Fan Noise Radiaion from Jet Engine Inlet: CAA Simulation and Physics

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Source Diagnostic Test (SDT) 2002
NASA Glenn Research Center

Outline

1. Code Development (using most advanced CAA methods)

    (a) Grid Design

    (b) Computational Model & Algorithm

    (c) Code Validation

2. Interesting/Puzzling Phenomena Observed
3. Mean Flow Results
4. Diffraction and Refraction
5. Parametric Study

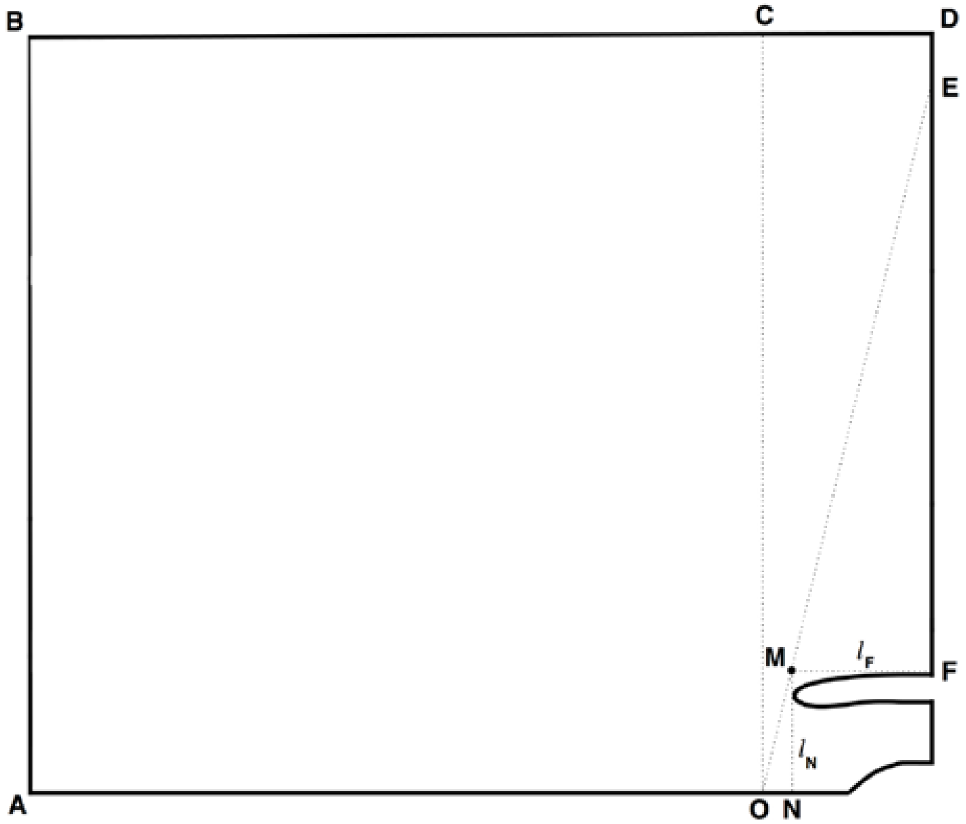
    (a) Forward Flight Effect

    (b) Frequency Effect

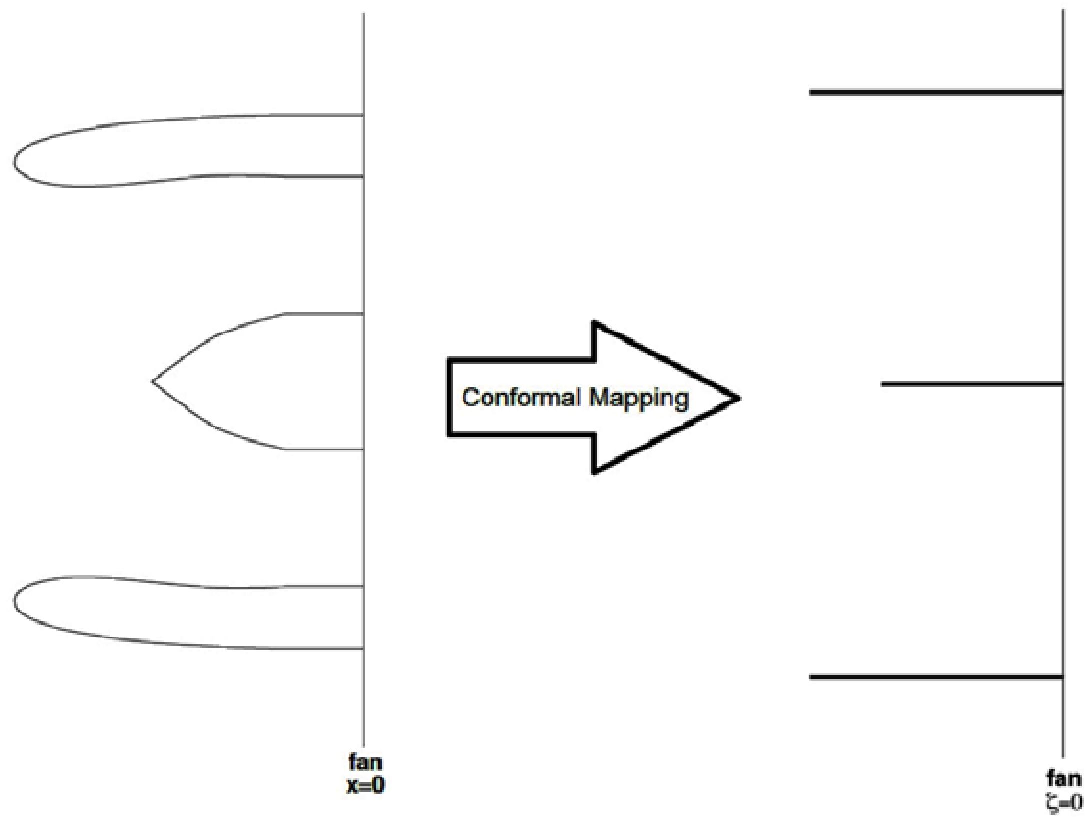
    (c) Azimuthal Mode Number Effect

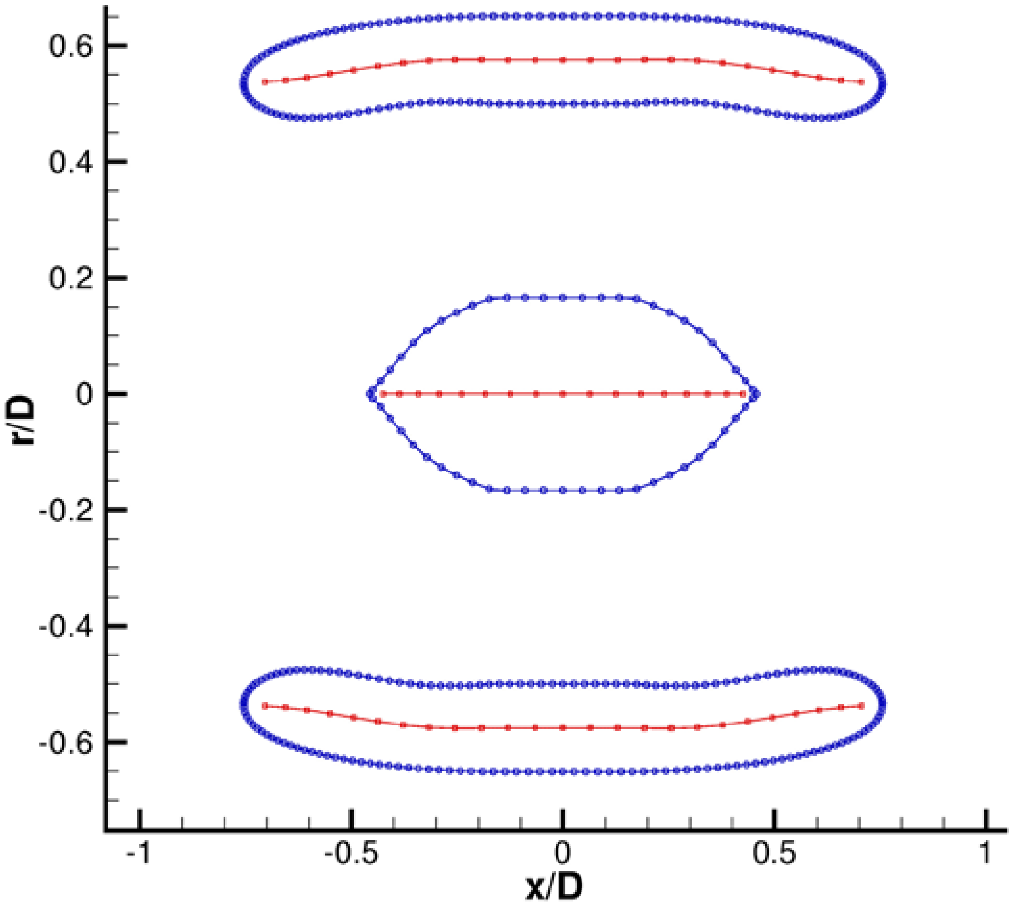
6. Summary and Conclusions

Grid Design

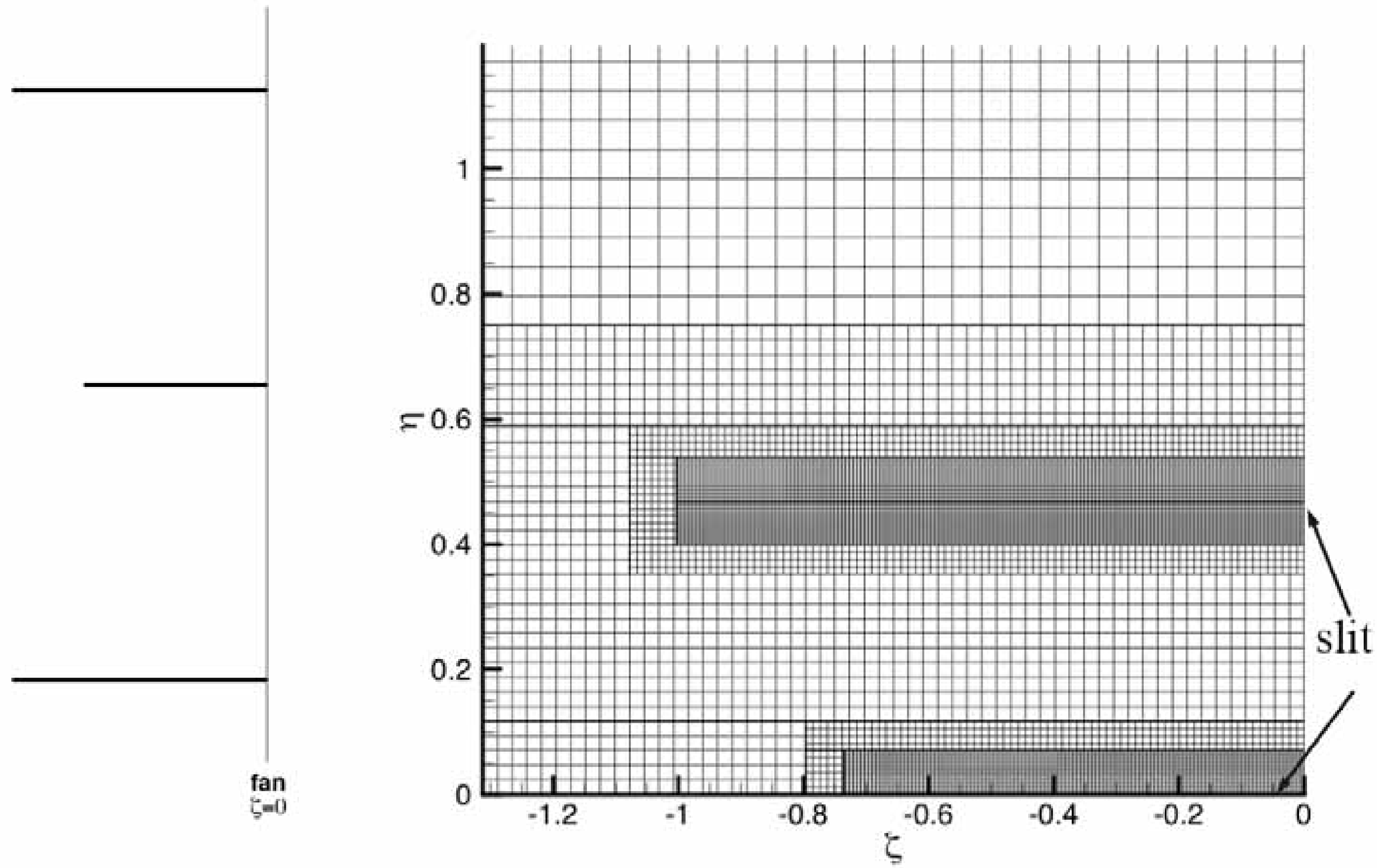


Source Diagnostic Test (SDT) 2002
NASA Glenn Research Center

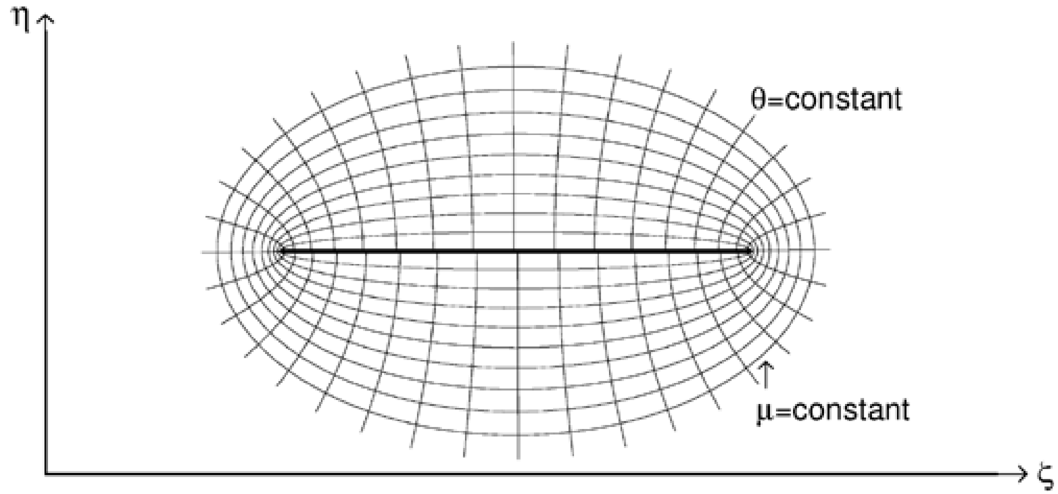

Conformal Mapping
fan
x=0
fan
ζ=0

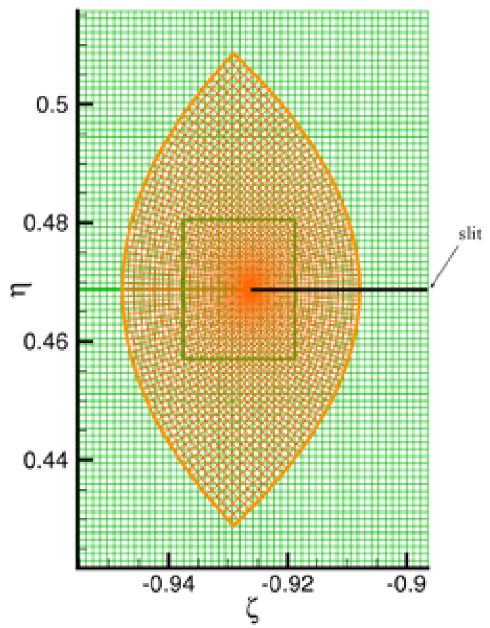


Source points and enforcement points



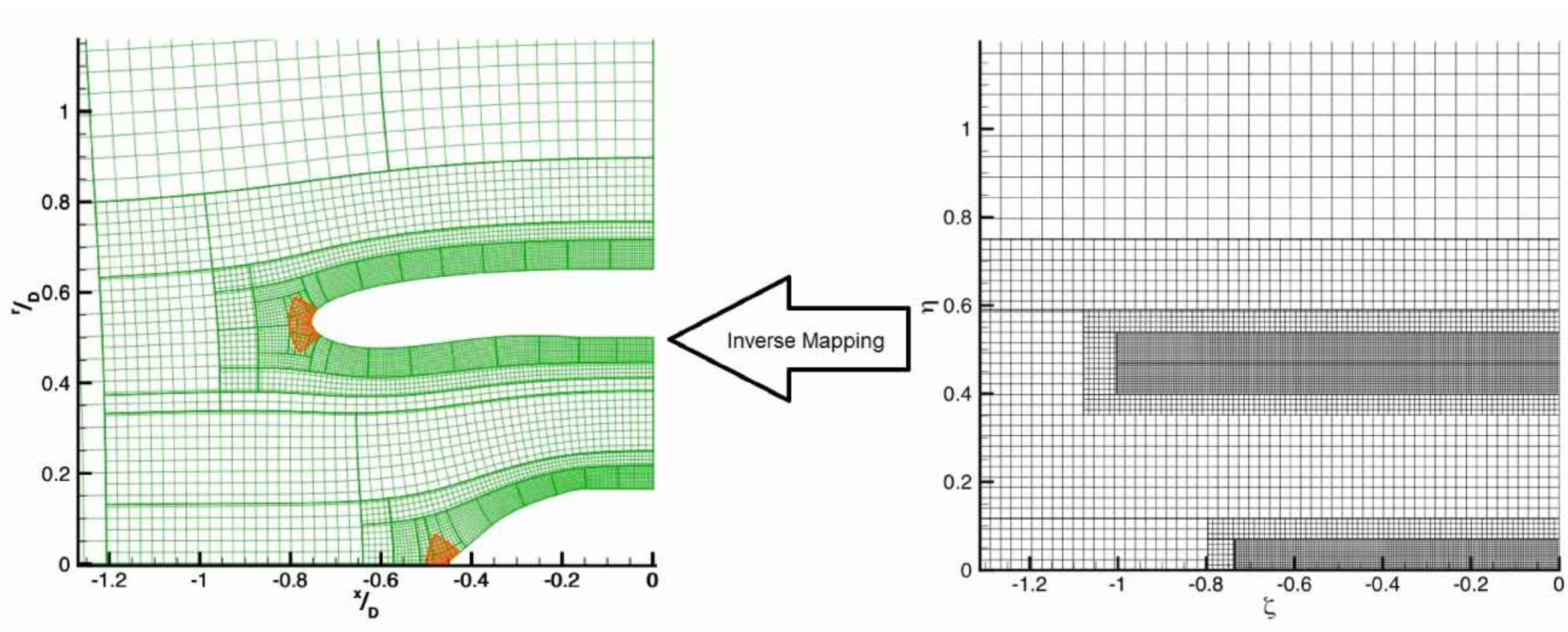
Computational plane with multi-size grids





Overset method is used to transfer data from elliptic grid to Cartesian grid.

