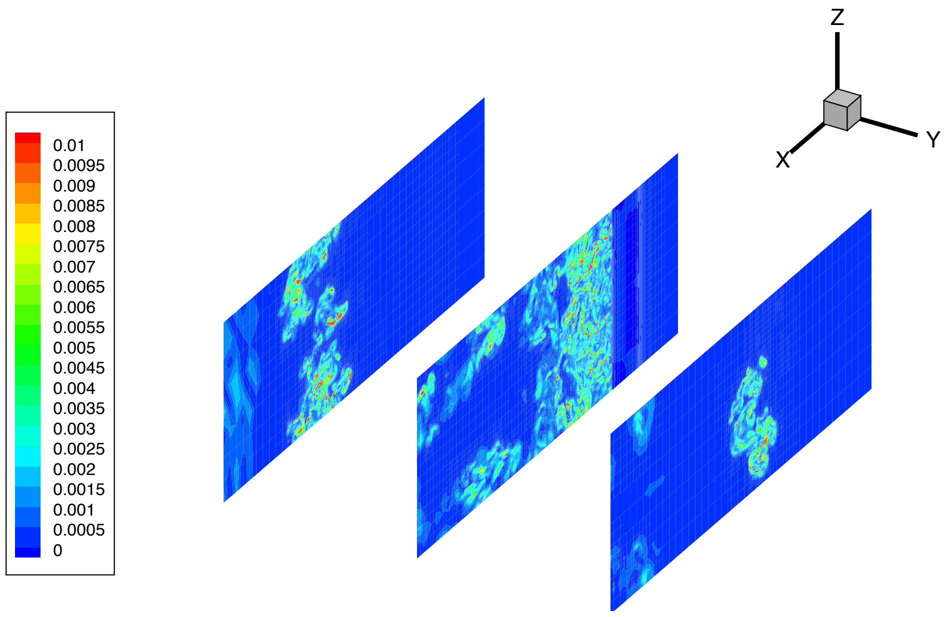


SGS energy vanishes inside of the obstacle (penalization)

$$z = -1.4, 0, 1.4$$

$$y = -1.5, 0, 1.5$$





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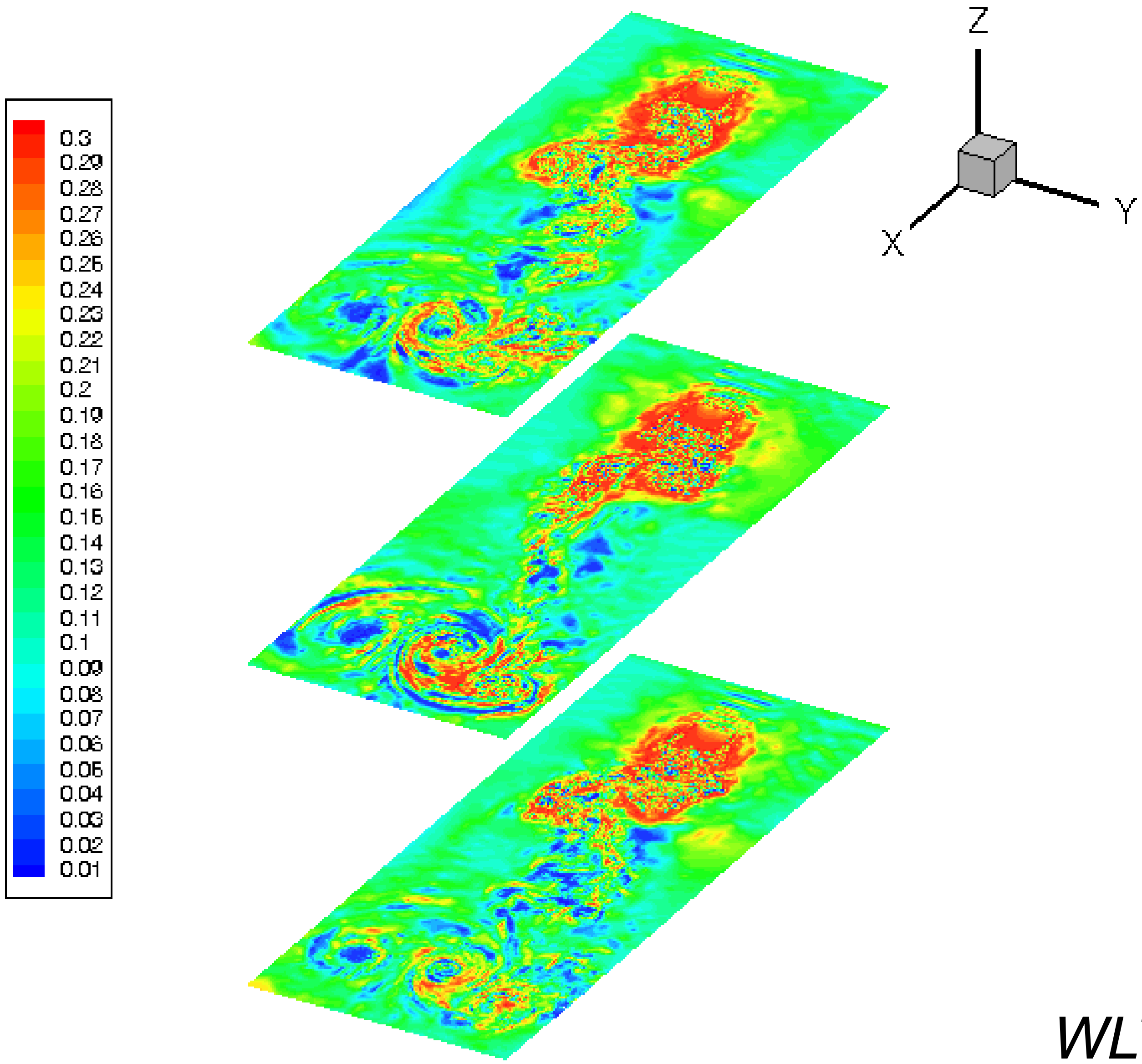
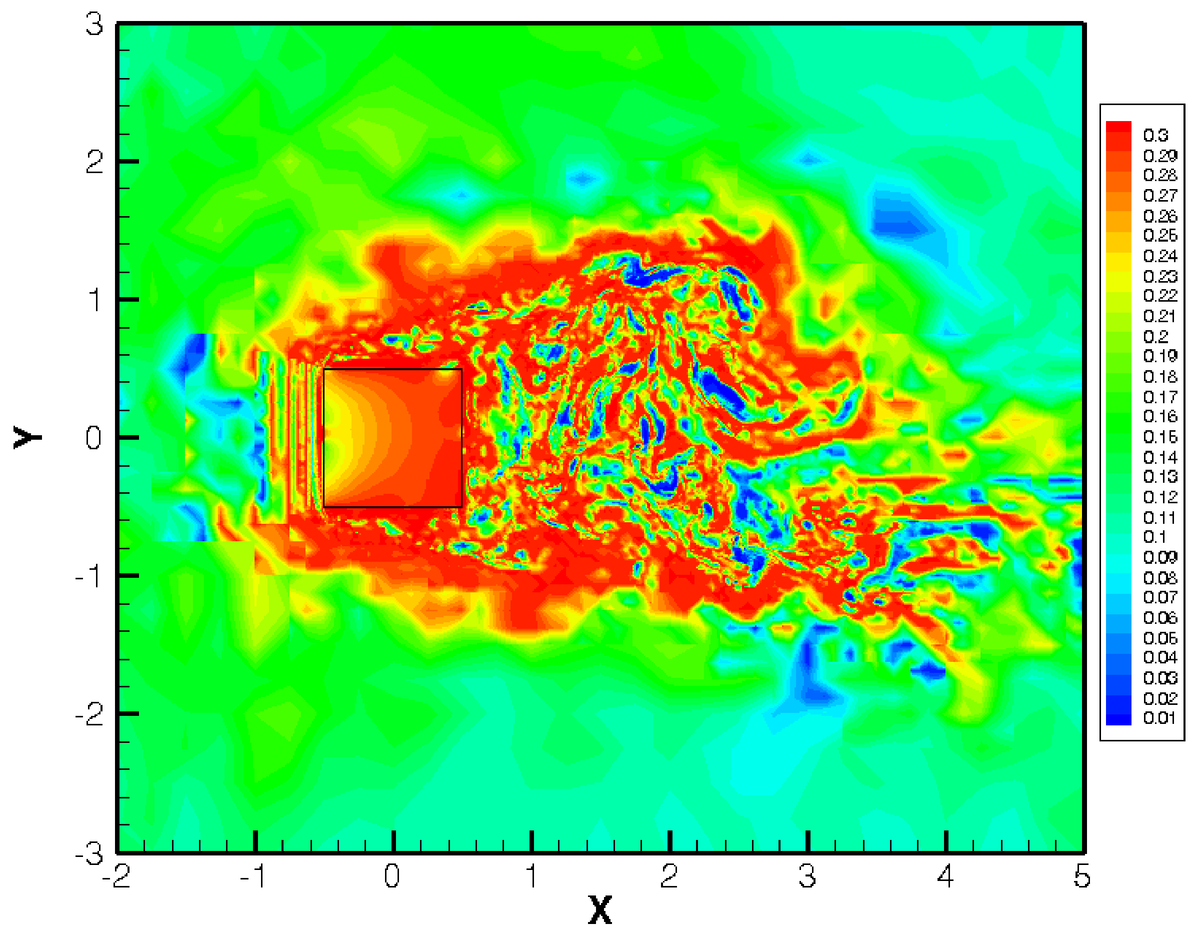
Variable thresholding