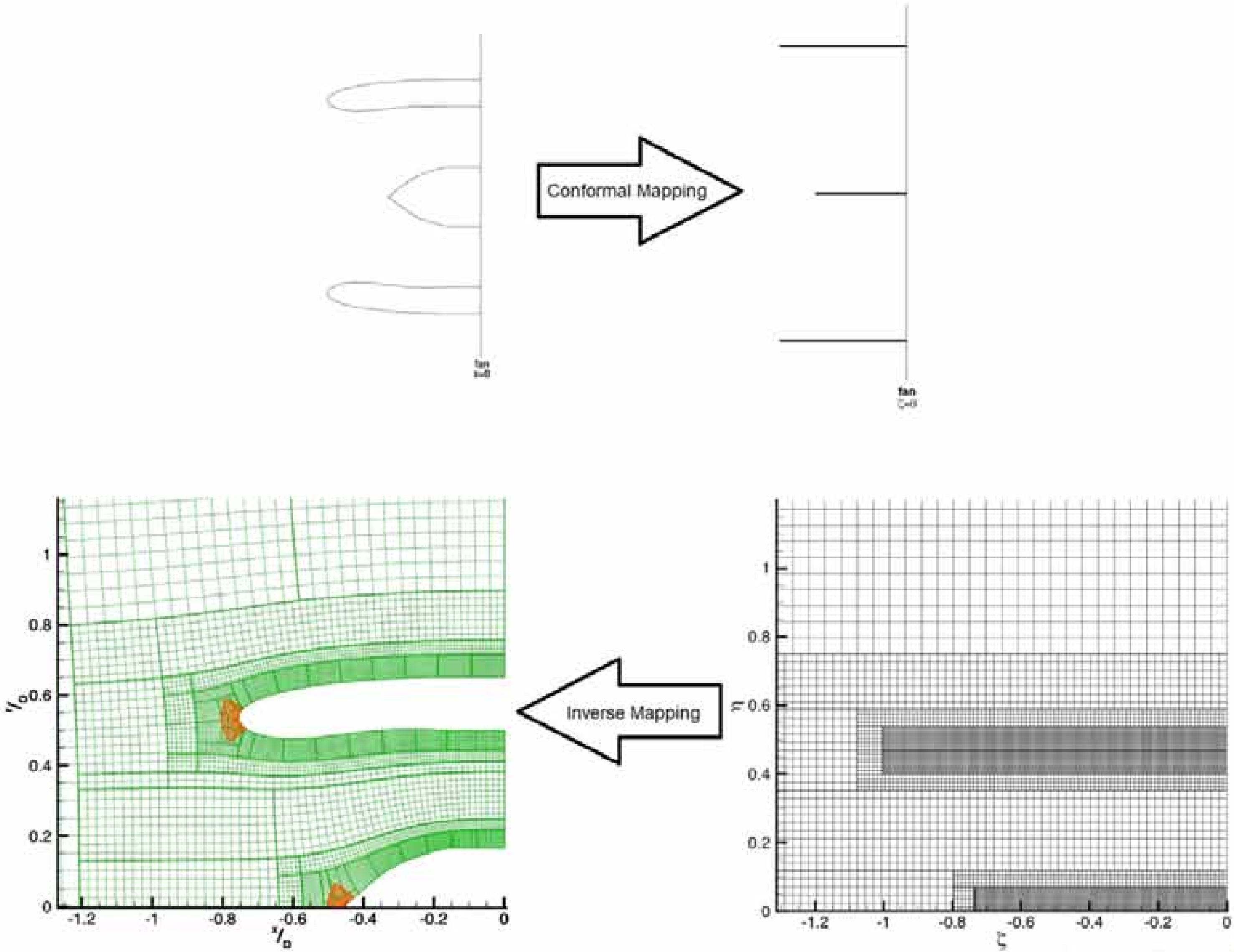
Elliptic mesh to provide a high resolution grid at the tip of casing and hub

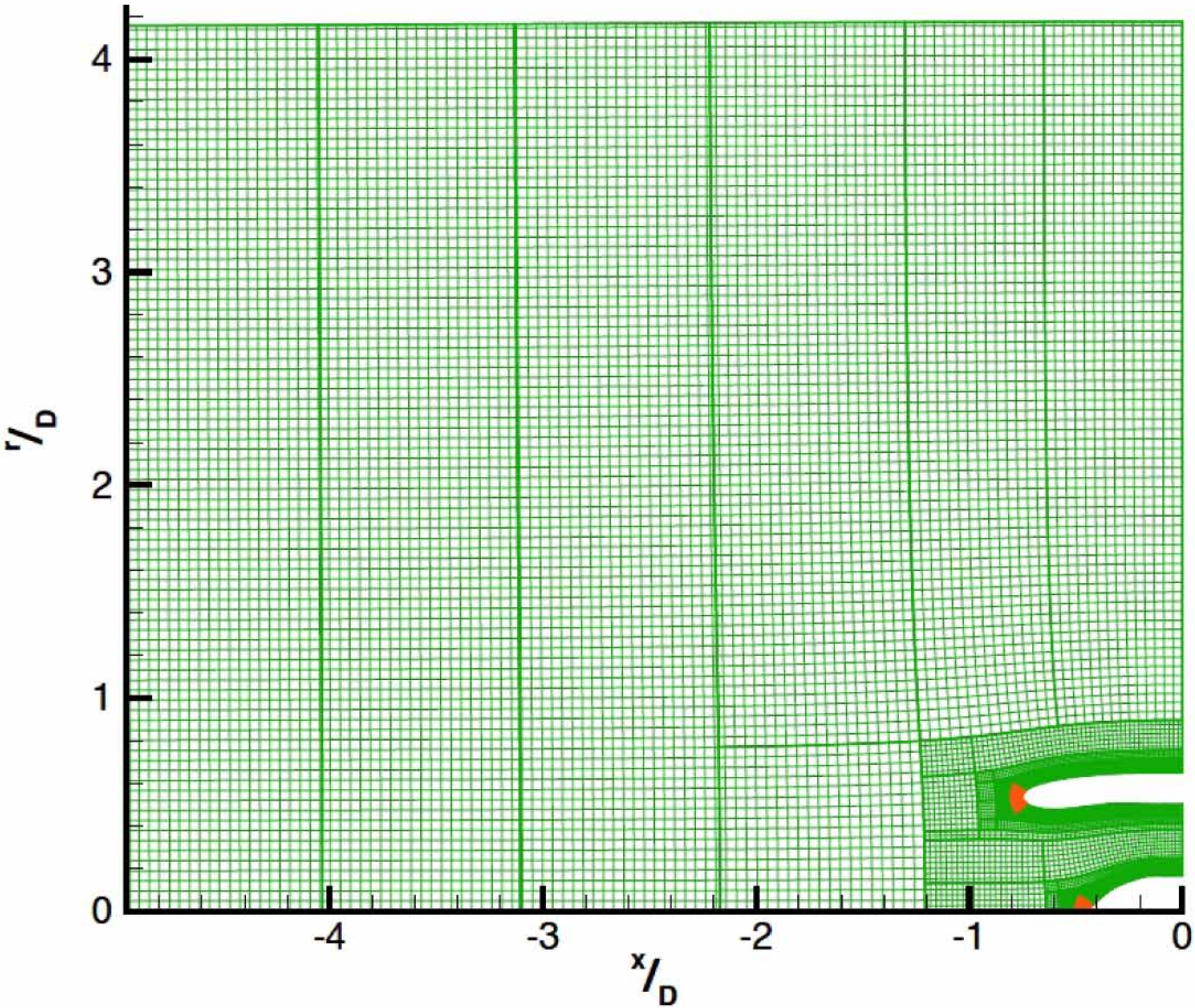


Physical Plane

Computational Plane

Grid Generation





The physical domain - domain extends 5 D for directivity computation

Computational Model and Algorithm

Mean flow computation

Full Euler equations

Acoustic Computation

Linearized Euler equations

Time-marching algorithm

Multi-size-mesh Multi-time-step DRP scheme

Mean Flow Calculation

Need mean flow to compute acoustics

- solve full Euler equations
- choose appropriate boundary conditions

Mean Flow Calculation

Physical variables

ρ density

u horizontal velocity

v vertical velocity

p pressure

Full Euler equations

$$\frac{\partial \rho}{\partial t} + v\left(\frac{\partial \rho}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial \rho}{\partial \eta}\frac{\partial \eta}{\partial r}\right) + u\left(\frac{\partial \rho}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial \rho}{\partial \eta}\frac{\partial \eta}{\partial x}\right) + \rho\left(\frac{\partial v}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x}\right) = 0$$

$$\frac{\partial u}{\partial t} + v\left(\frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial r}\right) + u\left(\frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x}\right) + \frac{1}{\rho}\left(\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial x}\right) = 0$$

$$\frac{\partial v}{\partial t} + v\left(\frac{\partial v}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial r}\right) + u\left(\frac{\partial v}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial x}\right) + \frac{1}{\rho}\left(\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial r}\right) = 0$$

$$\frac{\partial p}{\partial t} + v\left(\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial r}\right) + u\left(\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial x}\right) + \gamma p\left(\frac{\partial v}{\partial \zeta}\frac{\partial \zeta}{\partial r} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x}\right) = 0$$

Mean Flow Calculation

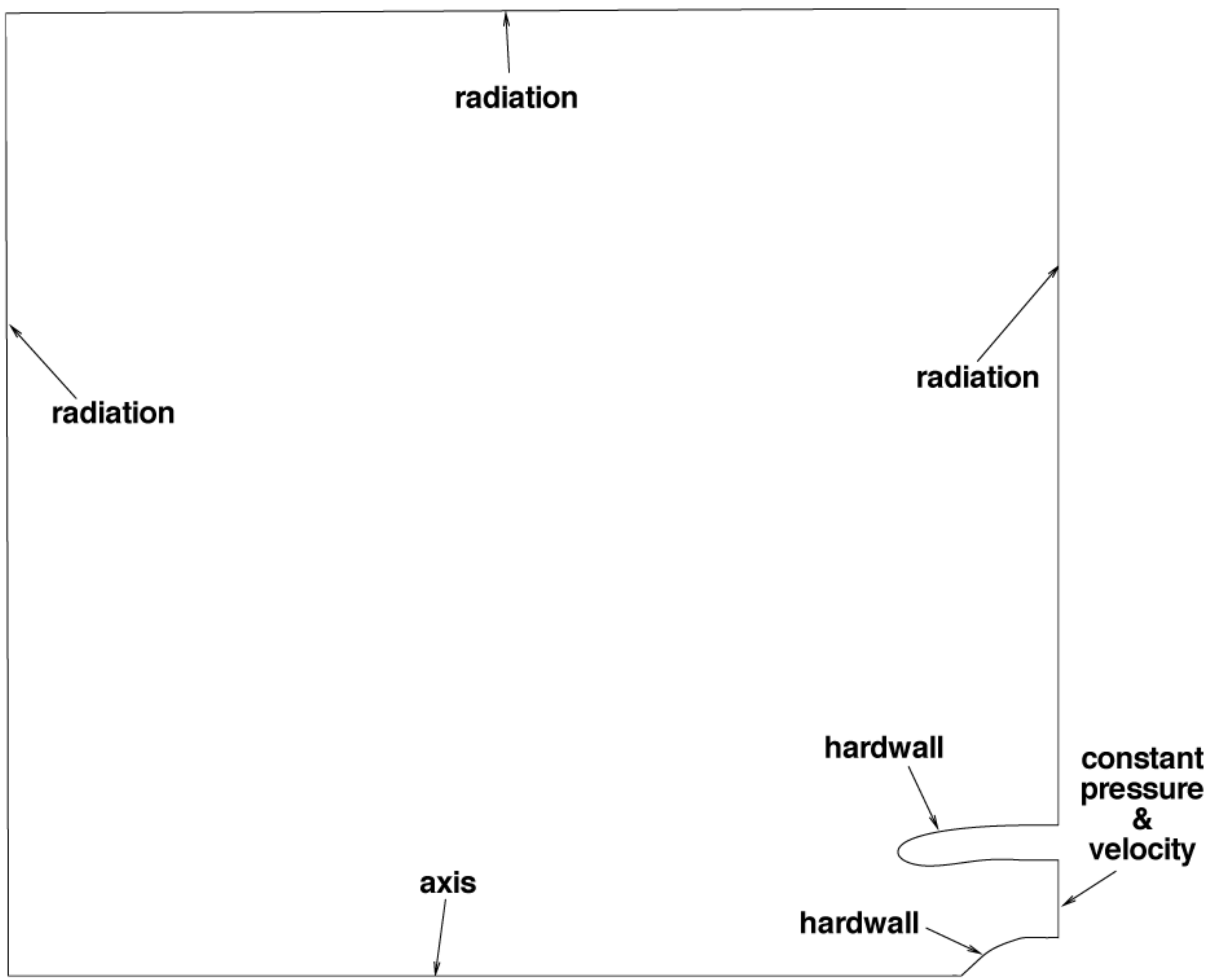
Full Euler equations (overset grids)

$$\frac{\partial\rho}{\partial t}+v\left[\frac{\partial\rho}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial\rho}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)\right]+u\left[\frac{\partial\rho}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right.$$
$$\left.+\frac{\partial\rho}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]+\rho\left[\frac{\partial v}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial v}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{v}{r}\right.$$
$$\left.+\frac{\partial u}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)+\frac{\partial u}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]=0$$

$$\frac{\partial u}{\partial t}+v\left[\frac{\partial u}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial u}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)\right]+u\left[\frac{\partial u}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right.$$
$$\left.+\frac{\partial u}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]+\frac{1}{\rho}\left[\frac{\partial p}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)+\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]=0$$

$$\frac{\partial v}{\partial t}+v\left[\frac{\partial v}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial v}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)\right]+u\left[\frac{\partial v}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right.$$
$$\left.+\frac{\partial v}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]+\frac{1}{\rho}\left[\frac{\partial p}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)\right]=0$$

$$\frac{\partial p}{\partial t}+v\left[\frac{\partial p}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)\right]+u\left[\frac{\partial p}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right.$$
$$\left.+\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]+\gamma p\left[\frac{\partial v}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{\partial v}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial r}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial r}\right)+\frac{v}{r}\right.$$
$$\left.+\frac{\partial u}{\partial\mu}\left(\frac{\partial\mu}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\mu}{\partial\eta}\frac{\partial\eta}{\partial x}\right)+\frac{\partial u}{\partial\theta}\left(\frac{\partial\theta}{\partial\zeta}\frac{\partial\zeta}{\partial x}+\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]=0$$



Boundary conditions

Acoustics Computation

Acoustics

- solve linearized Euler equations
  - use computed mean flow
  - use computed mean flow derivatives
- choose appropriate boundary conditions

Acoustics Computation

Physical variables

ρ density

u horizontal x-velocity

v vertical r-velocity

w ϕ-velocity

p pressure

$\bar{\rho}$ mean flow density

$\bar{u}$ mean flow horizontal velocity

$\bar{v}$ mean flow vertical velocity

$\bar{p}$ mean flow pressure

Acoustics Computation

Linearized Euler equations

$$\frac{\partial\rho}{\partial t} + \bar{v}\left(\frac{\partial\rho}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial\rho}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + v\frac{\partial\bar{\rho}}{\partial r} + \bar{u}\left(\frac{\partial\rho}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial\rho}{\partial\eta}\frac{\partial\eta}{\partial x}\right) + u\frac{\partial\bar{\rho}}{\partial x}$$
$$+\bar{\rho}\left[\left(\frac{\partial v}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + \frac{v}{r} + \frac{im}{r}w + \left(\frac{\partial u}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]$$
$$+\rho\left(\frac{\partial\bar{v}}{\partial r} + \frac{\bar{v}}{r} + \frac{\partial\bar{u}}{\partial x}\right) = 0$$

$$\frac{\partial u}{\partial t} + \bar{v}\left(\frac{\partial u}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + v\frac{\partial\bar{u}}{\partial r} + \bar{u}\left(\frac{\partial u}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}\right) + u\frac{\partial\bar{u}}{\partial x}$$
$$+\frac{1}{\bar{\rho}}\left(\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial p}{\partial\eta}\frac{\partial\eta}{\partial x}\right) - \frac{\rho}{\bar{\rho}^2}\frac{\partial\bar{p}}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \bar{v}\left(\frac{\partial v}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + v\frac{\partial\bar{v}}{\partial r} + \bar{u}\left(\frac{\partial v}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial x}\right) + u\frac{\partial\bar{v}}{\partial x}$$
$$+\frac{1}{\bar{\rho}}\left(\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial p}{\partial\eta}\frac{\partial\eta}{\partial r}\right) - \frac{\rho}{\bar{\rho}^2}\frac{\partial\bar{p}}{\partial r} = 0$$

$$\frac{\partial w}{\partial t} + \bar{v}\left(\frac{\partial w}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial w}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + \bar{u}\left(\frac{\partial w}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial w}{\partial\eta}\frac{\partial\eta}{\partial x}\right) + \frac{\bar{v}w}{r} + \frac{1}{\bar{\rho}}\frac{im}{r}p = 0$$

$$\frac{\partial p}{\partial t} + \bar{v}\left(\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial p}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + v\frac{\partial\bar{p}}{\partial r} + \bar{u}\left(\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial p}{\partial\eta}\frac{\partial\eta}{\partial x}\right) + u\frac{\partial\bar{p}}{\partial x}$$
$$+\gamma\bar{p}\left[\left(\frac{\partial v}{\partial\zeta}\frac{\partial\zeta}{\partial r} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial r}\right) + \frac{v}{r} + \frac{im}{r}w + \left(\frac{\partial u}{\partial\zeta}\frac{\partial\zeta}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\right]$$
$$+\gamma p\left(\frac{\partial\bar{v}}{\partial r} + \frac{\bar{v}}{r} + \frac{\partial\bar{u}}{\partial x}\right) = 0$$

Acoustics Computation

Linearized Euler equations (overset grids)

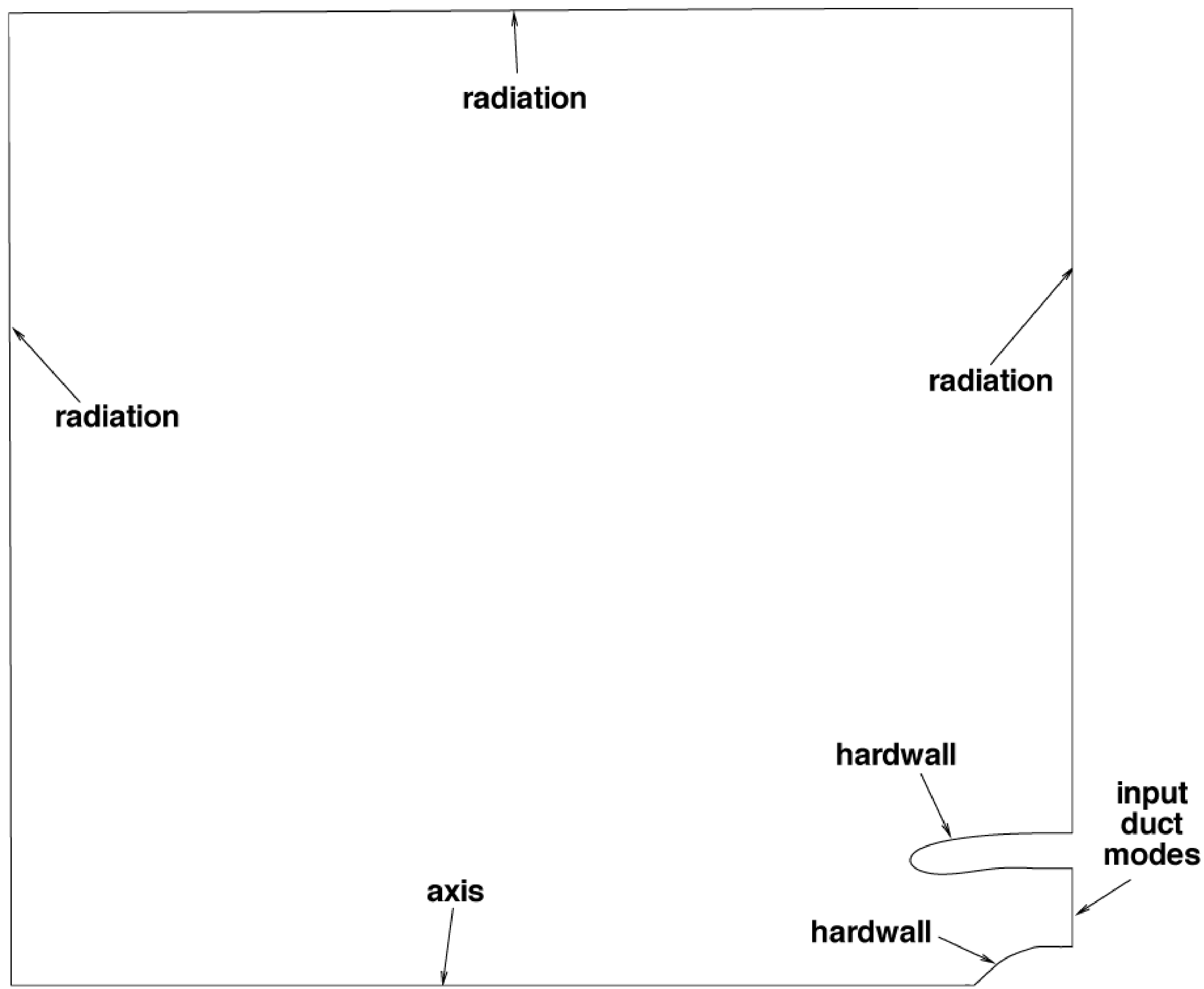
$$
\begin{aligned}
&\frac{\partial \rho}{\partial t}+\bar{v}\left[\frac{\partial \rho}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial \rho}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]+v\frac{\partial \bar{\rho}}{\partial r}+\bar{u}\left[\frac{\partial \rho}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right.\\
&\quad\left.+\frac{\partial \rho}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]+u\frac{\partial \bar{\rho}}{\partial x}+\bar{\rho}\left\{\left[\frac{\partial v}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial v}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]\right.\\
&\quad\left.+\frac{v}{r}+\frac{im}{r}w+\left[\frac{\partial u}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)+\frac{\partial u}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]\right\}+\rho\left(\frac{\partial \bar{v}}{\partial r}+\frac{\bar{v}}{r}+\frac{\partial \bar{u}}{\partial x}\right)=0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial u}{\partial t}+\bar{v}\left[\frac{\partial u}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial u}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]+v\frac{\partial \bar{u}}{\partial r}+\bar{u}\left[\frac{\partial u}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right.\\
&\quad\left.+\frac{\partial u}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]+u\frac{\partial \bar{u}}{\partial x}+\frac{1}{\bar{\rho}}\left[\frac{\partial p}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)+\frac{\partial p}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]-\frac{\rho}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x}=0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial v}{\partial t}+\bar{v}\left[\frac{\partial v}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial v}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]+v\frac{\partial \bar{v}}{\partial r}+\bar{u}\left[\frac{\partial v}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right.\\
&\quad\left.+\frac{\partial v}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]+u\frac{\partial \bar{v}}{\partial x}+\frac{1}{\bar{\rho}}\left[\frac{\partial p}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial p}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]-\frac{\rho}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial r}=0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial w}{\partial t}+\bar{v}\left[\frac{\partial w}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial w}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]+\bar{u}\left[\frac{\partial w}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right.\\
&\quad\left.+\frac{\partial w}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]+\frac{\bar{v}w}{r}+\frac{1}{\bar{\rho}}\frac{im}{r}p=0
\end{aligned}
$$

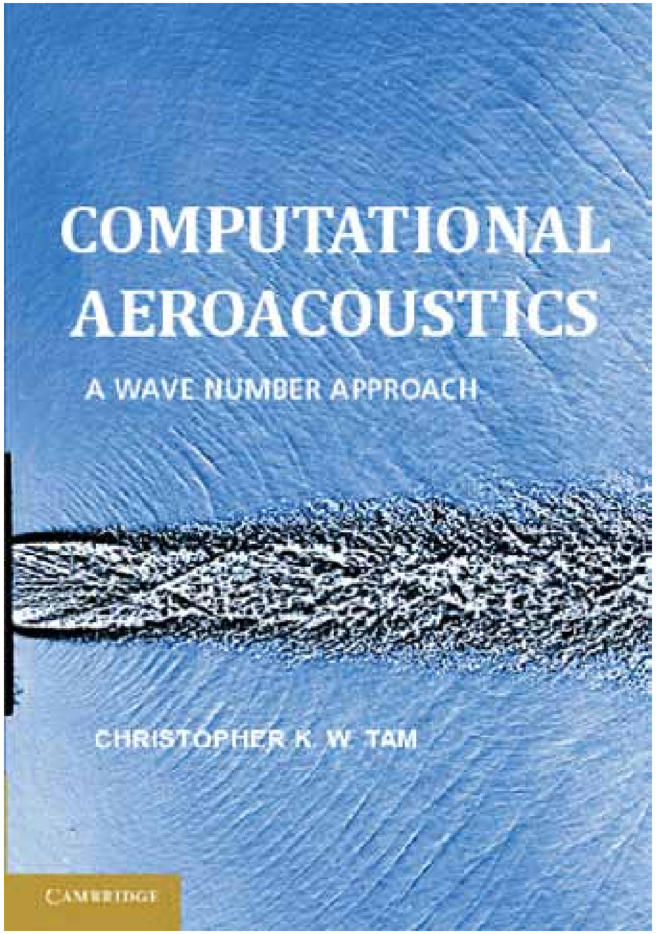
$$
\begin{aligned}
&\frac{\partial p}{\partial t}+\bar{v}\left[\frac{\partial p}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial p}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]+v\frac{\partial \bar{p}}{\partial r}+\bar{u}\left[\frac{\partial p}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right.\\
&\quad\left.+\frac{\partial p}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]+u\frac{\partial \bar{p}}{\partial x}+\gamma\bar{p}\left\{\left[\frac{\partial v}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial r}\right)+\frac{\partial v}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial r}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial r}\right)\right]\right.\\
&\quad\left.+\frac{v}{r}+\frac{im}{r}w+\left[\frac{\partial u}{\partial \mu}\left(\frac{\partial \mu}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial x}\right)+\frac{\partial u}{\partial \theta}\left(\frac{\partial \theta}{\partial \zeta}\frac{\partial \zeta}{\partial x}+\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\right]\right\}+\gamma p\left(\frac{\partial \bar{v}}{\partial r}+\frac{\bar{v}}{r}+\frac{\partial \bar{u}}{\partial x}\right)=0
\end{aligned}
$$



radiation
radiation
radiation
hardwall
input
duct
modes
axis
hardwall

Time marching computation

Governing equations are discretized and time marched according to the multi-size-mesh multi-time-step dispersion-relation-preserving (DRP) scheme.



References:

Tam & Kurbatskii 2003 “multi-size-mesh multi-time-step DRP scheme for multiple scales aeroacoustics problems. International Journal of CFD. Vol. 17, 119-132.

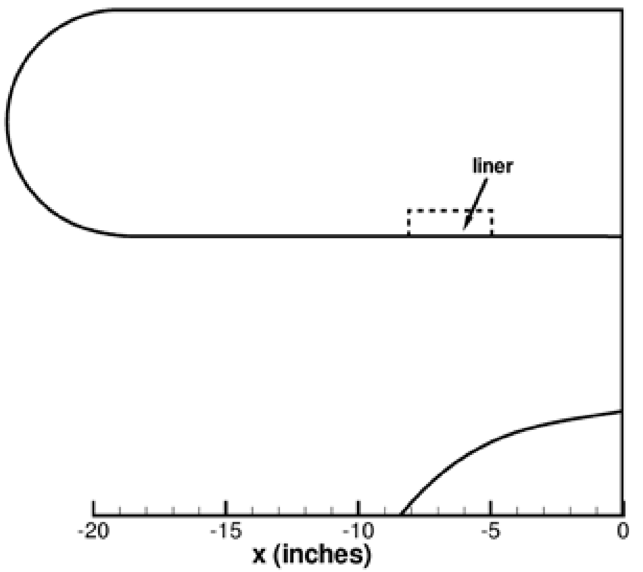
‘Computational Aeroacoustics: a wave number approach’ by C.K.W. Tam Cambridge University Press 2012 (Chapter 12. Multiscales Problems)

Code Validation

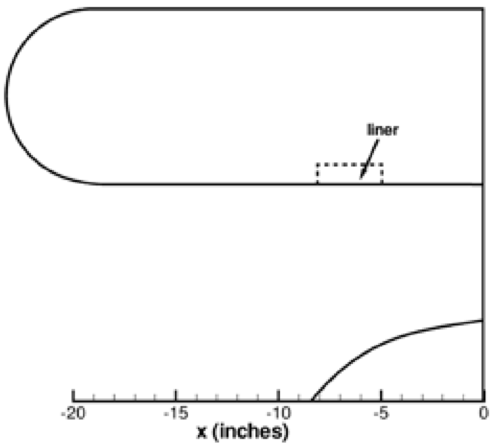
JT-15D Static Test data

L.J. Heidelberg, E.J. Rice, L. Homyak, Acoustic performance of inlet suppressors on an engine generating a single mode, American Institute of Aeronautics and Astronautics Paper 81-1965, 1981.

K.J. Baumeister, S.J. Horowitz, Finite element-integral acoustic simulation of JT15D turbofan engine, Journal of Vibration, Acoustics, Stress, and Reliability in Design 106 (1984) 405–413.



JT15D engine inlet geometry



Test conditions

28 blades, 41 rods , blade passage frequency 3150 Hz

Cut-off ratio = 1.05

$M_{\text{fan face}} = 0.147$, $M_{\text{flight}} = 0$

Only one propagating mode with $m = 13$, $n = 1$

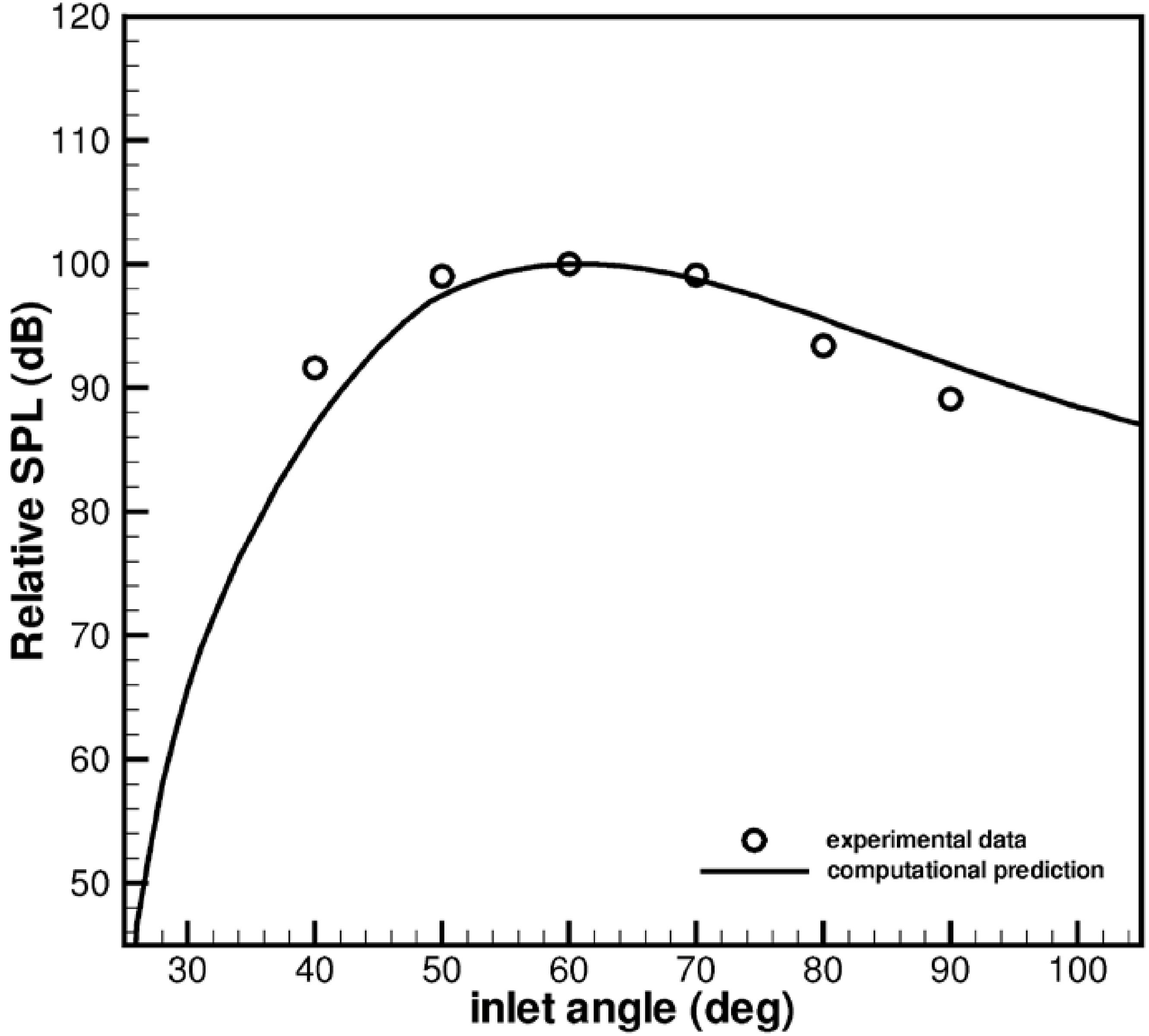
3 Test cases

(a) Hard wall

(b) Liner $Z = 0.638 + i\,0.5$

$Z = 1.136 + i\,0.5$

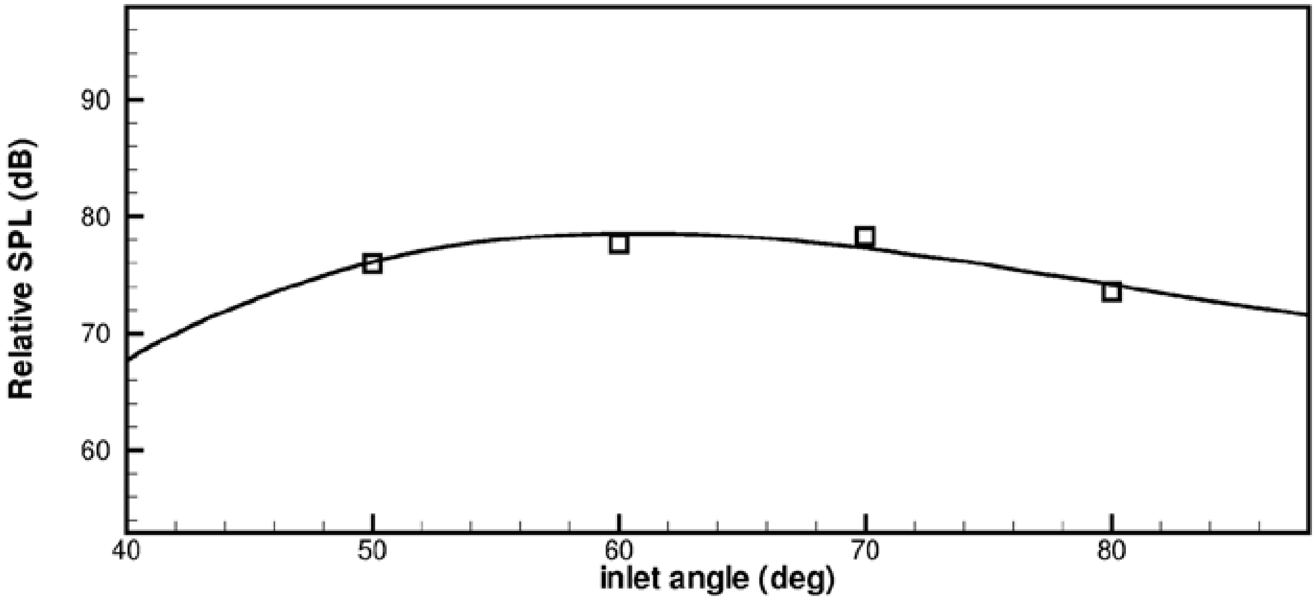
Hard Wall



120
110
100
90
80
70
60
50
Relative SPL (dB)
30
40
50
60
70
80
90
100
inlet angle (deg)
experimental data
computational prediction

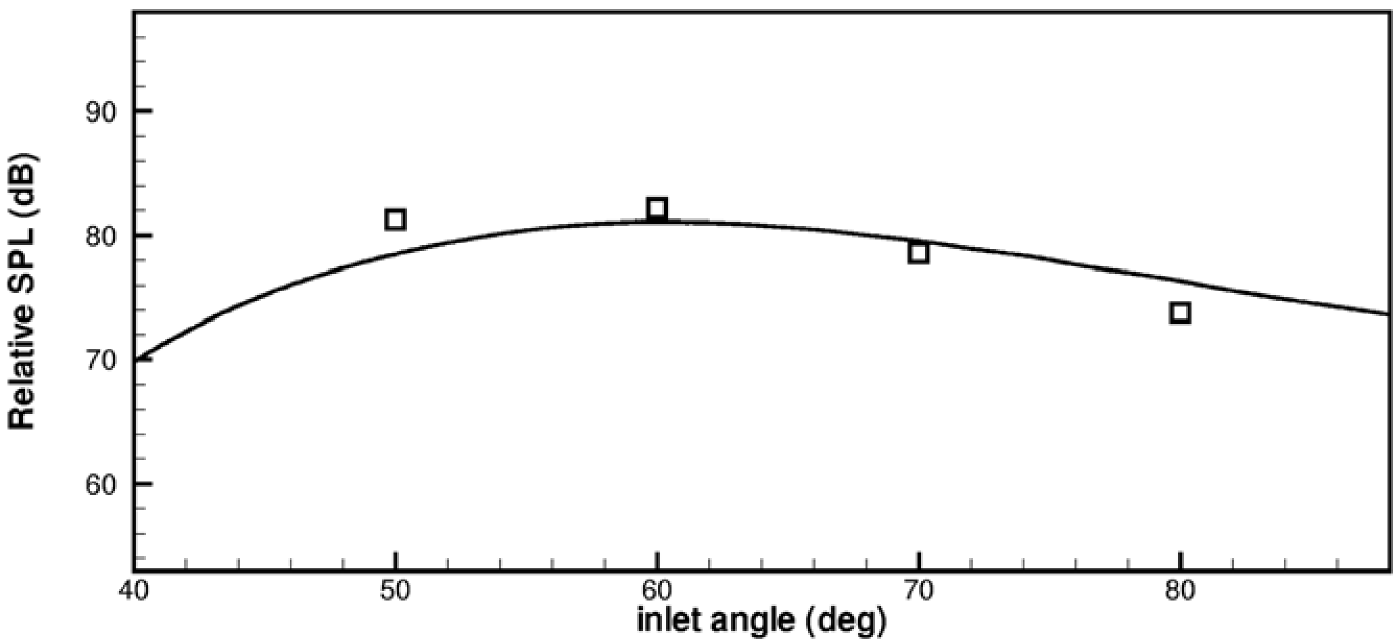
Acoustic Liner

$$Z = 0.638 + i\,0.5$$



Acoustic Liner

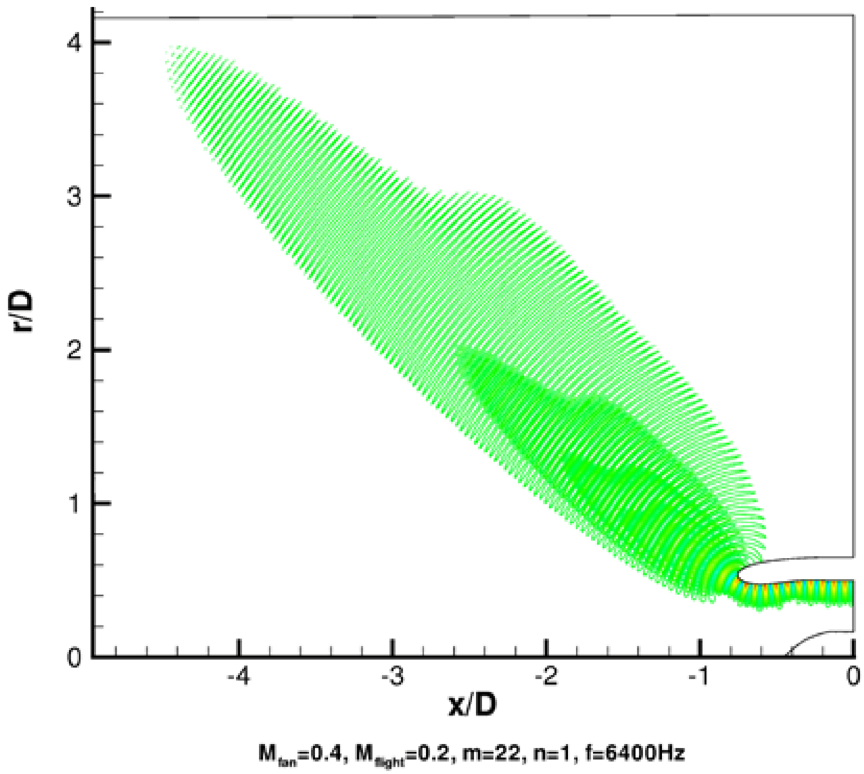
$$Z = 1.136 + i\,0.5$$

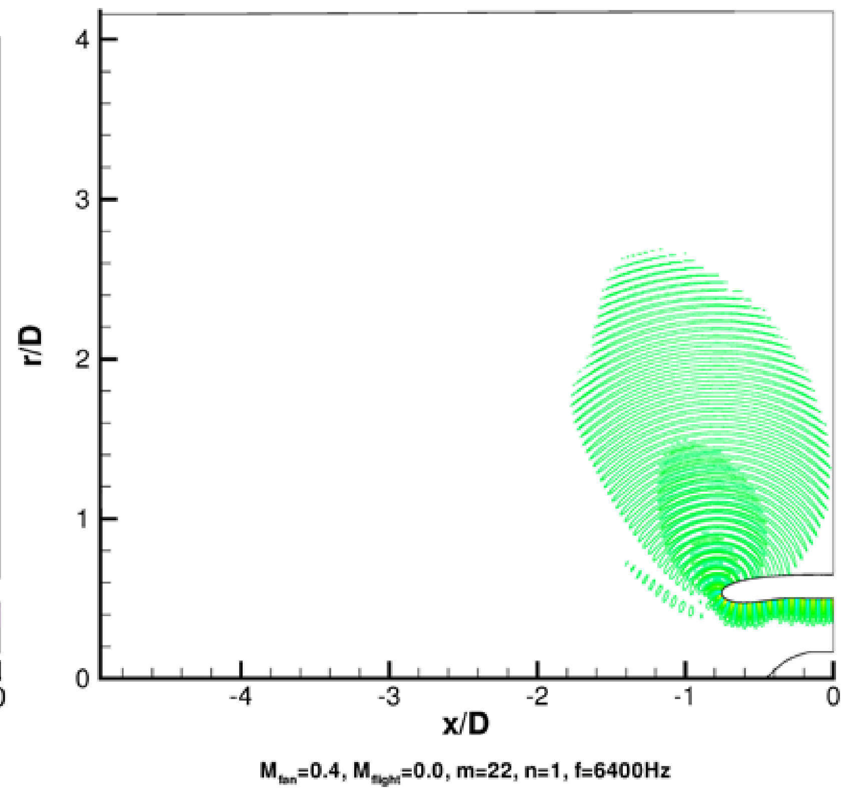


Interesting / Puzzling engine inlet radiation phenomena observed

QuickTime™ and a
mpeg4 decompressor
are needed to see this picture.

Azimuthal mode number, m=22; radial mode number, n=1; frequency, f=6400 Hz





Forward flight Mach number =0.2

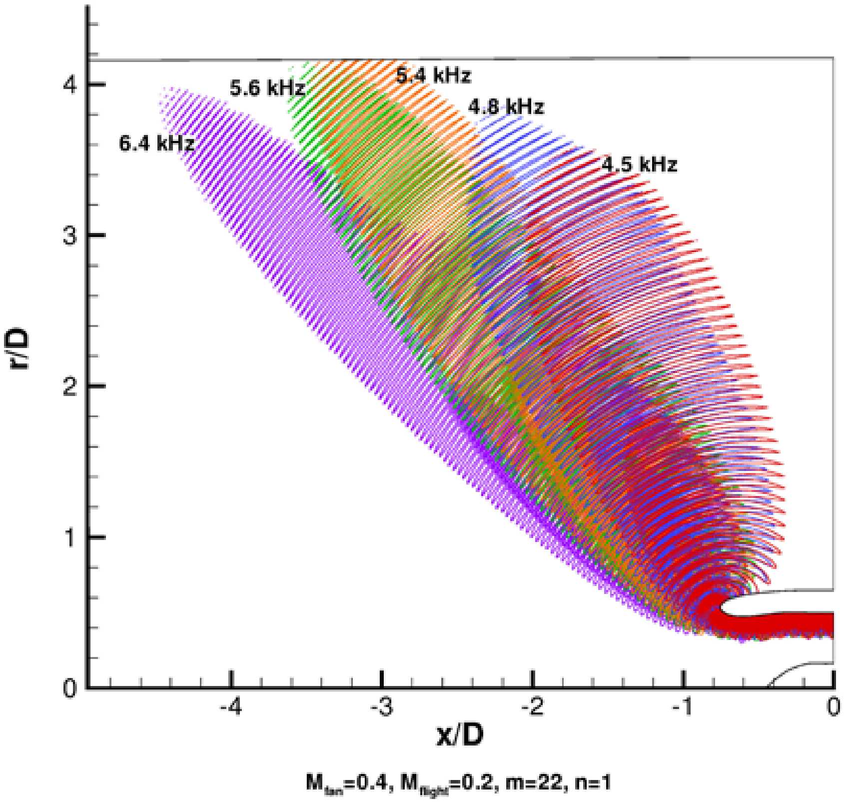
No forward flight

Why is there such a large change in directivity for a low flight Mach number of 0.2 ?