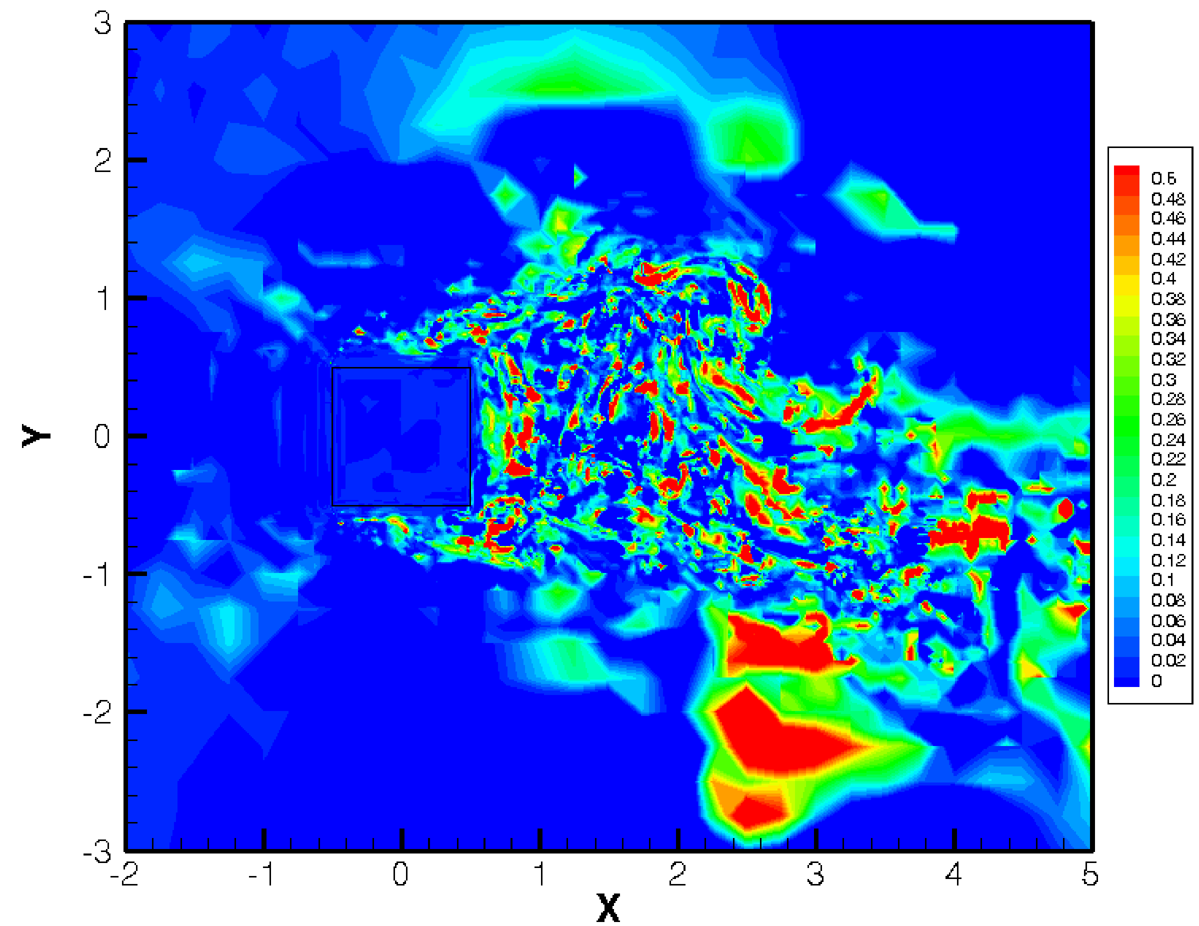


WLT threshold

Variable thresholding

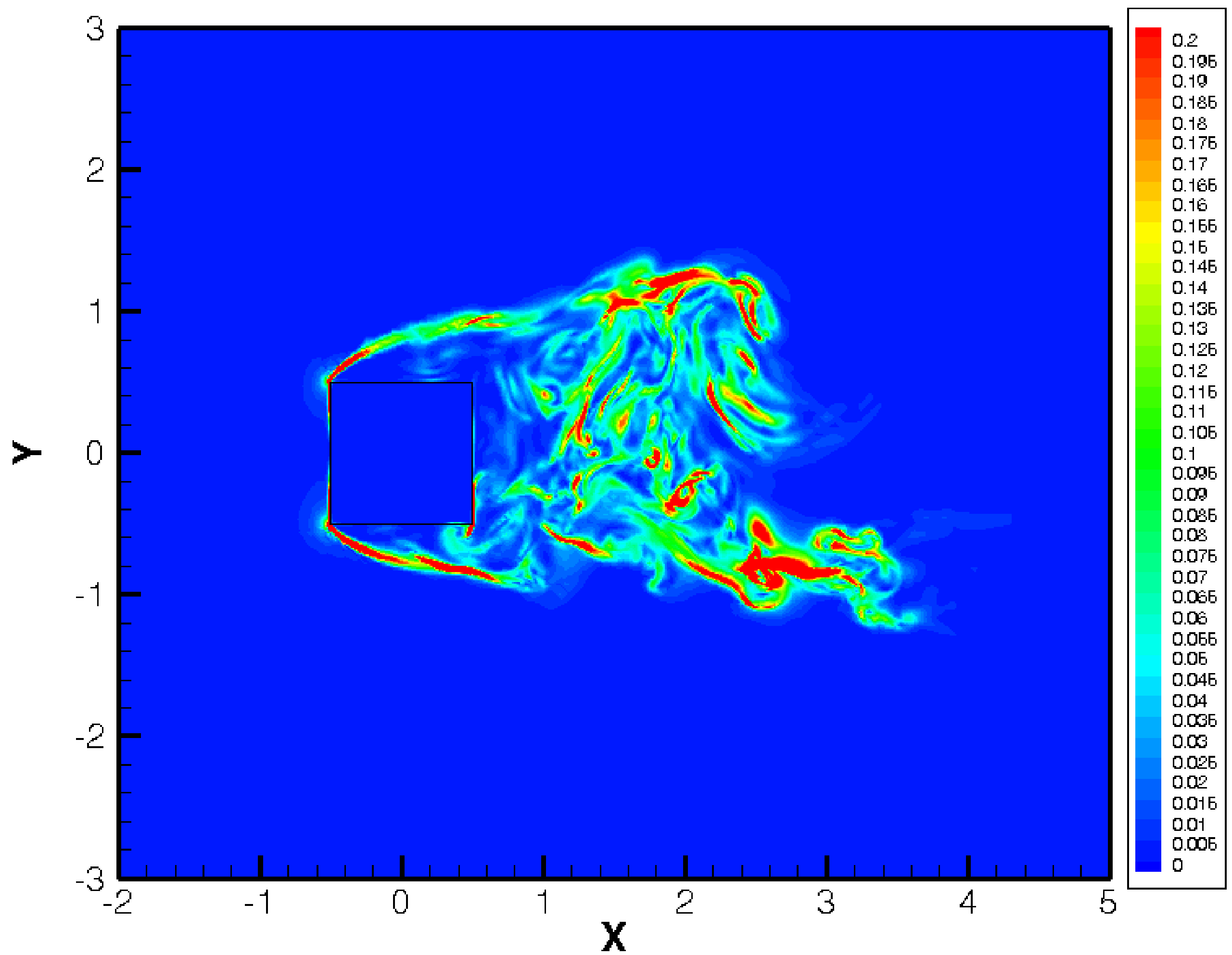


WLT threshold

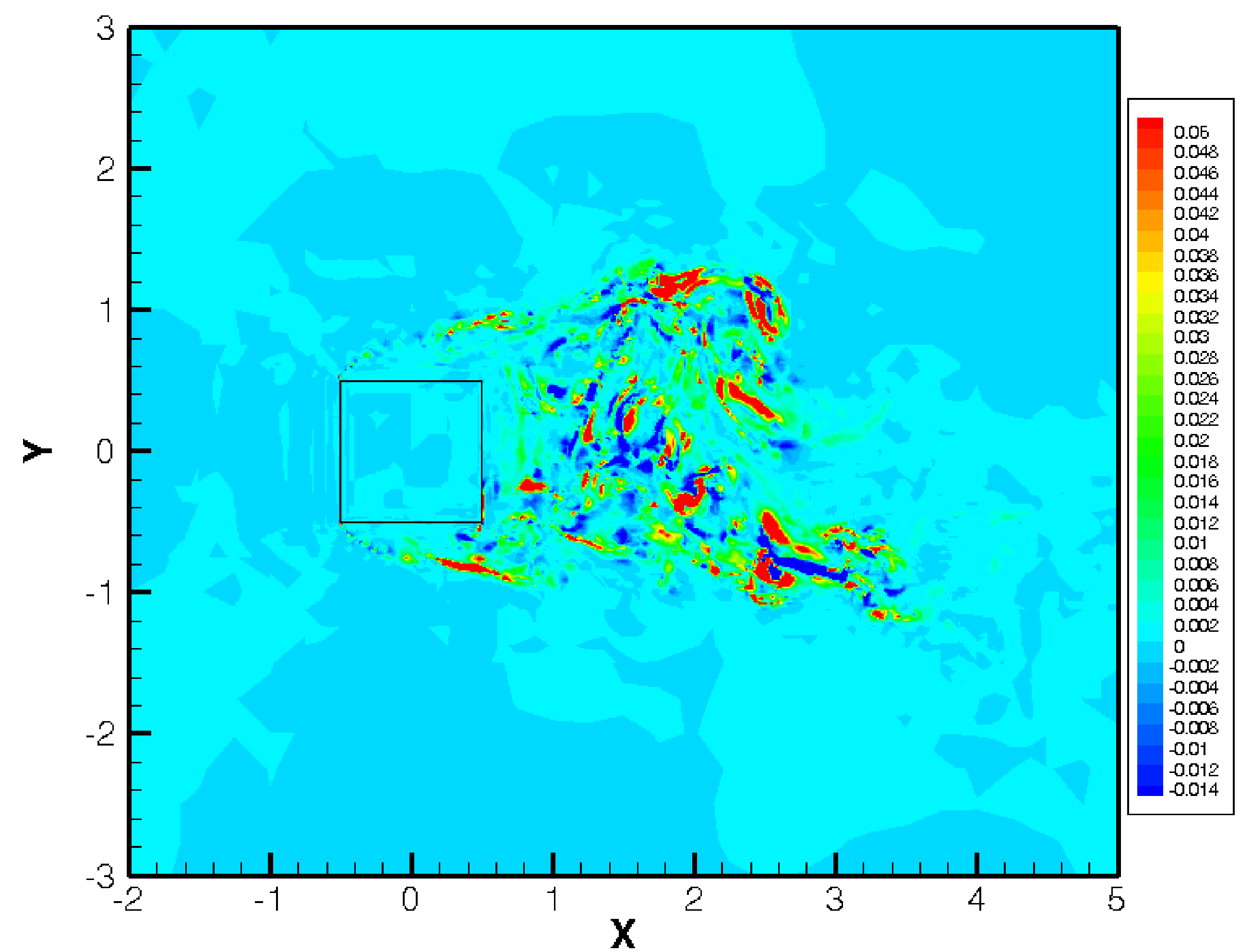


Fraction of SGS dissipation

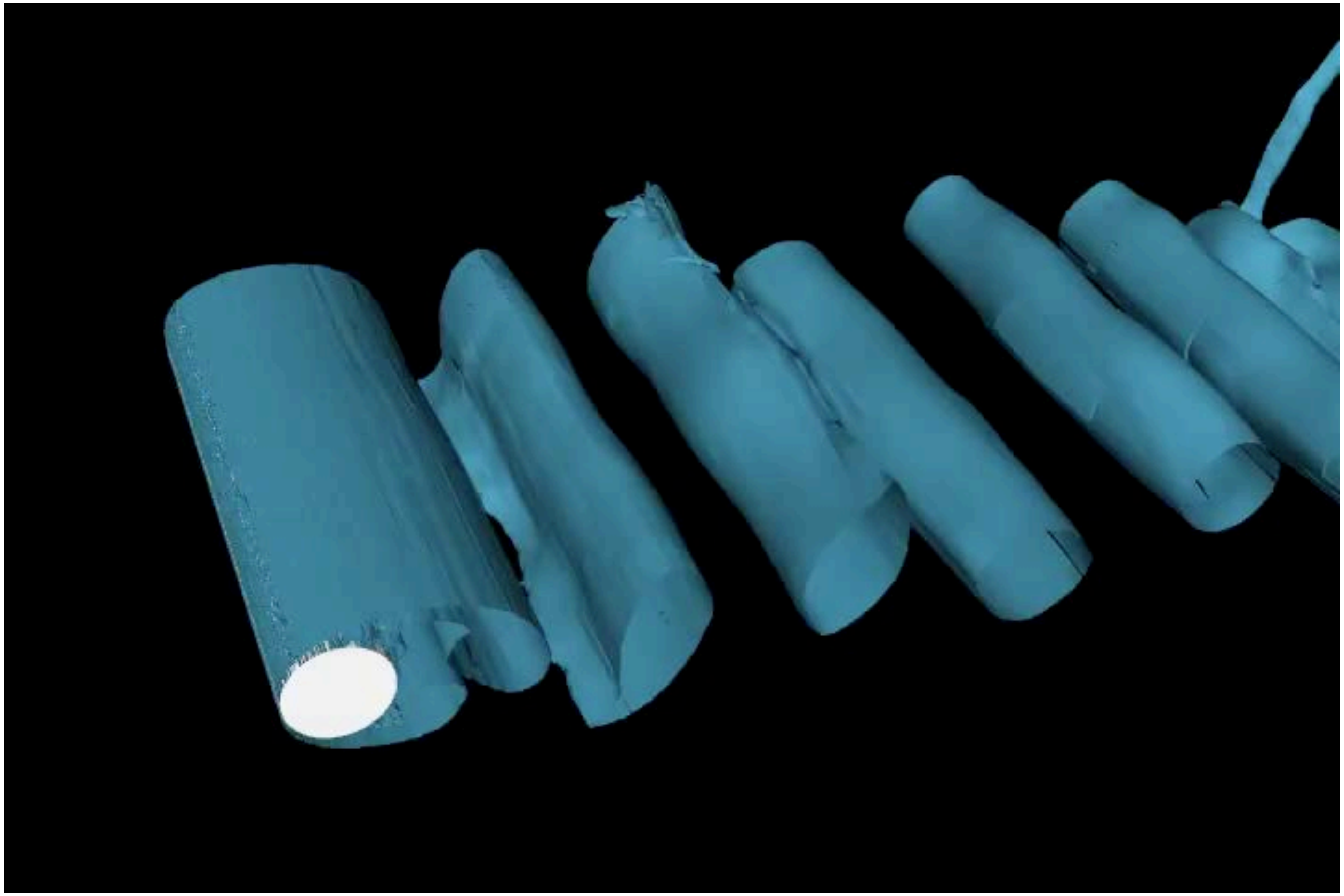