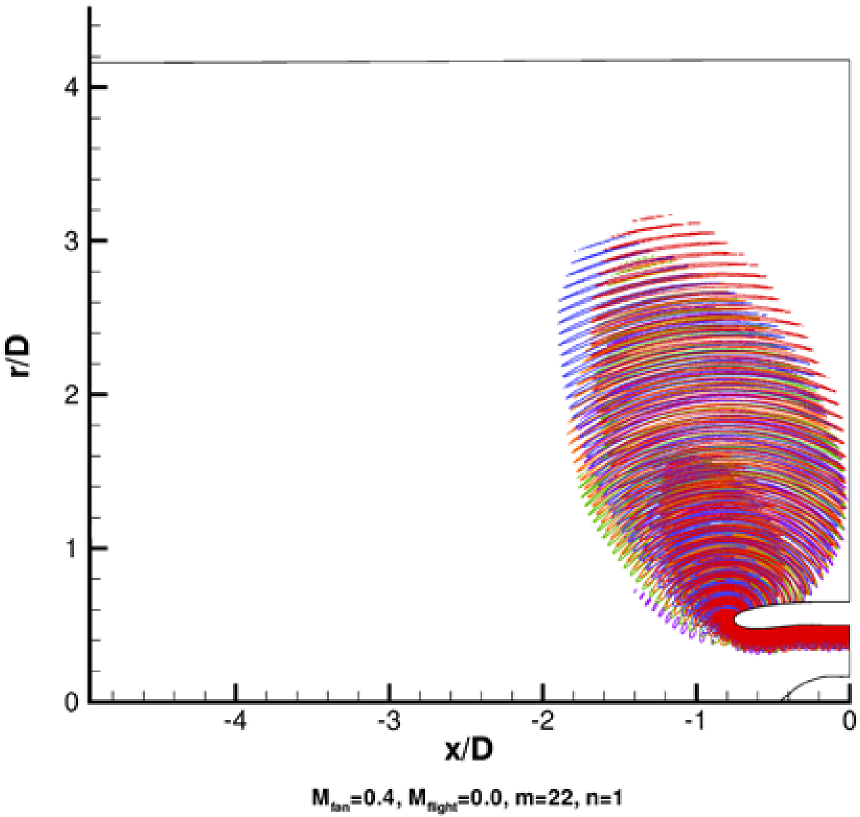
Another example

Consider the radiation of the same duct mode (with the same m and n) but at different frequencies.

Fan face Mach number, M_{fan} = 0.4; m = 22; n = 1



Flight Mach number = 0.2



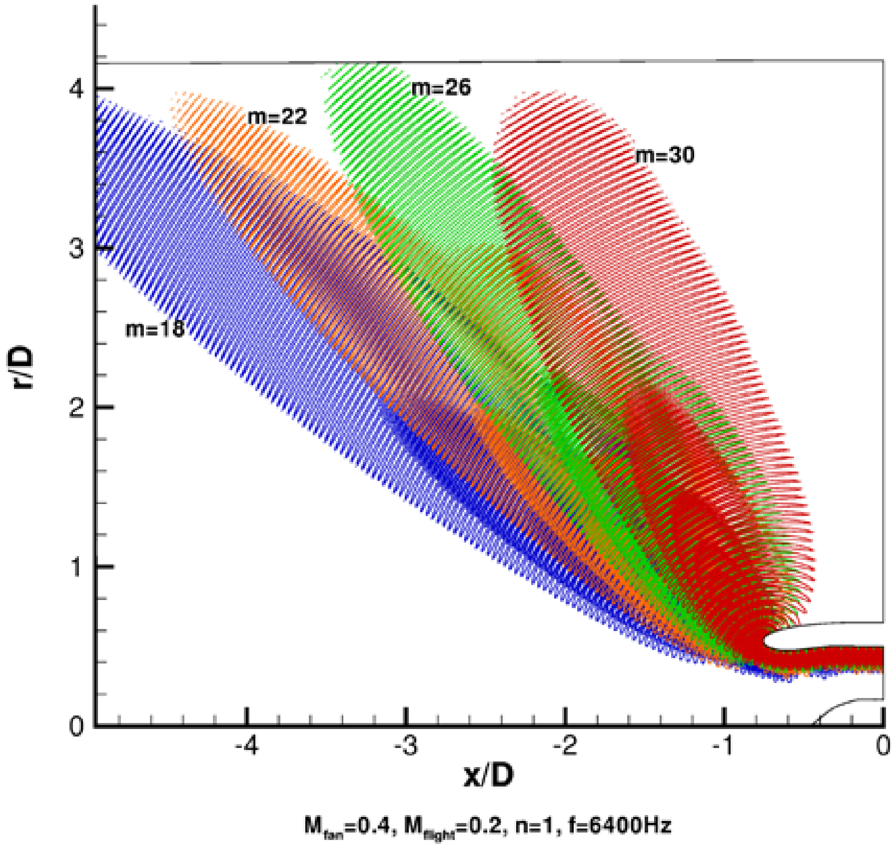
No forward flight

Why is there no frequency effect at static condition, but significant effect when there is forward flight ?

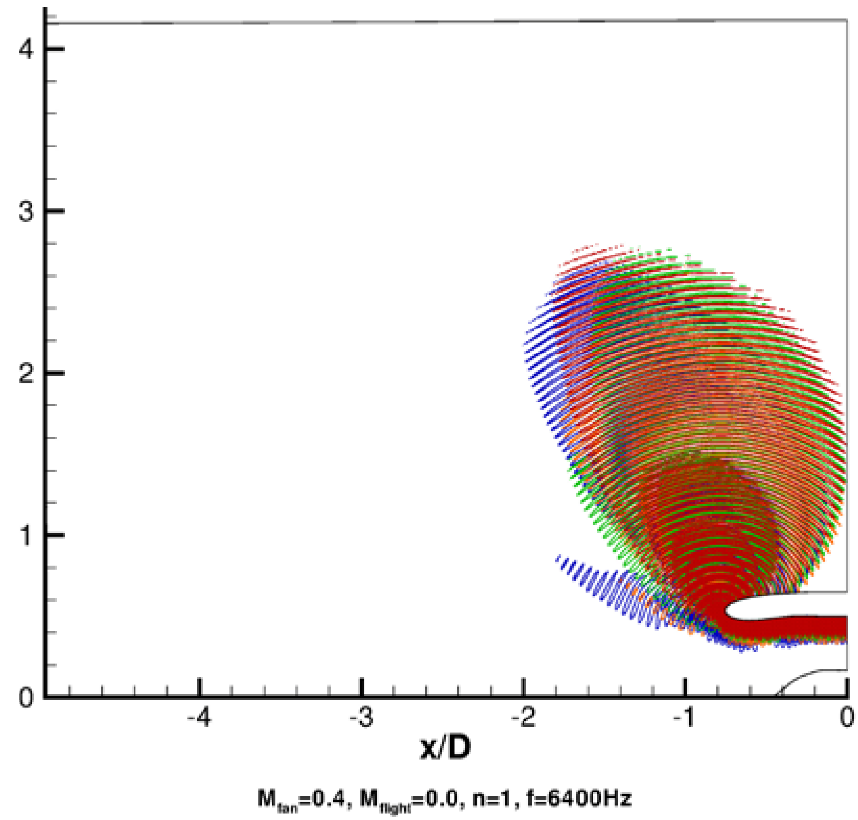
Another example of complexity

Consider the radiation of duct mode with the same frequency, the same radial mode number, but with different azimuthal mode number.

Fan Face Mach number = 0.4; f = 6400 Hz; n = 1



Flight Mach number = 0.2

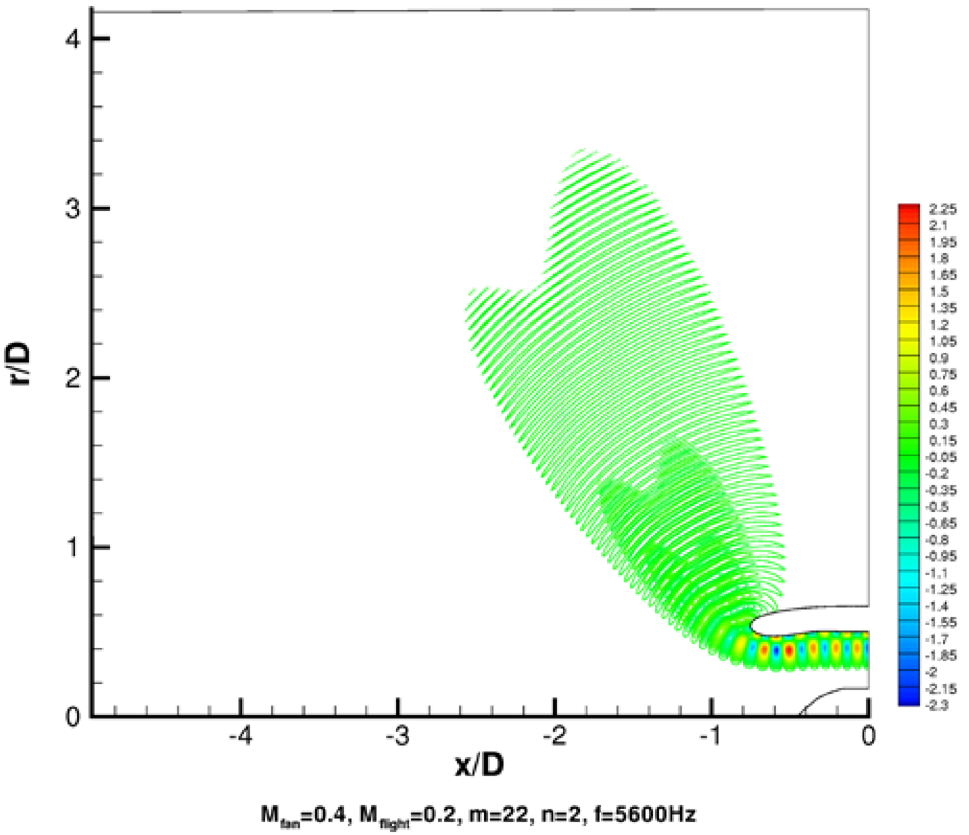


No forward flight

Why is there no azimuthal mode number effect at static condition, but significant effect when there is forward flight ?

Another example

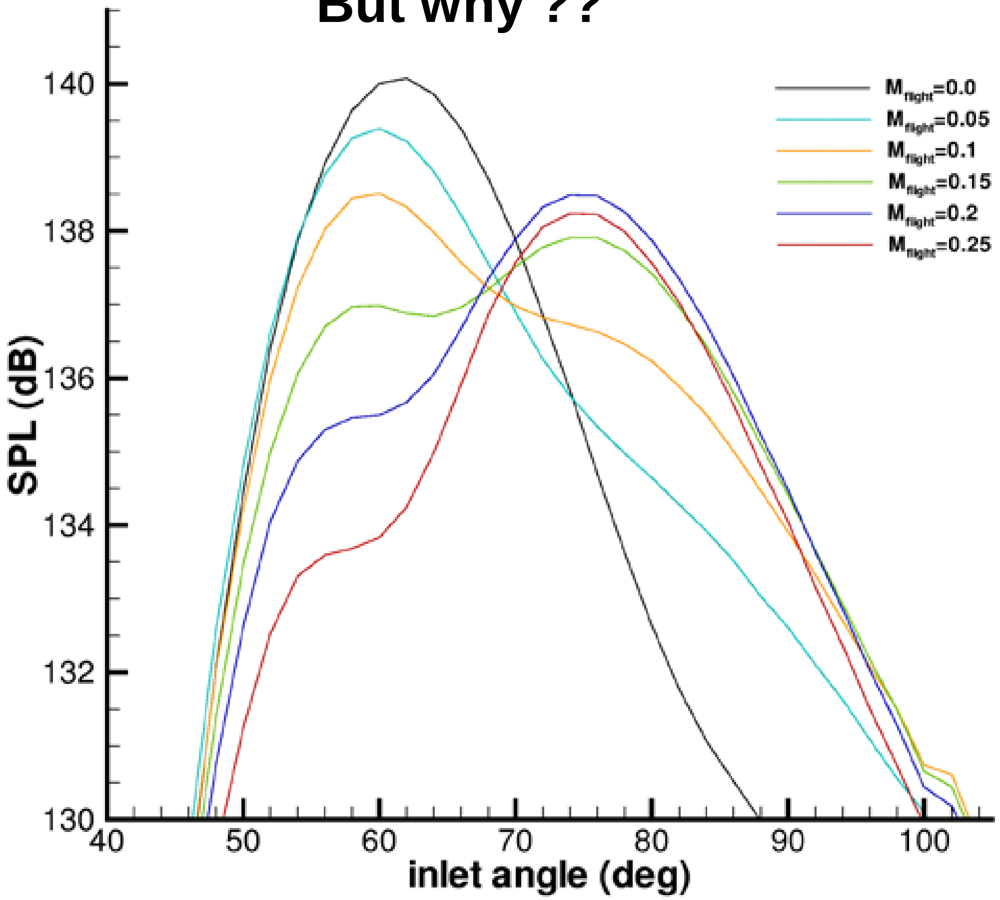
Radiation of duct modes with second radial mode number, n = 2.



Radiation splits into two beams

Relative intensity of the two beams are affected by forward flight.

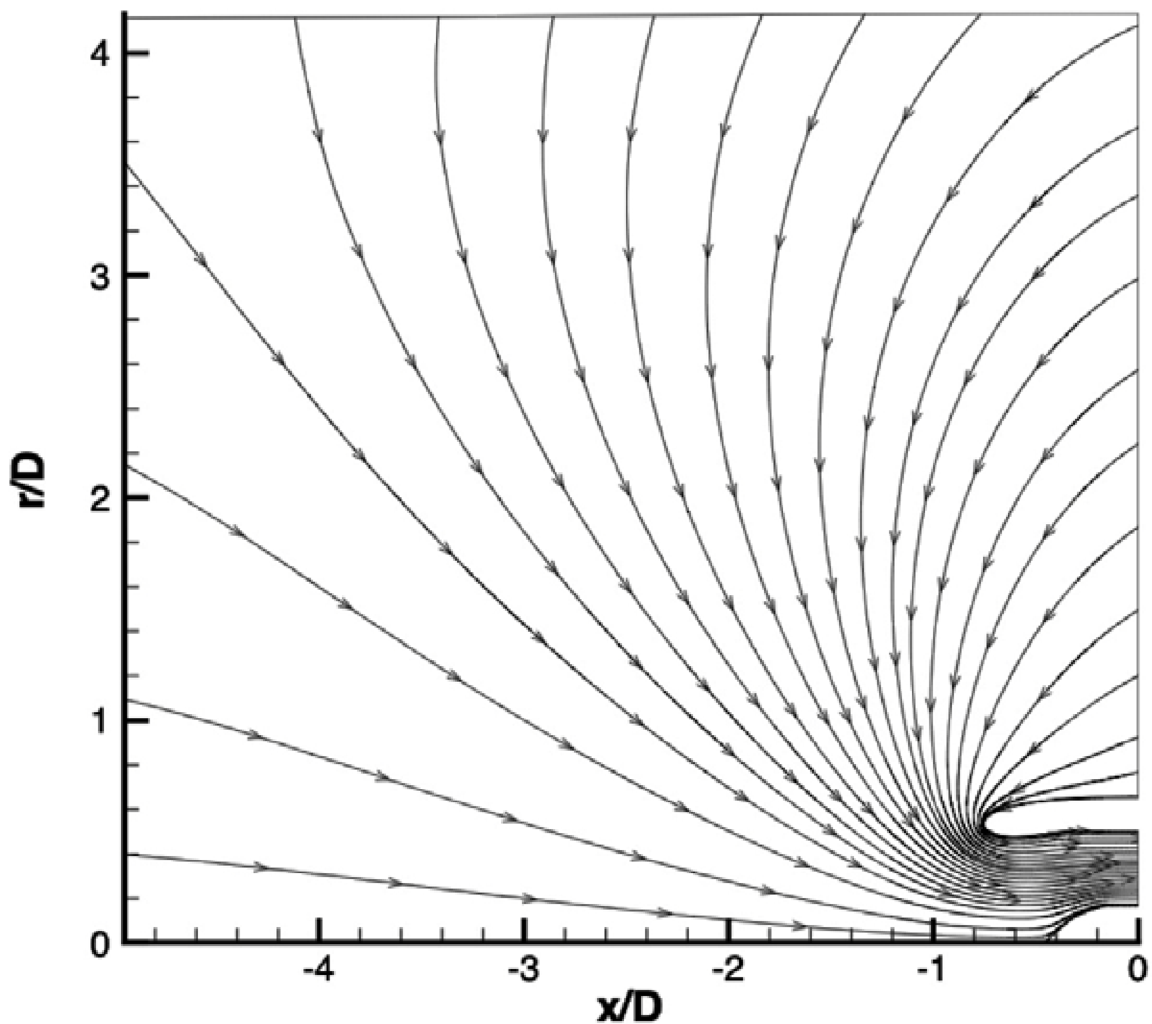
But why ??



M_{fan}=0.4, m=22, n=2, f=5600Hz

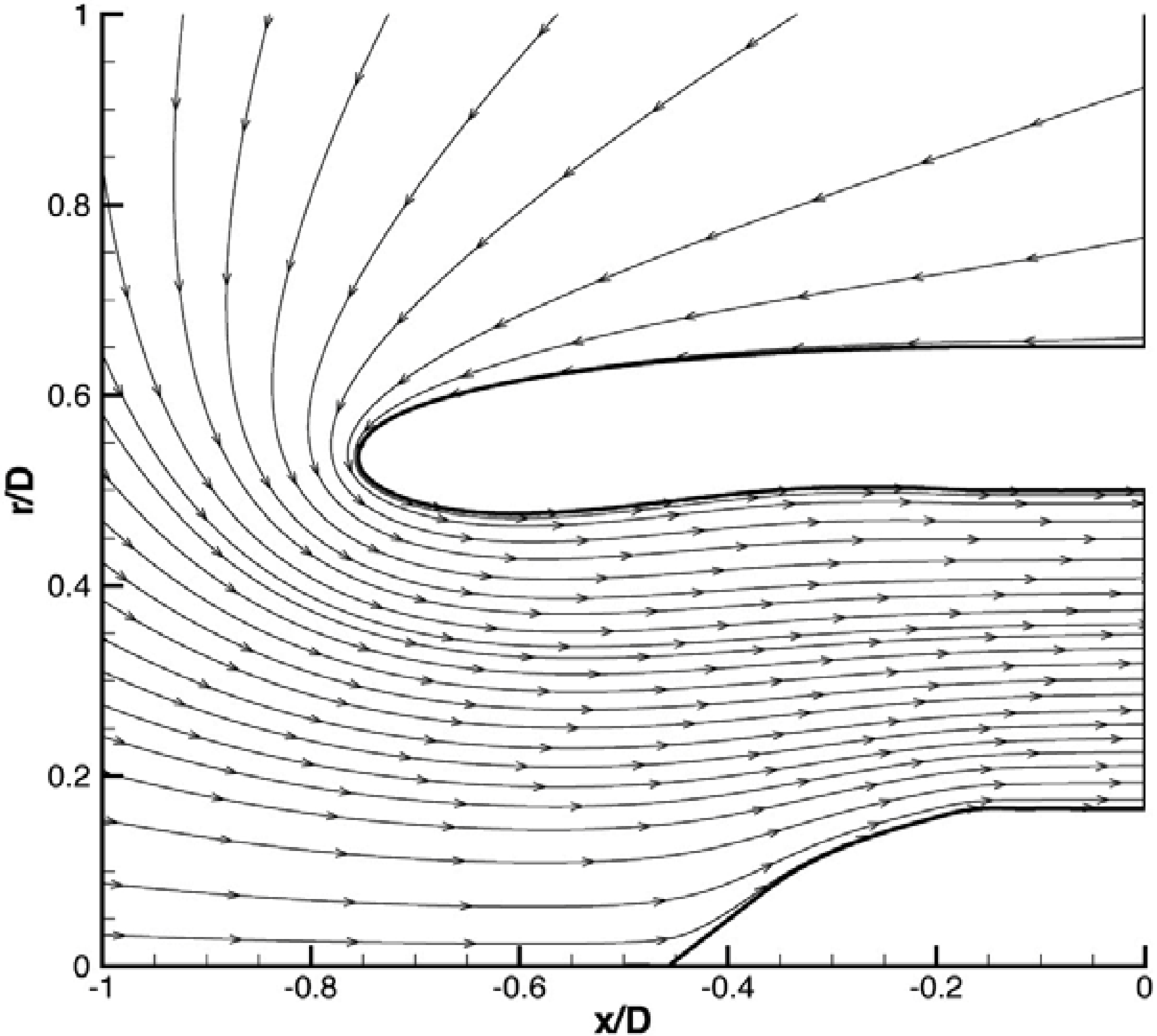
The mean flow around the engine casing turns out would play a crucial role in all the phenomena just shown.

Mean Flow Stream lines



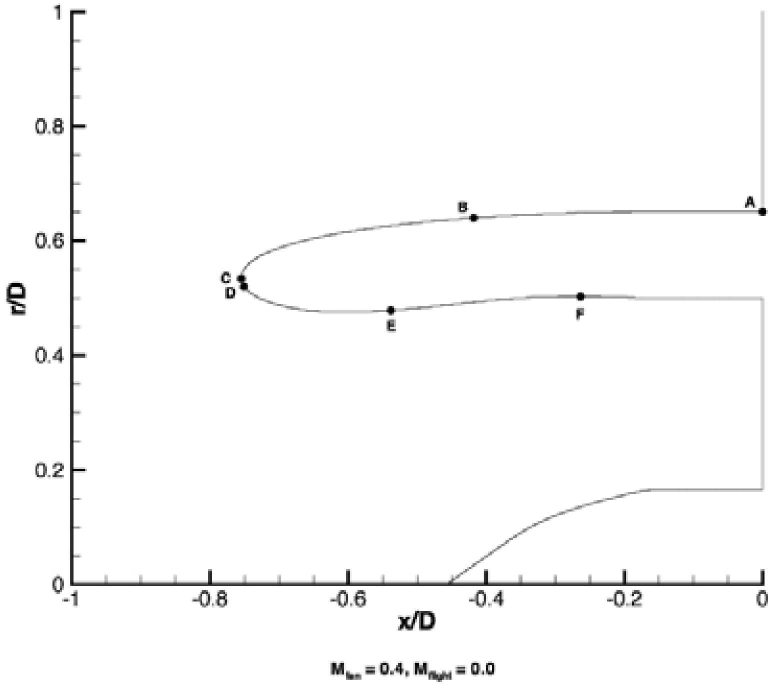
No forward flight

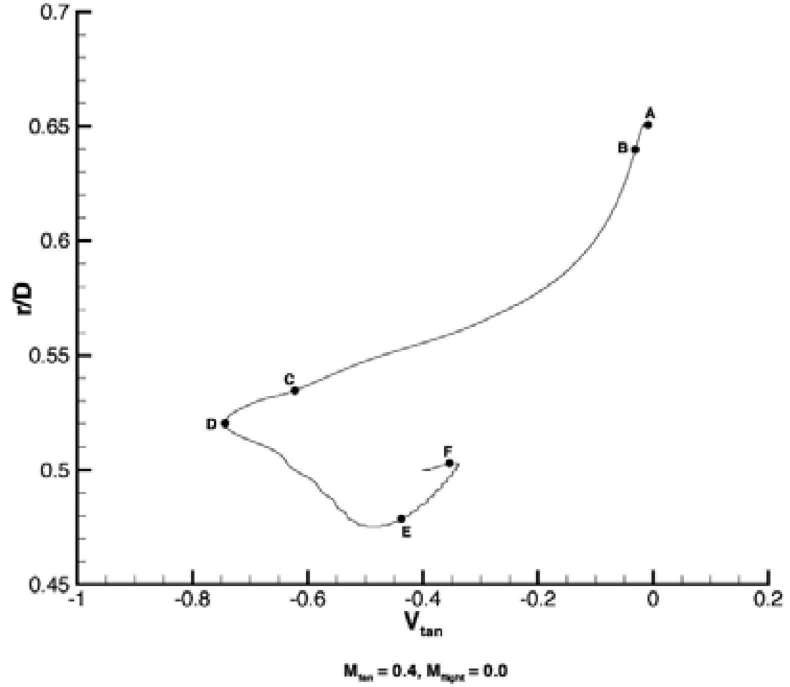
Mean Flow



No forward flight

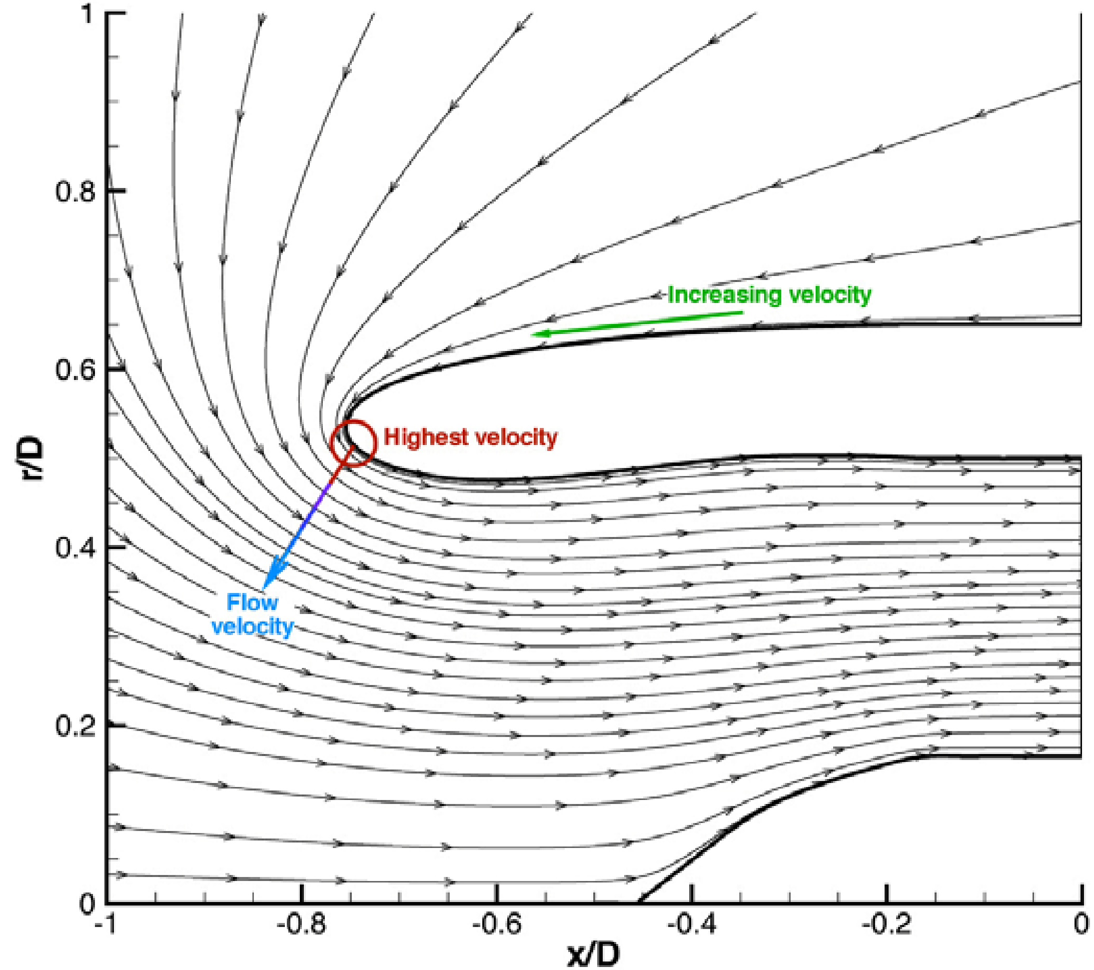
Mean Flow





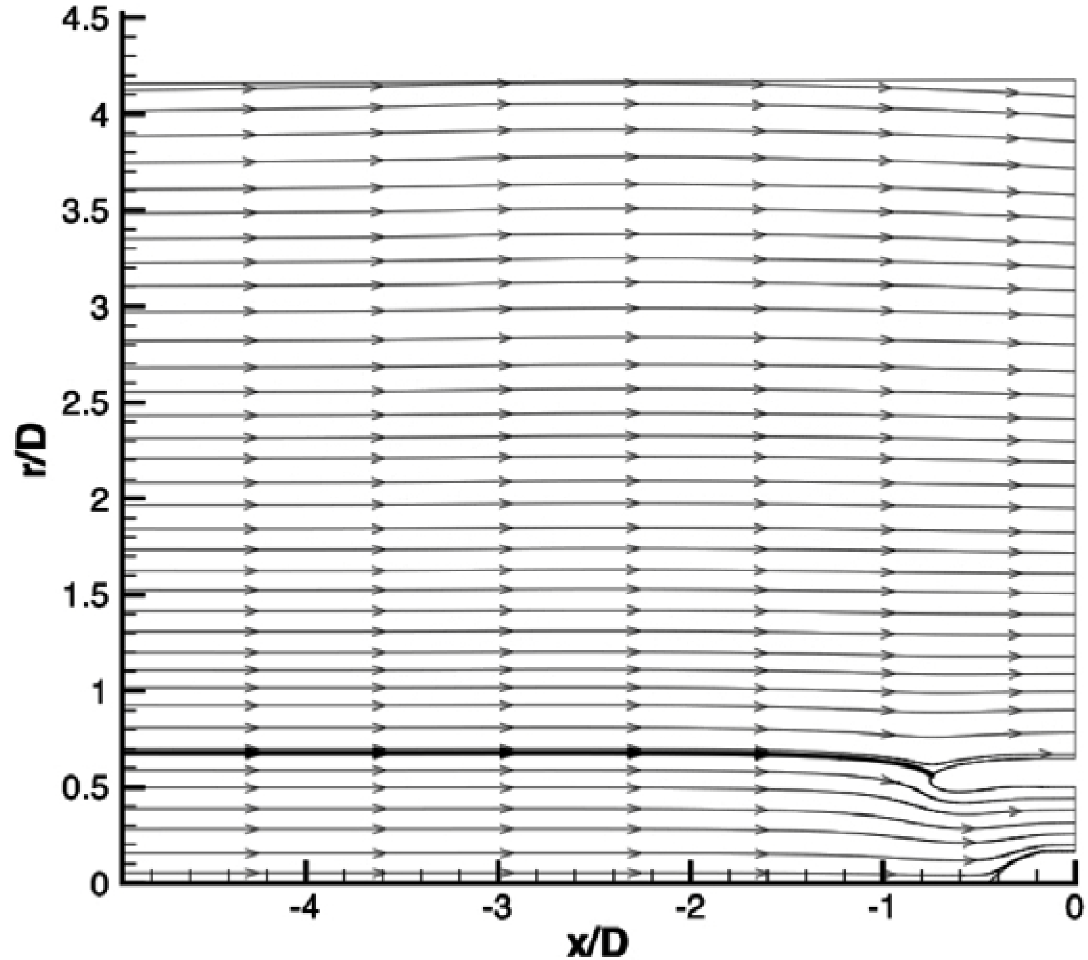
No forward flight

Mean Flow



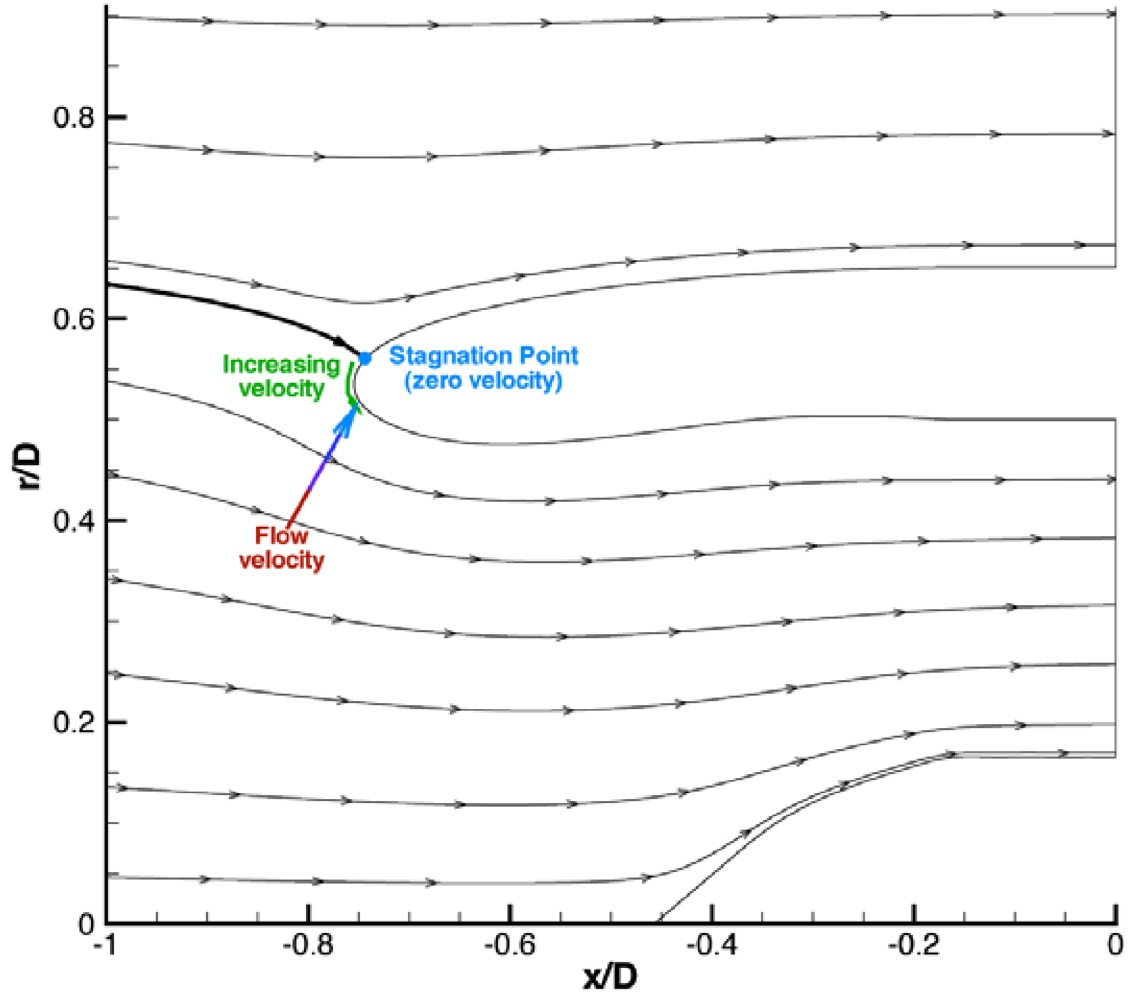
No forward flight

Mean Flow



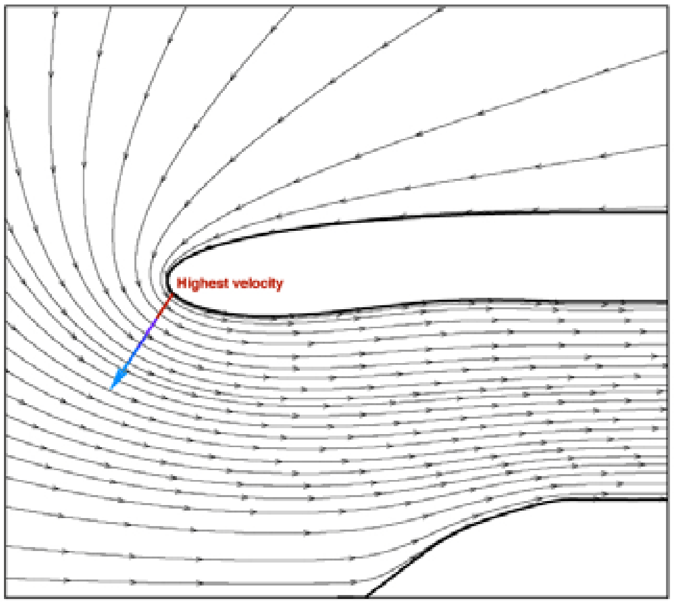
Forward flight

Mean Flow

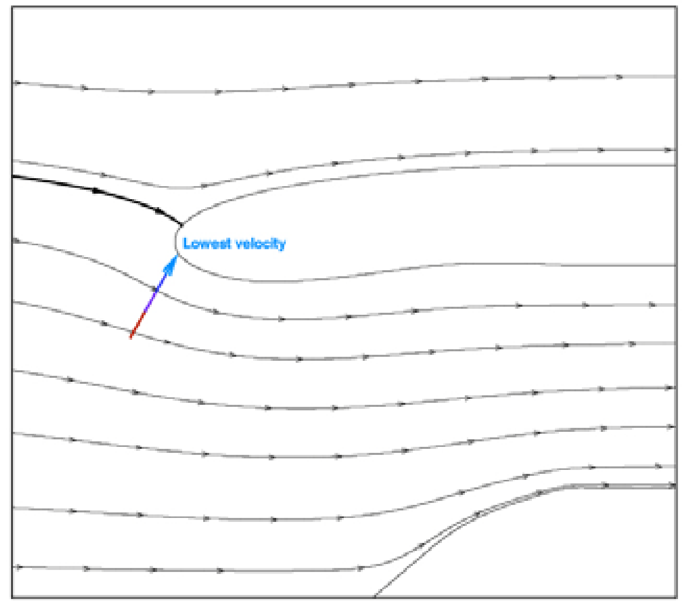


Forward flight

Mean Flow



Static



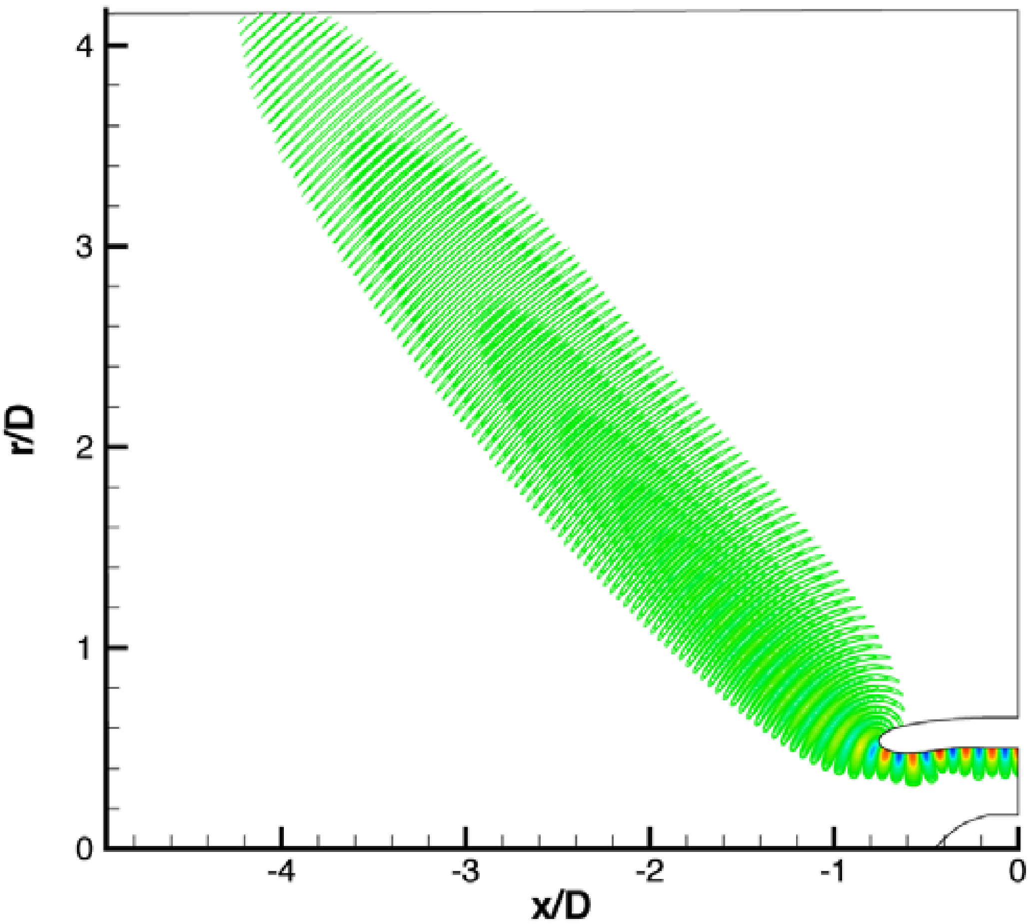
Flight

Physical Interpretation

Two physical processes control direction of radiation

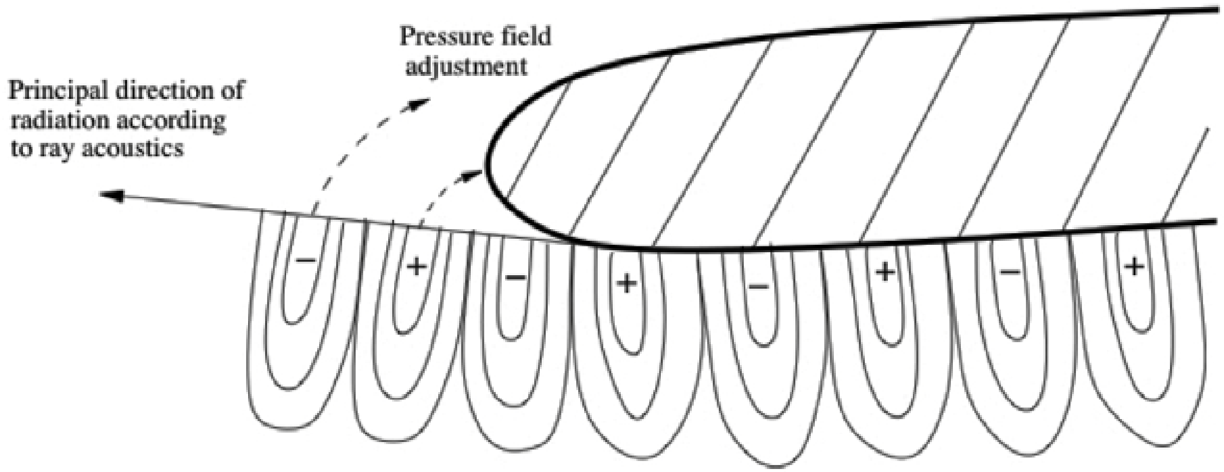
1. diffraction
   - the tendency of propagating sound waves to follow a curved surface of a solid body
2. refraction
   - the bending of direction of radiation by the velocity gradients of the flow