Variable thresholding



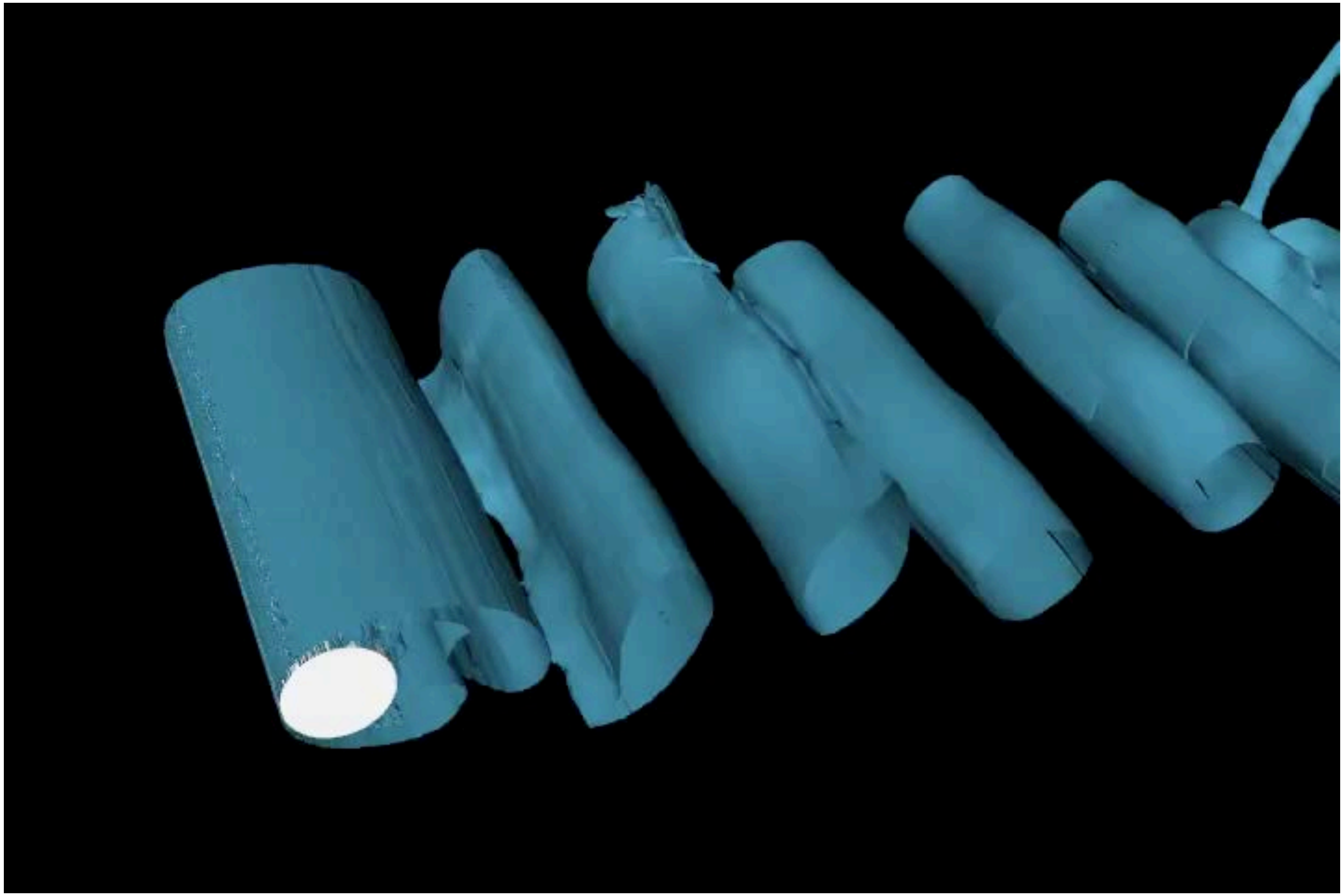
Resolved viscous dissipation



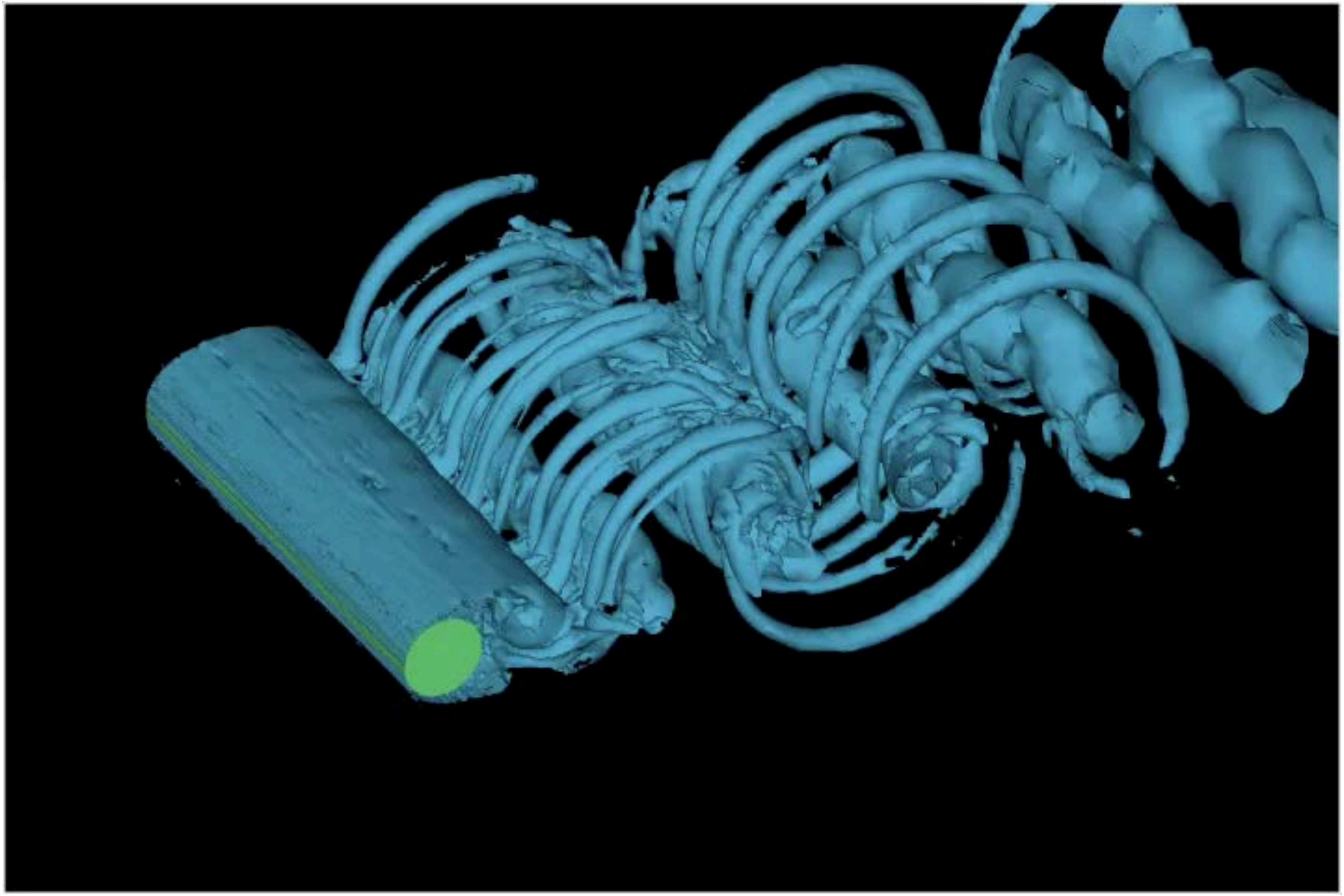
Modeled SGS dissipation



$Re = 350, Q = 0.1$

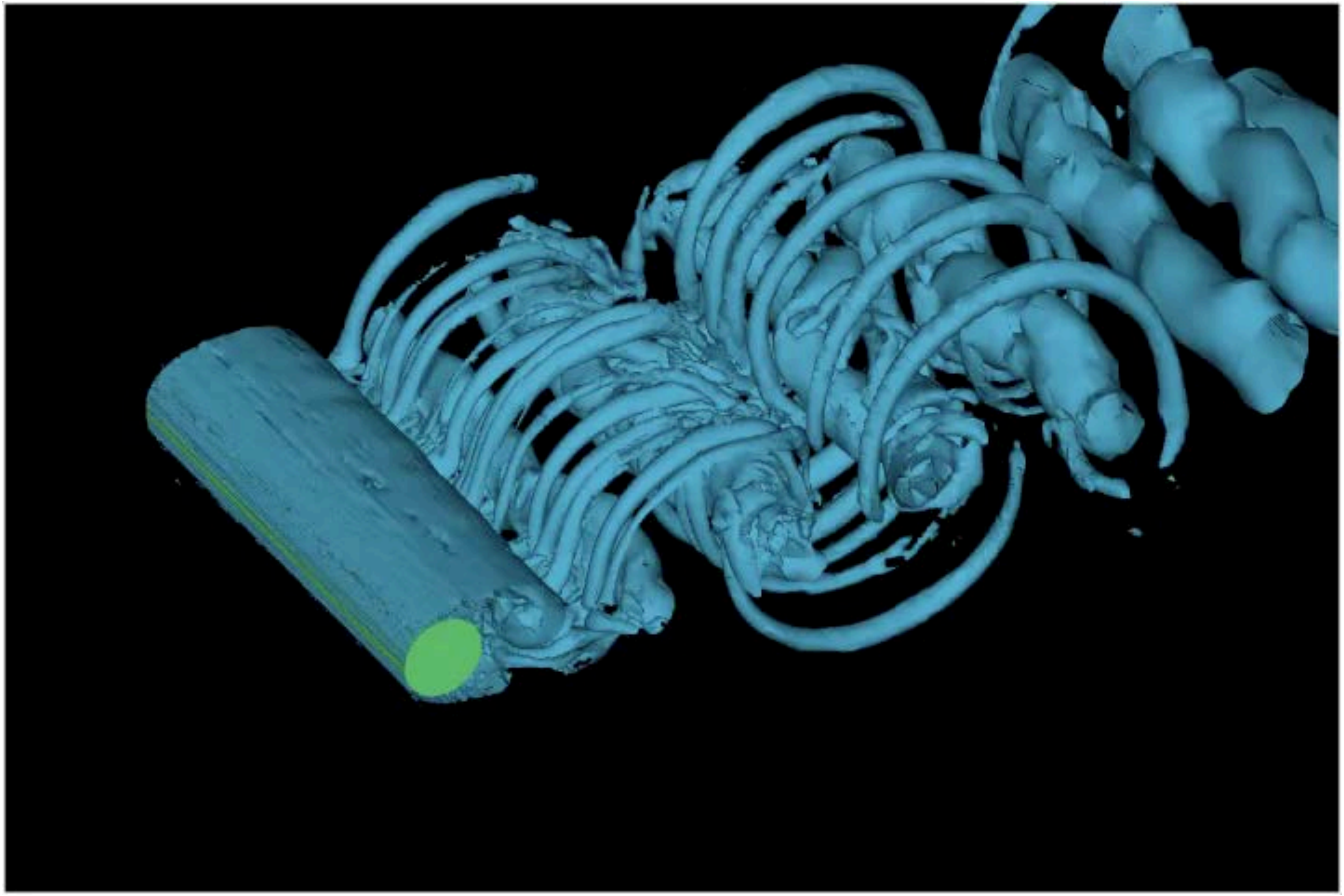