Physical Interpretation



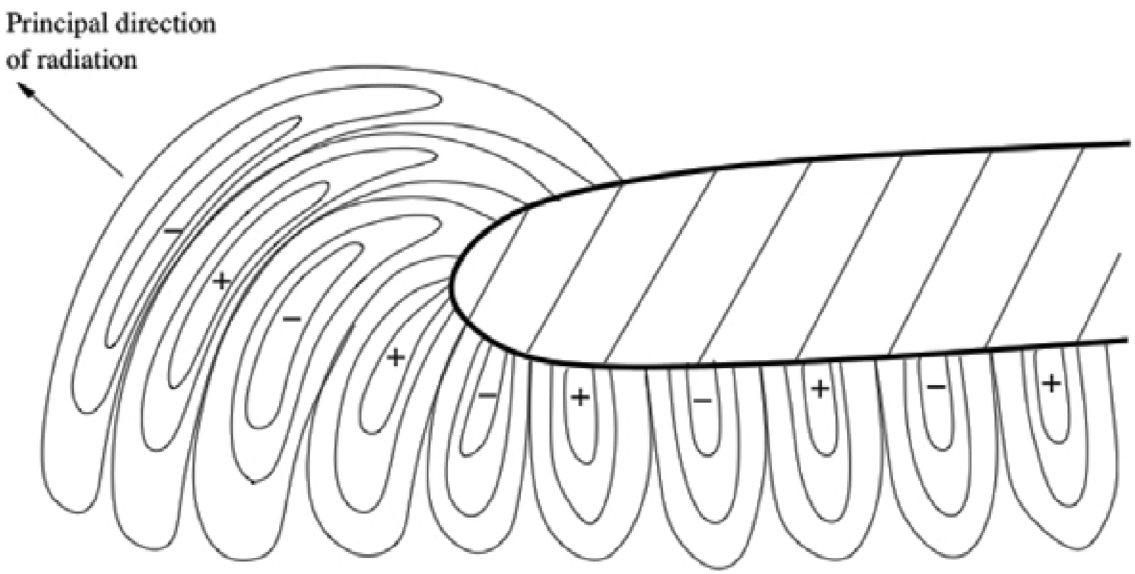
M_{fan}=0.0, M_{flight}=0.0, m=22, n=1, f=6400Hz

Physical Interpretation

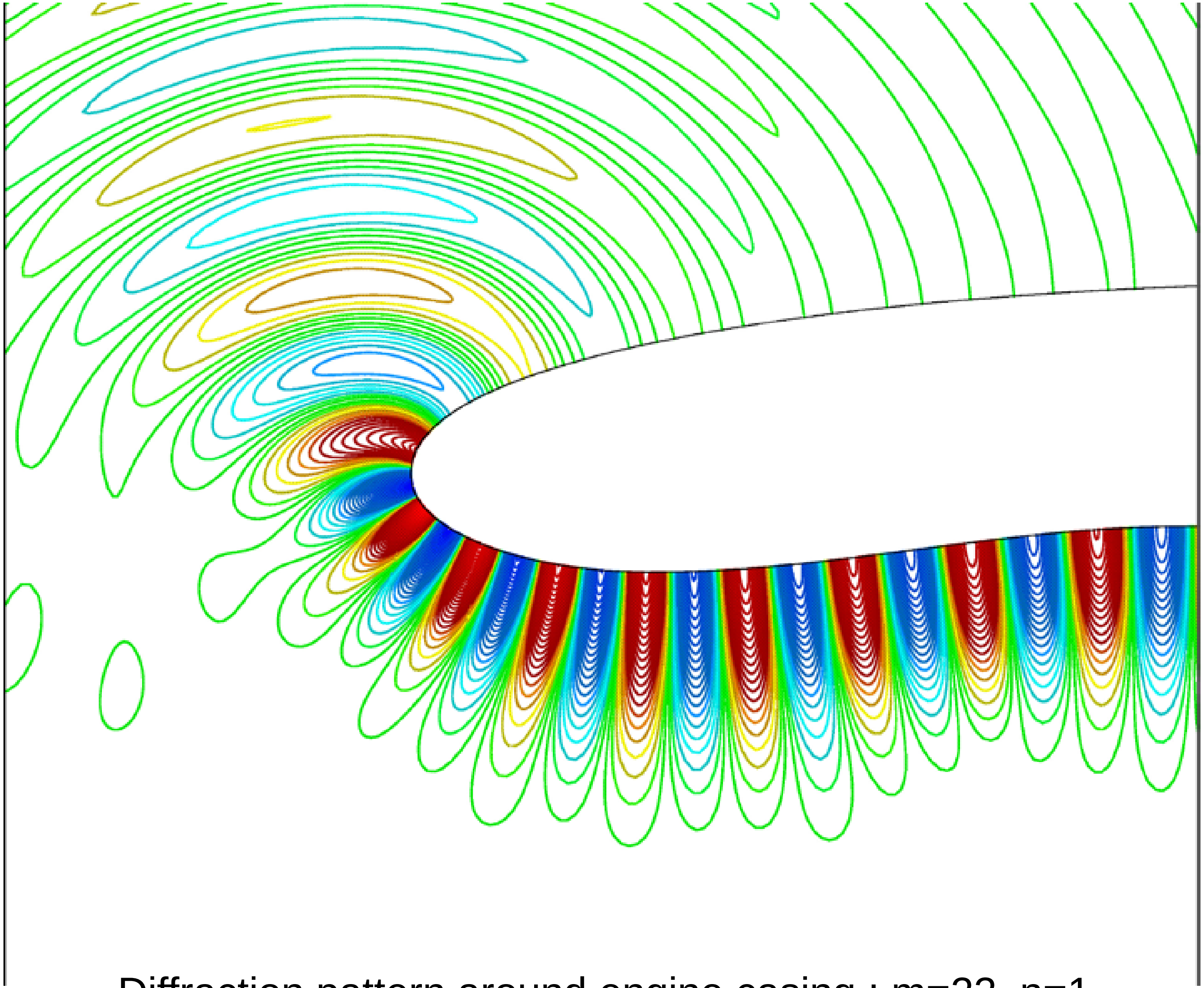


- pressure must balance
- solid wall can sustain high and low pressure

Physical Interpretation

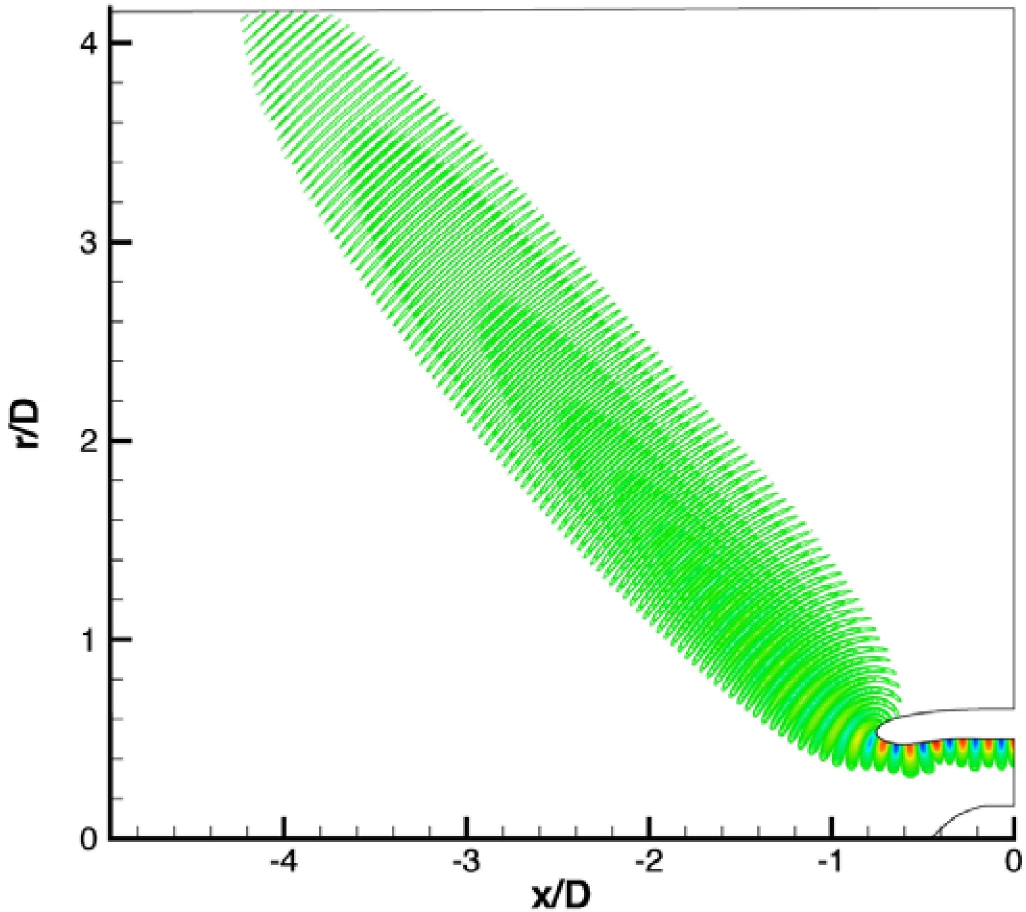


- pressure must balance
- solid wall can sustain high and low pressure



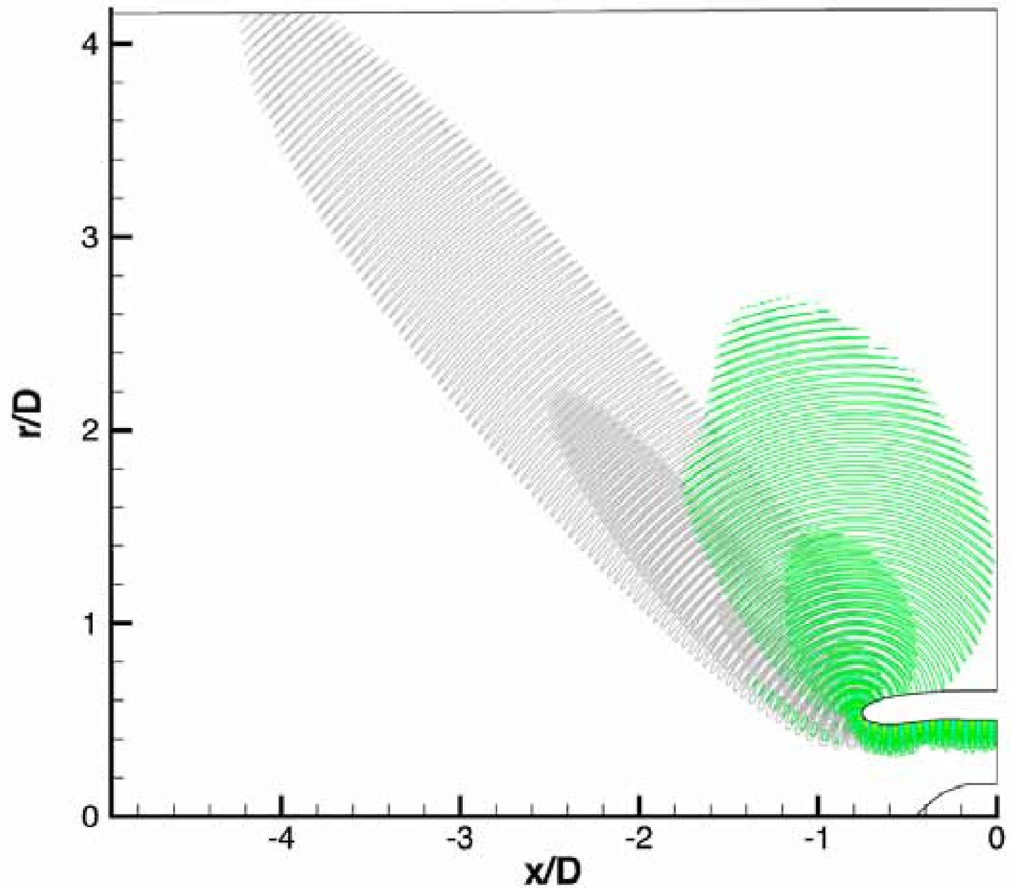
Diffraction pattern around engine casing : m=22, n=1,
frequency=6400 Hz, $M_{fan} = 0.4$

Physical Interpretation



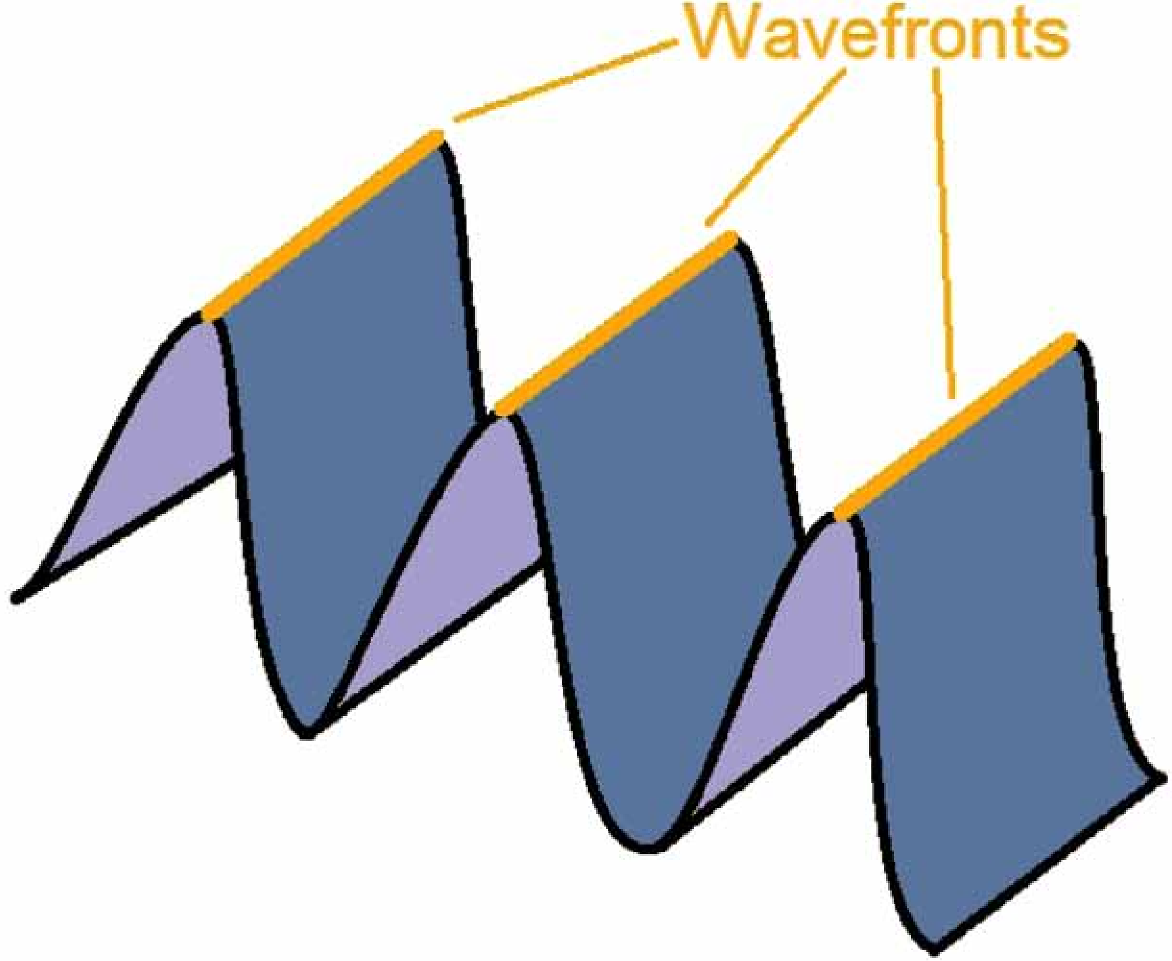
M_{fan}=0.0, M_{flight}=0.0, m=22, n=1, f=6400Hz

Physical Interpretation

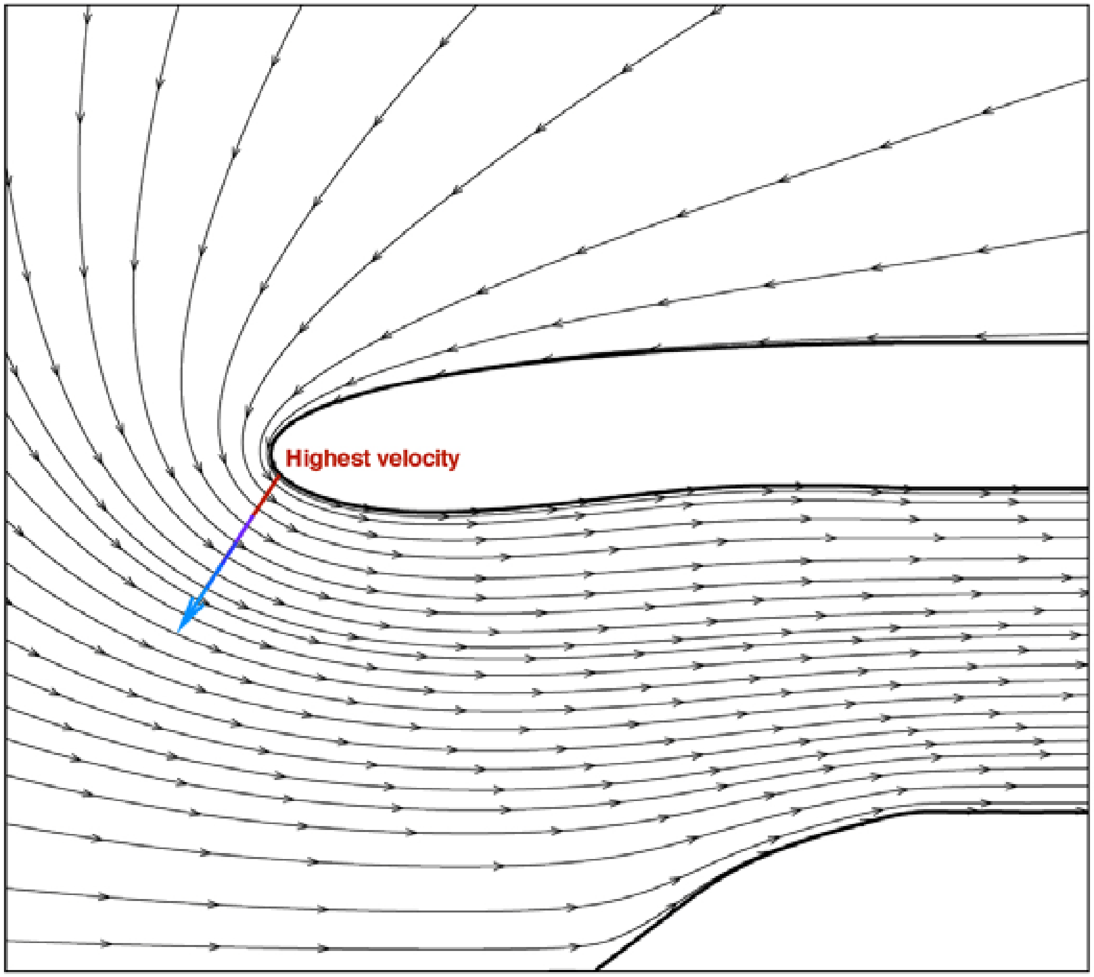


M_{fan}=0.4, M_{flight}=0.0, m=22, n=1, f=6400Hz

Physical Interpretation

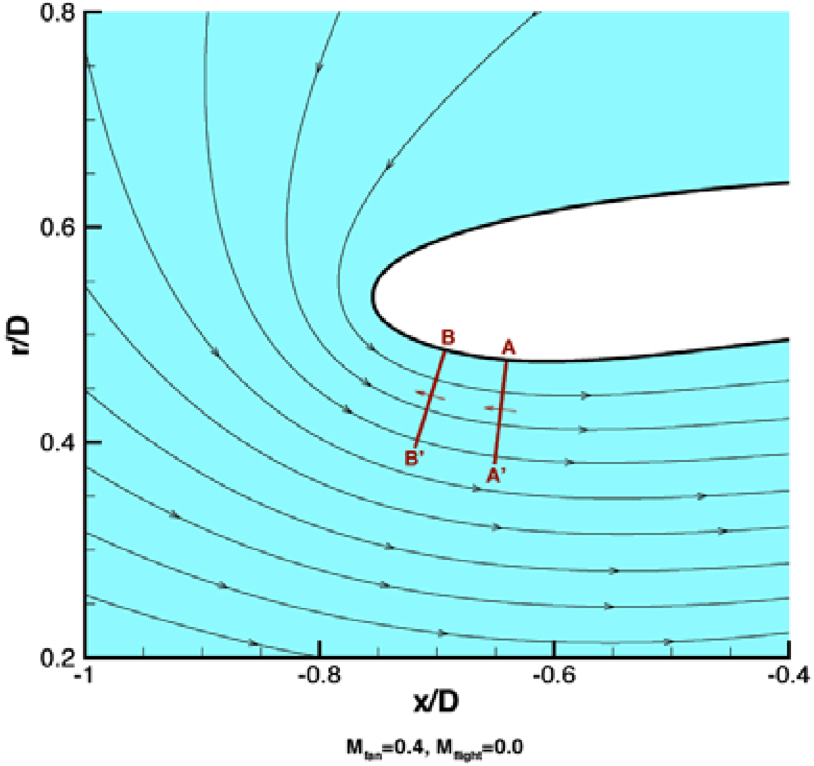


Physical Interpretation

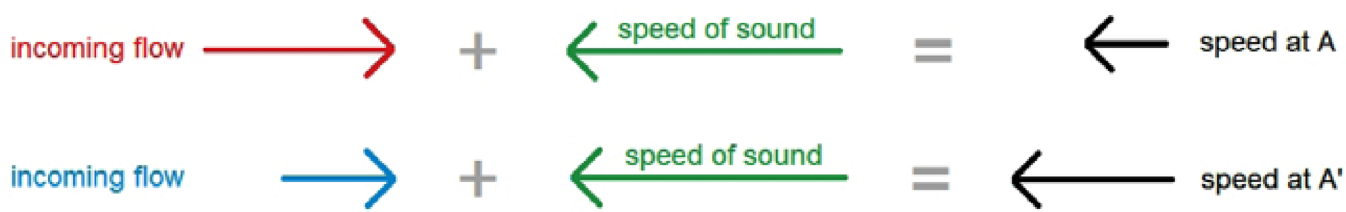


No forward flight

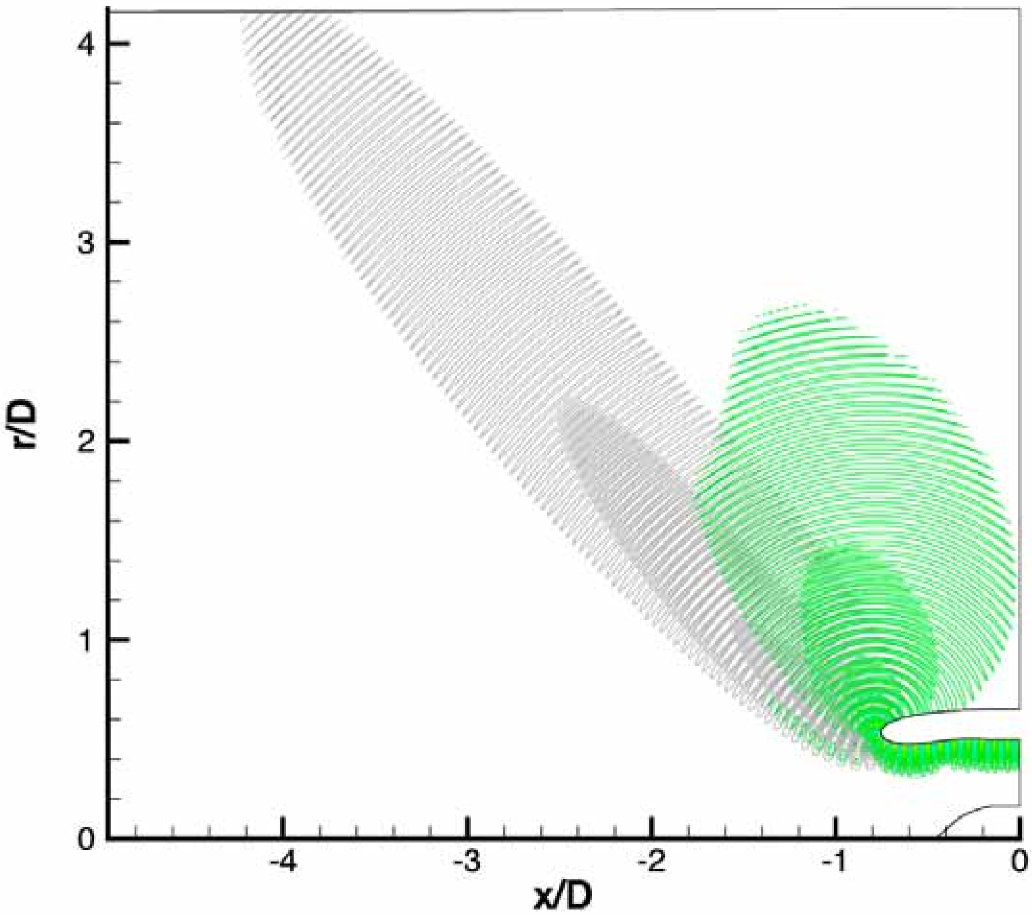
Physical Interpretation



$$\overline{AB} < \overline{A'B'}$$

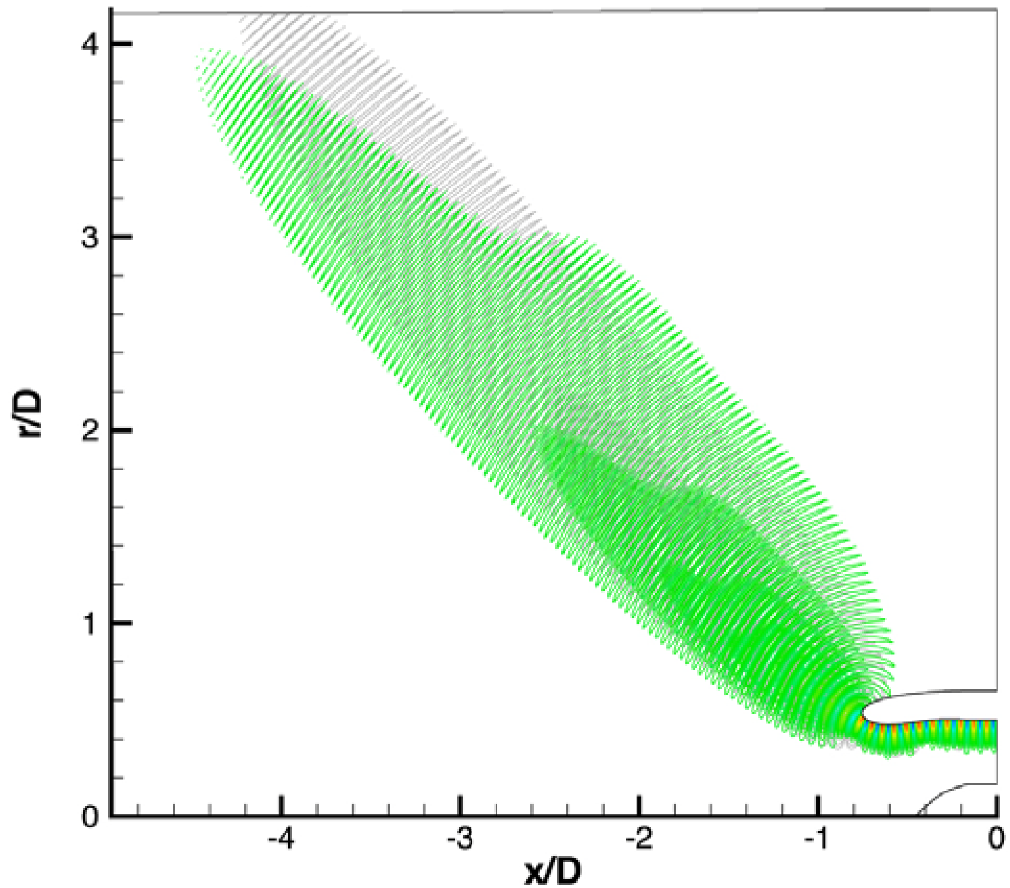


Physical Interpretation



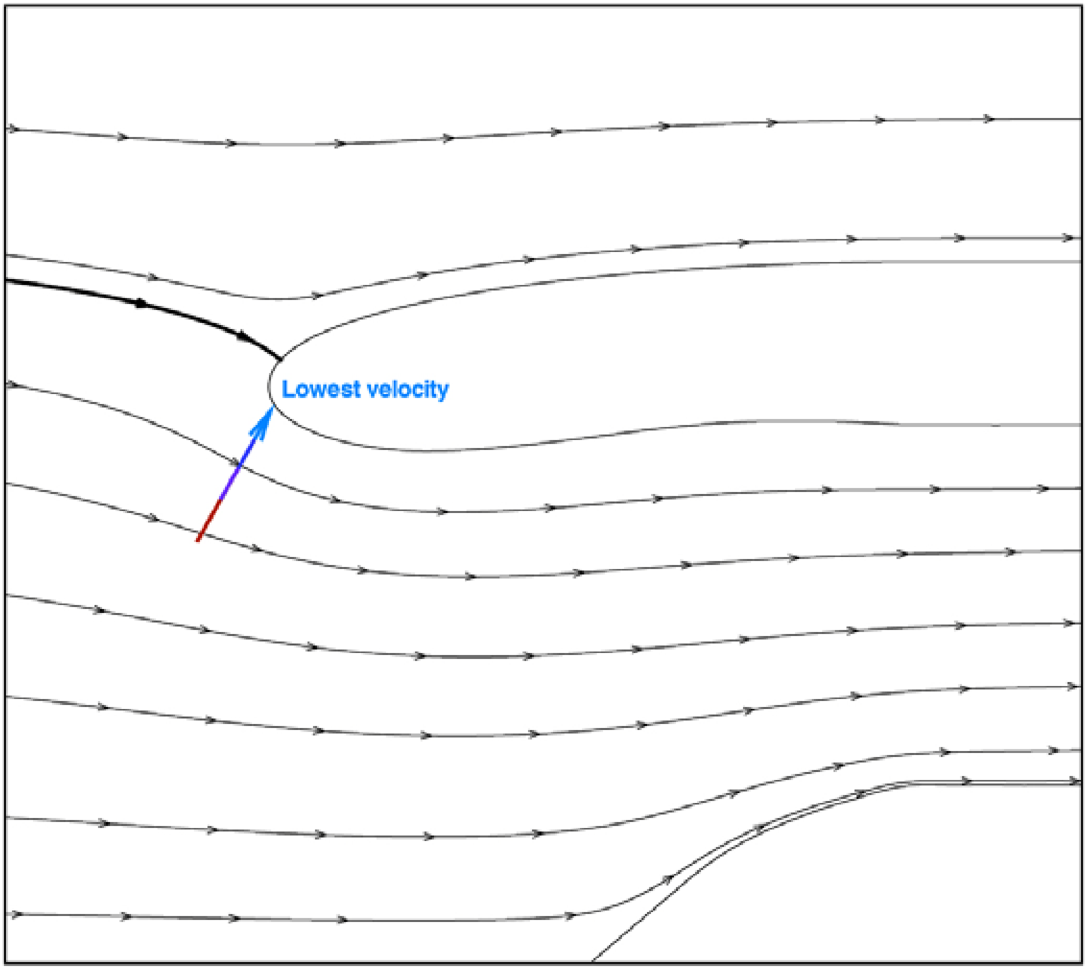
M_{fan}=0.4, M_{flight}=0.0, m=22, n=1, f=6400Hz

Physical Interpretation



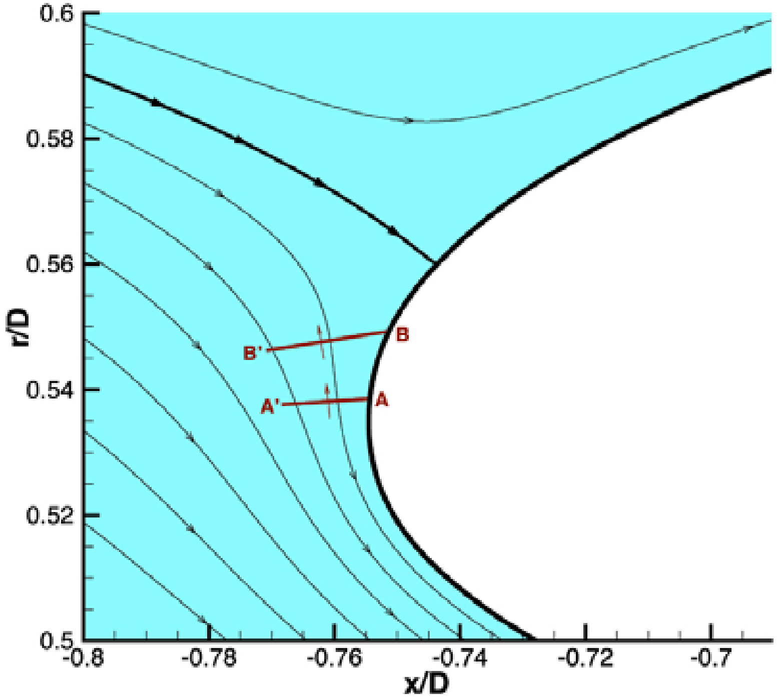
M_{fan}=0.4, M_{flight}=0.2, m=22, n=1, f=6400Hz

Physical Interpretation



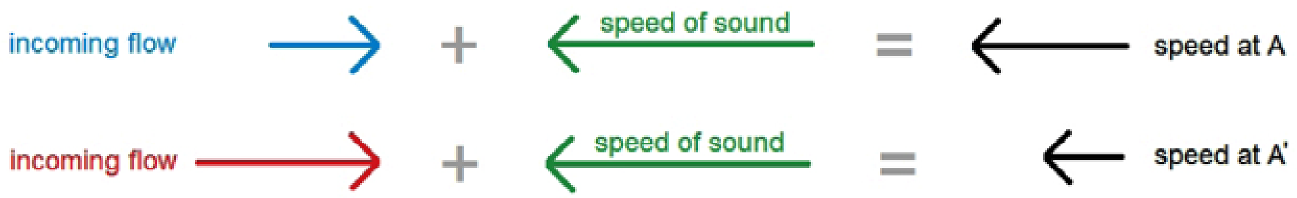
Forward flight

Physical Interpretation

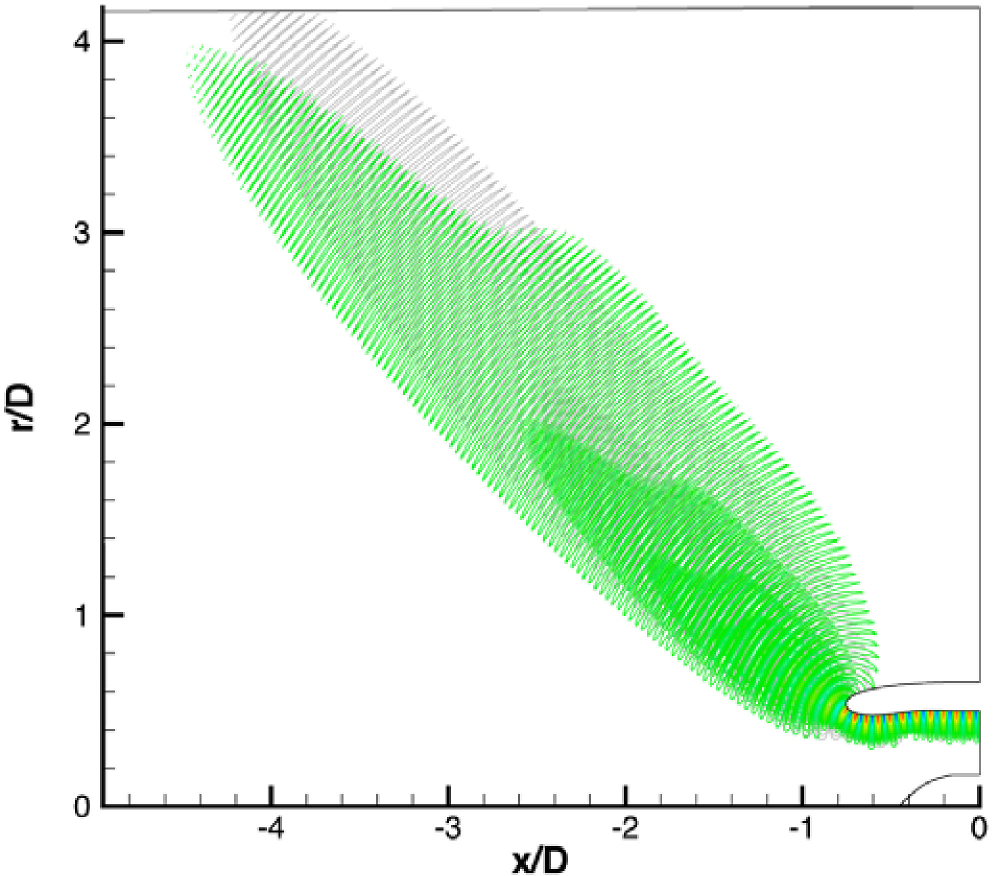


M_{fan}=0.4, M_{flight}=0.2

$$\overline{AB} > \overline{A'B'}$$



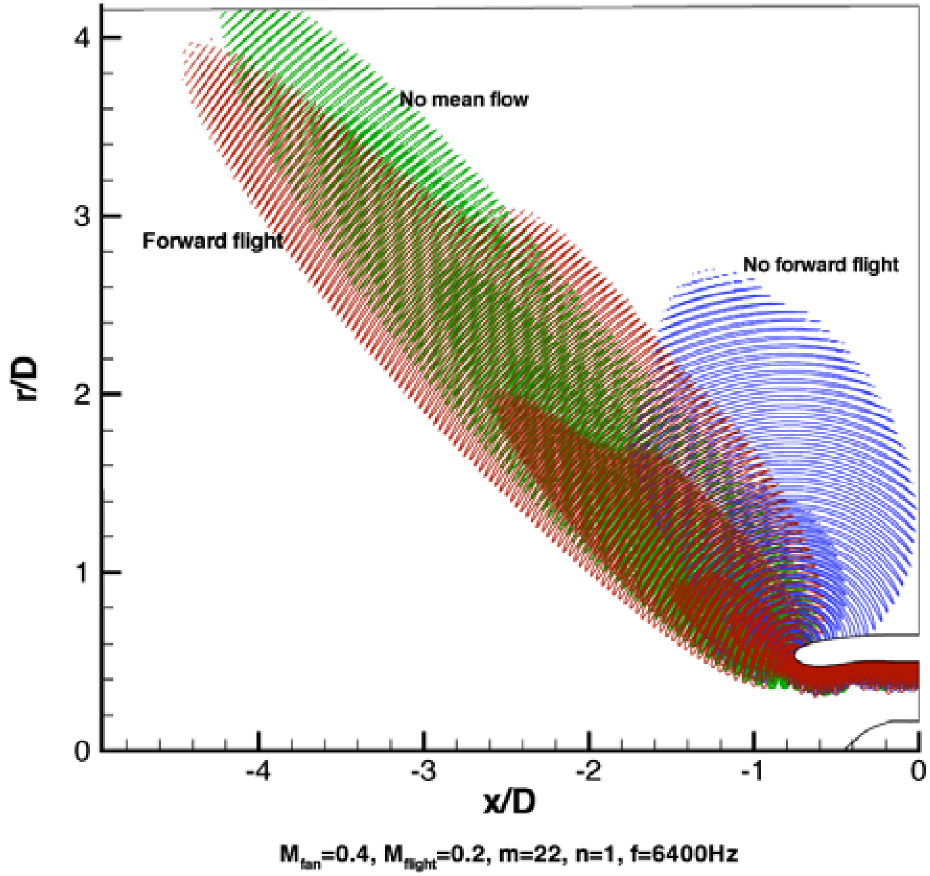
Physical Interpretation



M_{fan}=0.4, M_{flight}=0.2, m=22, n=1, f=6400Hz

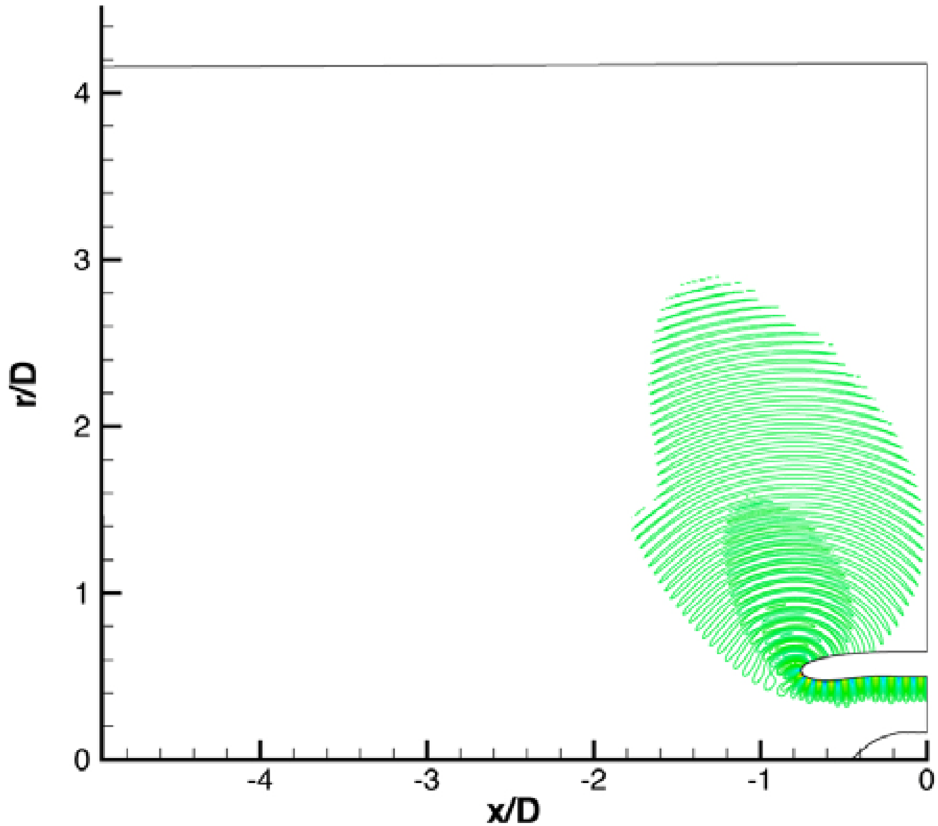
Diffraction effect is more dominant than refraction