$Re = 350, Q = 0.1$

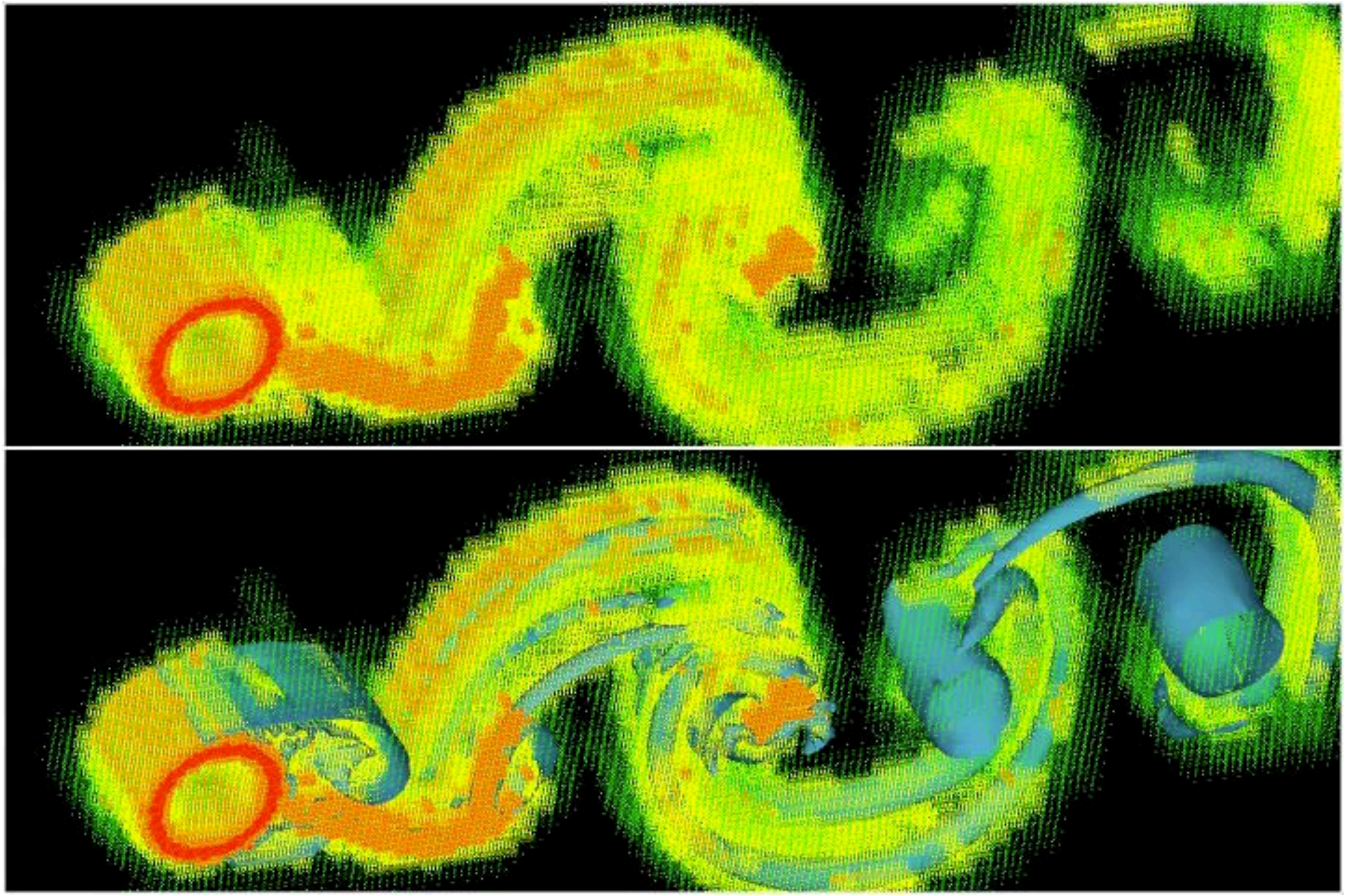


Re = 500, Q = 0.5

Ongoing Efforts: WDNS of Compressible Flow

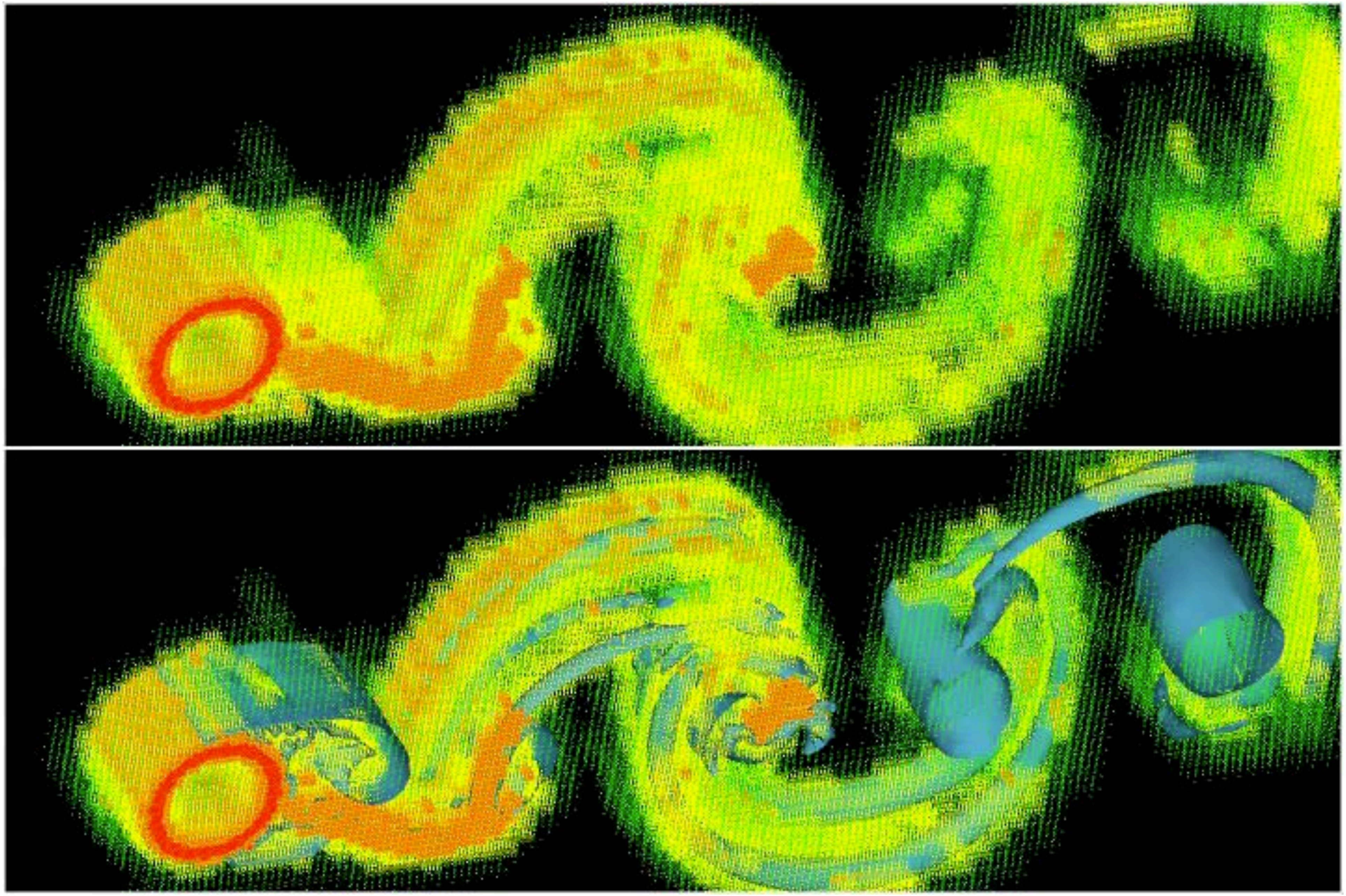


Re = 500, Q = 0.5



Adaptive levels $5 \leq j \leq 8$

Ongoing Efforts: WDNS of Compressible Flow



Adaptive levels $5 \le j \le 8$

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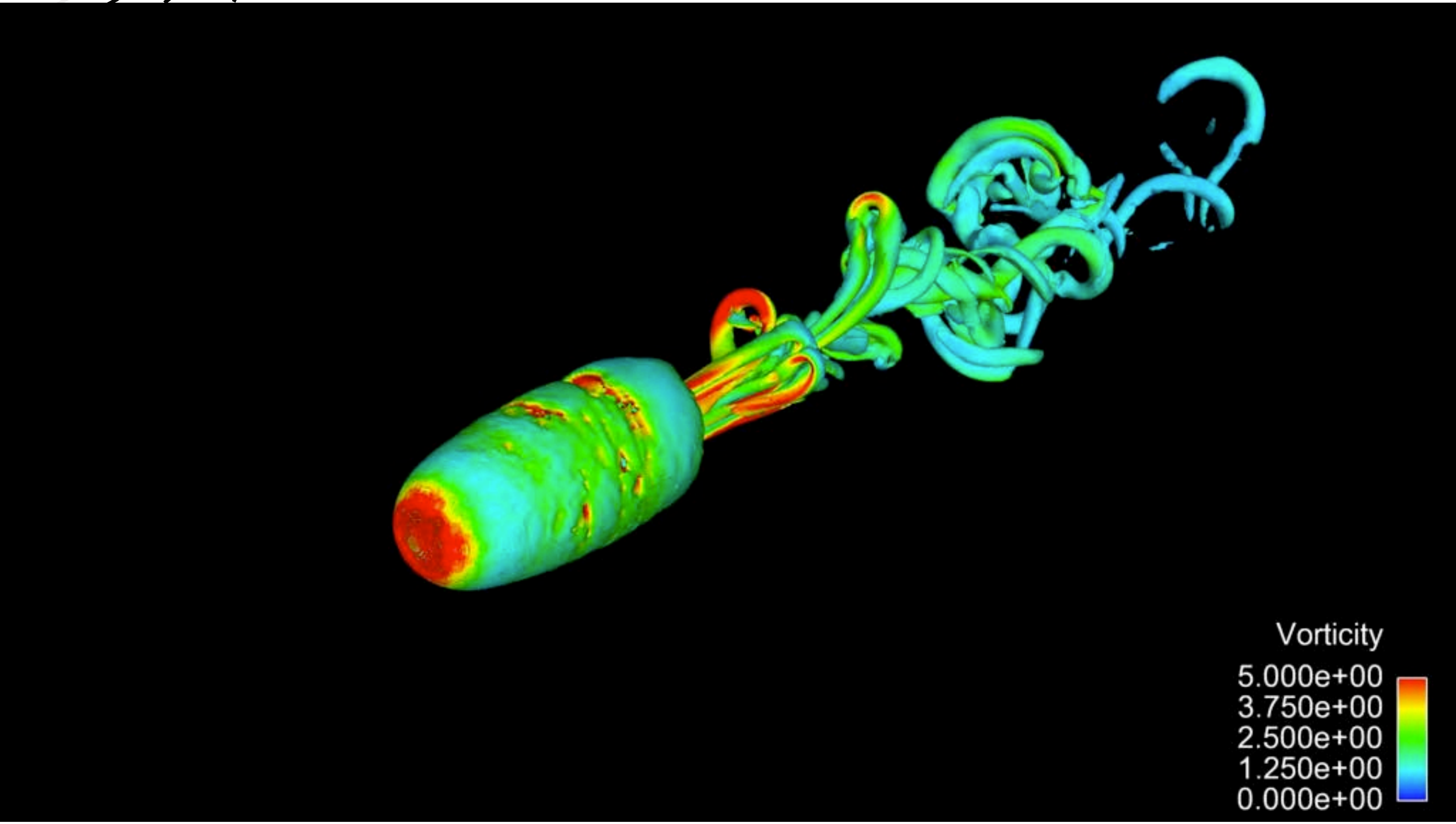
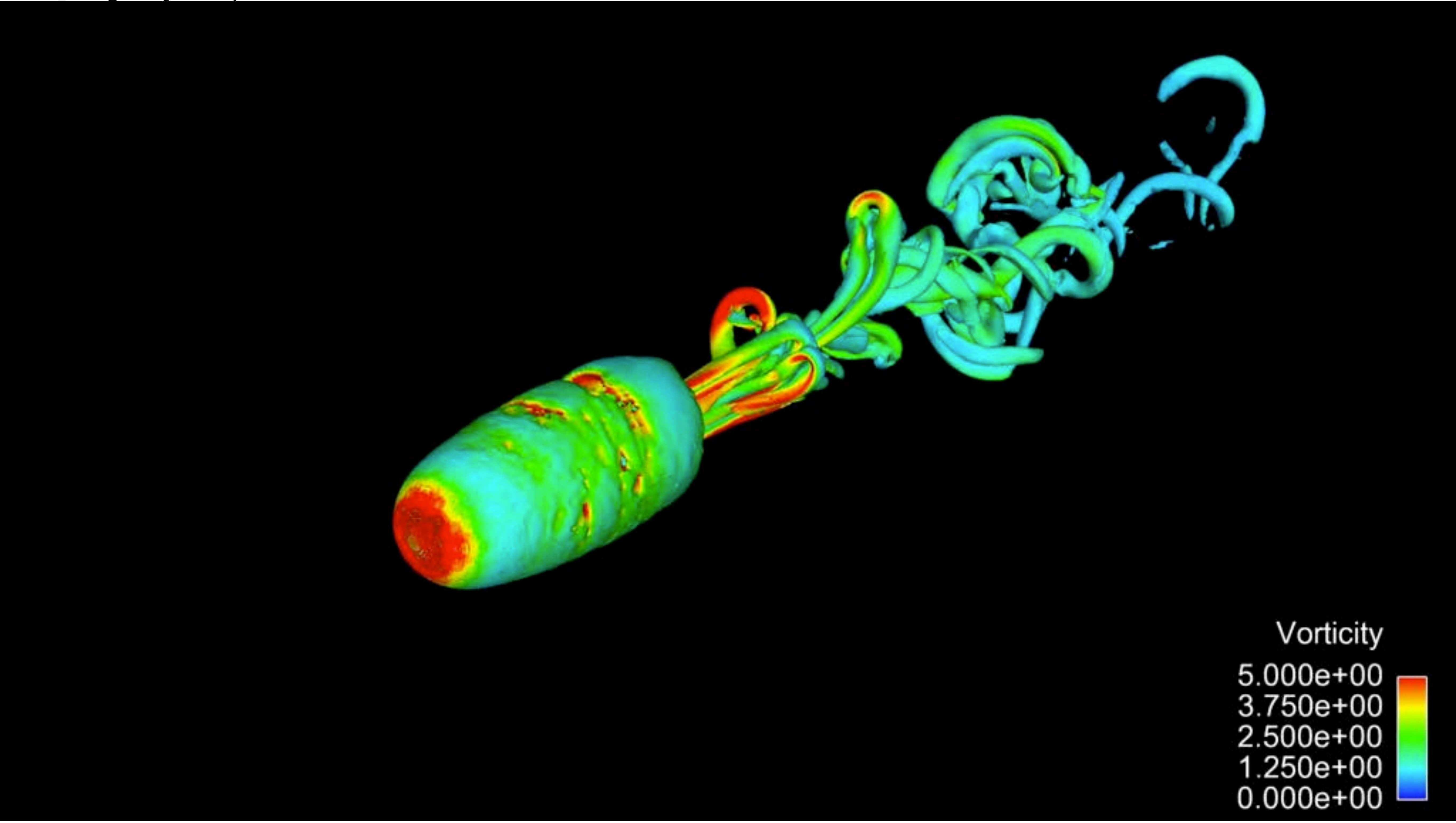
WDNS of Compressible Flow, Re=1000, M=0.7



WDNS of Compressible Flow, Re=1000, M=0.7



Conclusions and Perspectives

- Integrated framework for modeling and simulations of fluid flows:
  - highly *adaptive* numerical algorithm with robust *physics-based* grid adaptation and *active error* control
  - *tight integration* of numerics, physics based modeling, and *uncertainty quantification*
  - *unified hierarchy* of turbulence models of different fidelity
  - active control of *turbulence resolution*
  - spatially/temporarily varaible fidelity simulation
  - easy representation of flow geometry from auto CAD - penalization/immersed boundary approach

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http://scales.colorado.edu