Physical Interpretation



diffraction and refraction effects

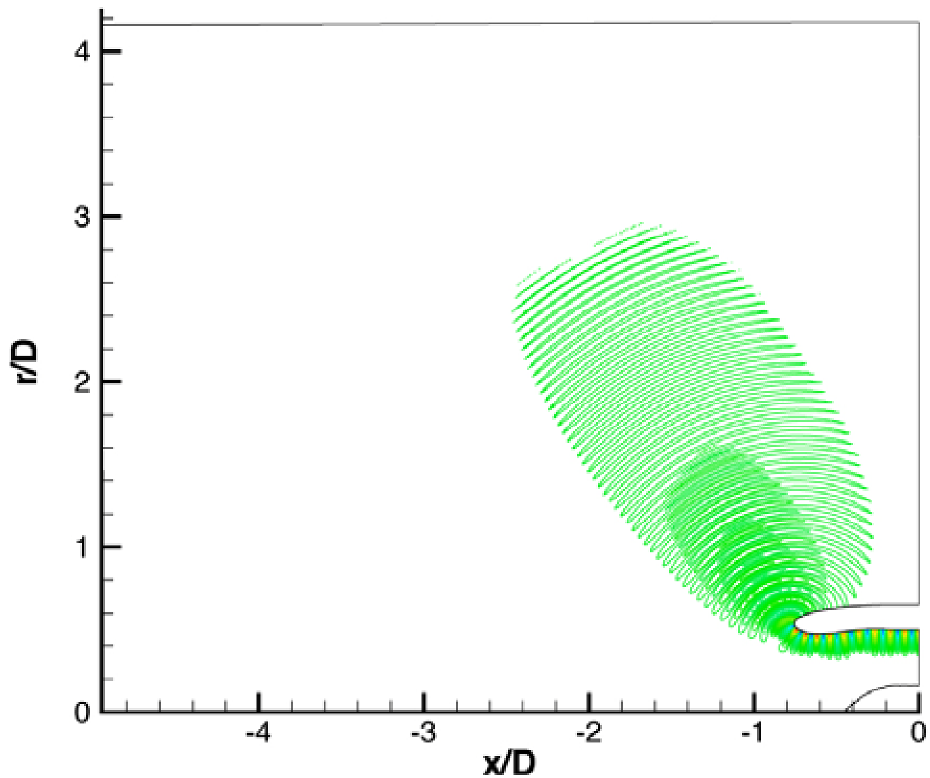
Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.0, m=22, n=1, f=5600Hz

Mach = 0.0

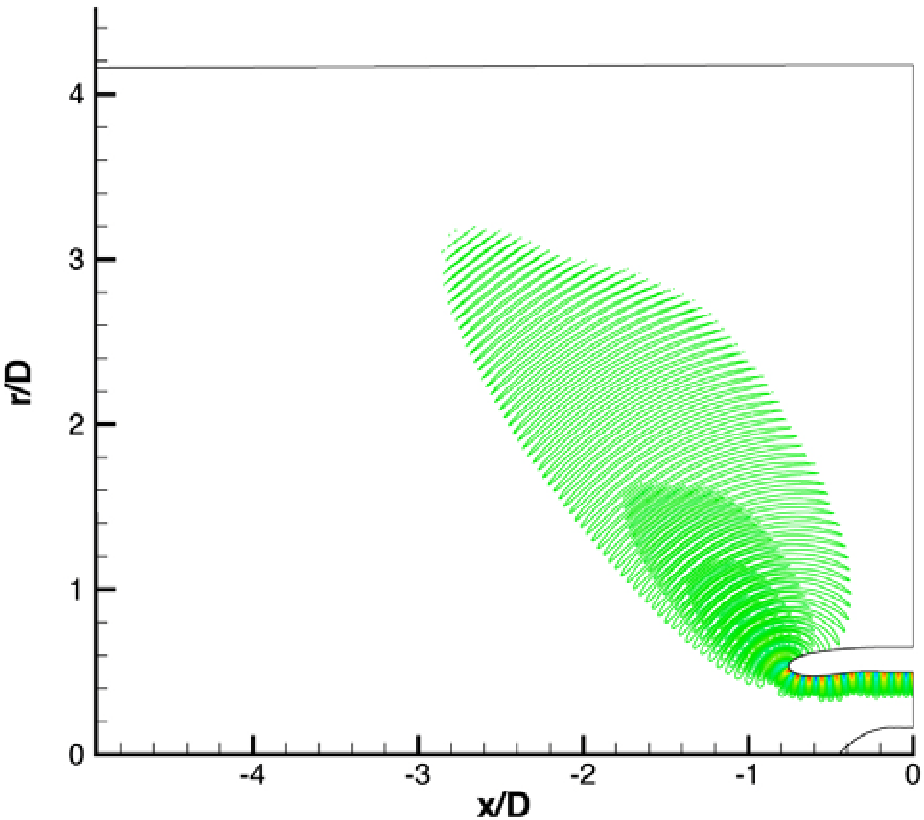
Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.05, m=22, n=1, f=5600Hz

Mach = 0.05

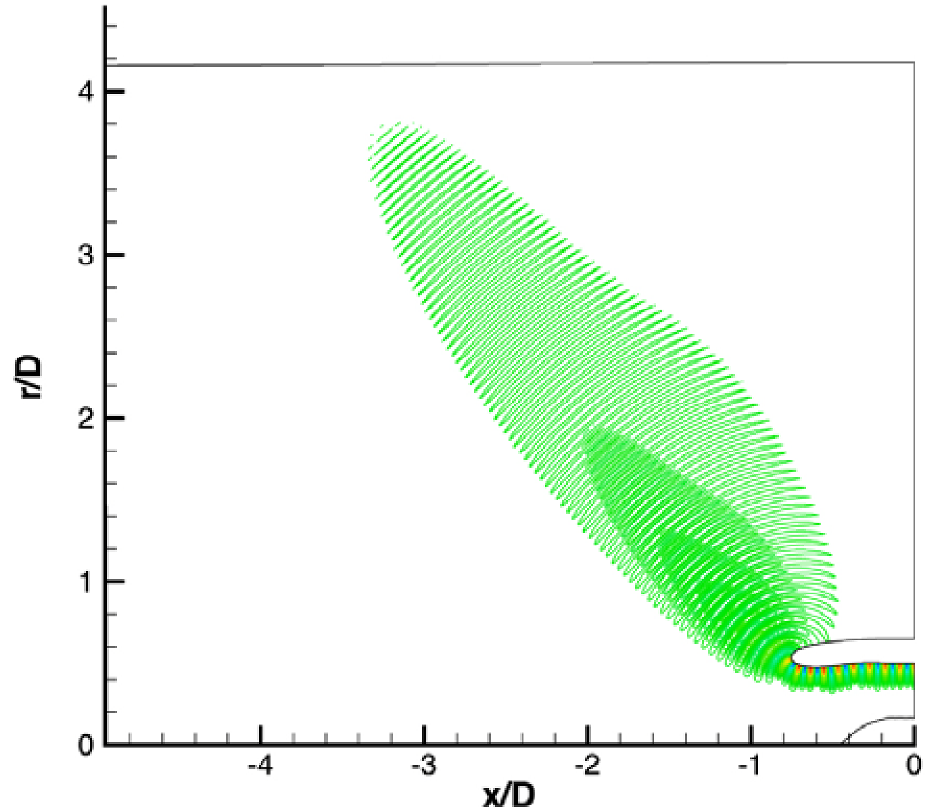
Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.1, m=22, n=1, f=5600Hz

Mach = 0.1

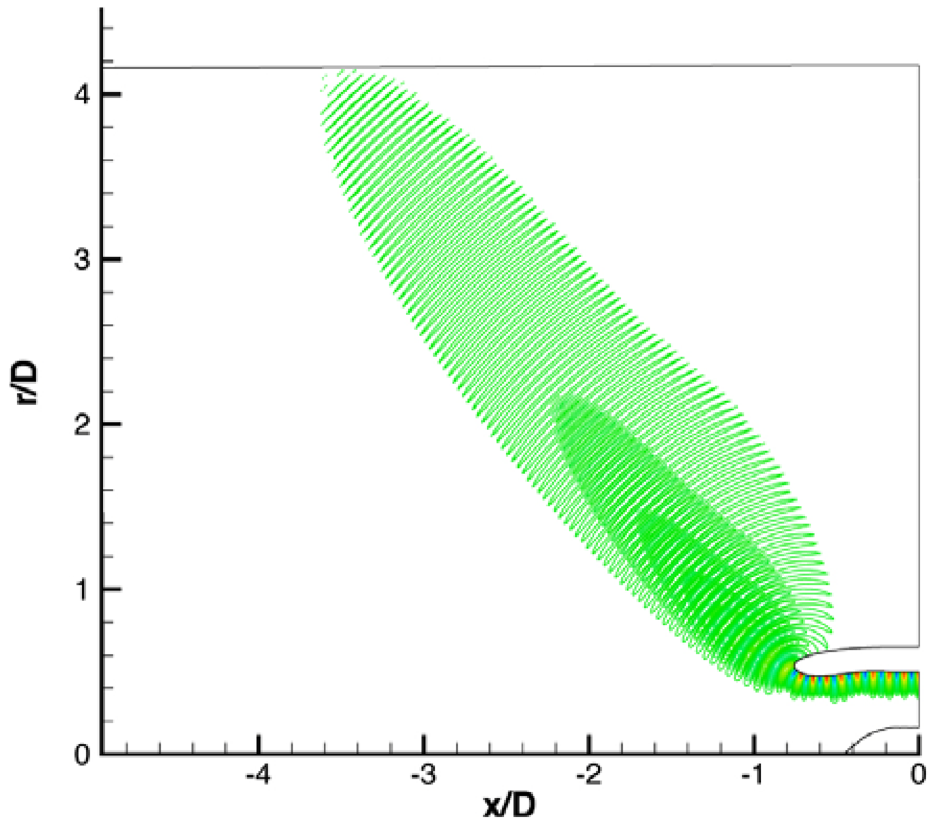
Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.15, m=22, n=1, f=5600Hz

Mach = 0.15

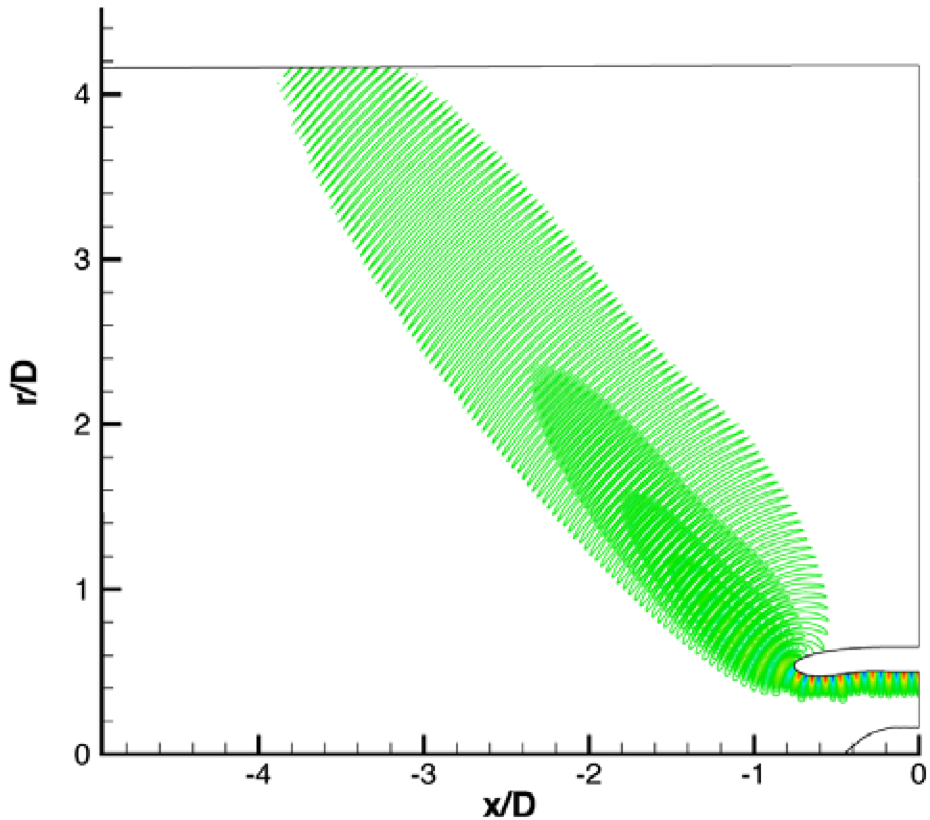
Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.2, m=22, n=1, f=5600Hz

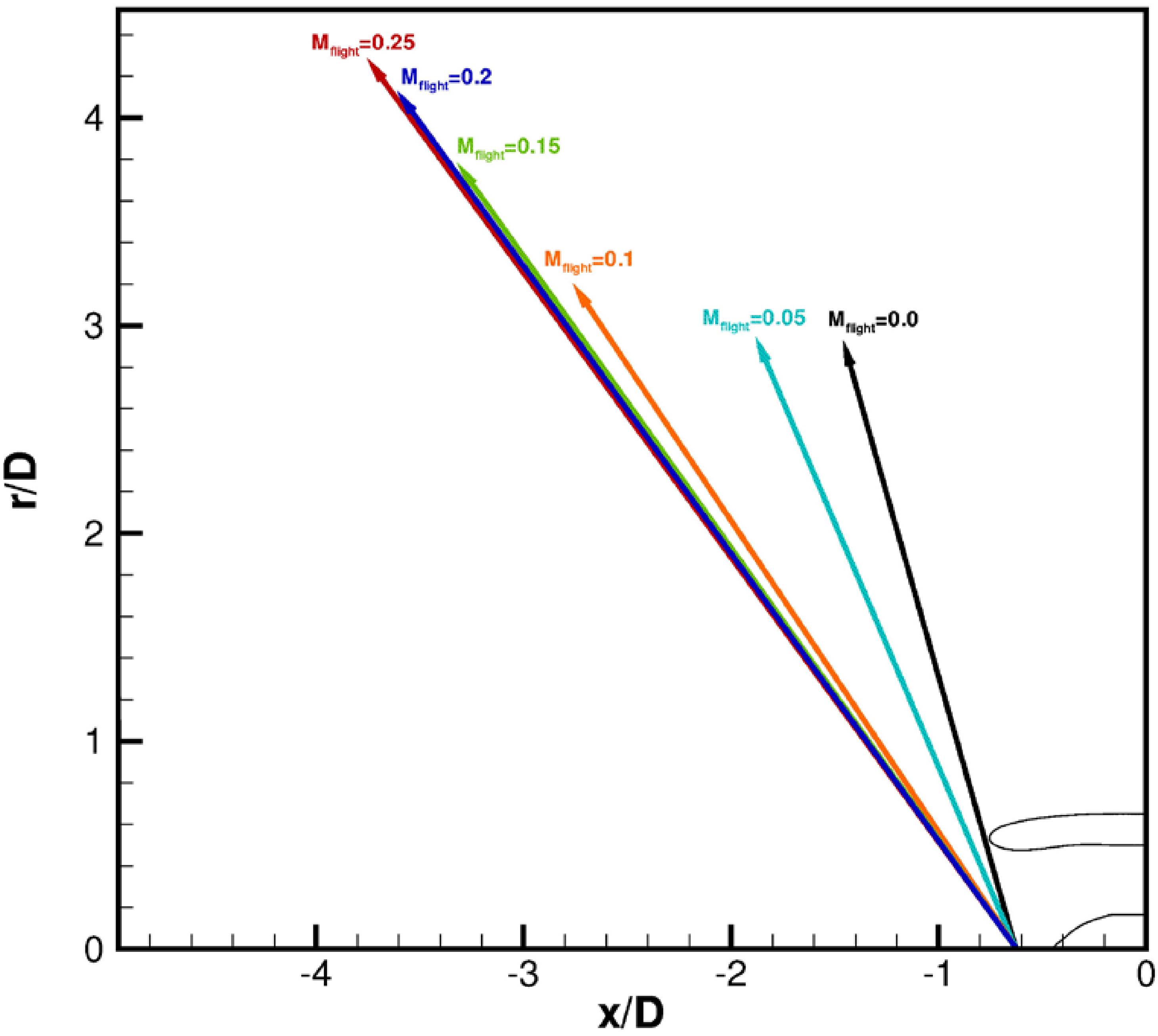
Mach = 0.2

Flight Mach Number Effect



M_{fan}=0.4, M_{flight}=0.25, m=22, n=1, f=5600Hz

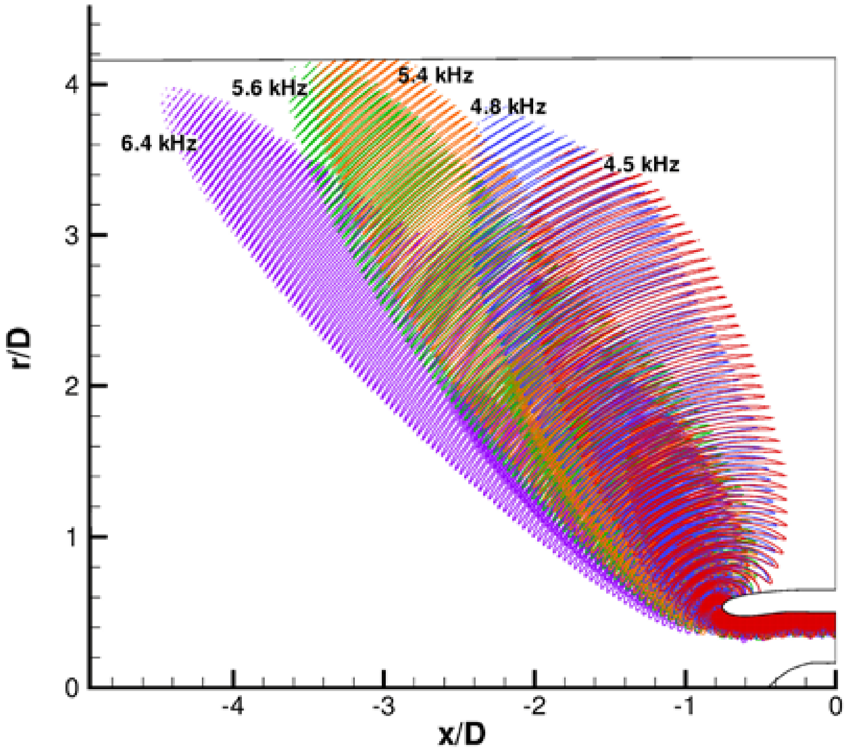
Mach = 0.25



M_flight=0.25
M_flight=0.2
M_flight=0.15
M_flight=0.1
M_flight=0.05
M_flight=0.0
r/D
x/D
0
1
2
3
4
-4
-3
-2
-1
0

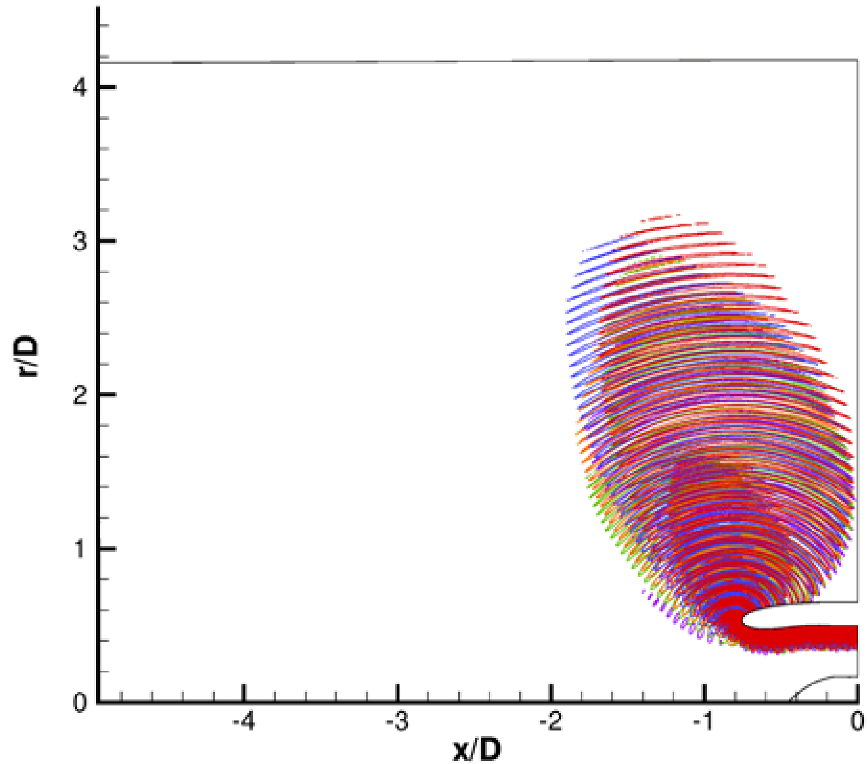
QuickTime™ and a
mpeg4 decompressor
are needed to see this picture.

Fan face Mach number, M_{fan} = 0.4; m = 22; n = 1



M_{fan}=0.4, M_{flight}=0.2, m=22, n=1

Flight Mach number = 0.2

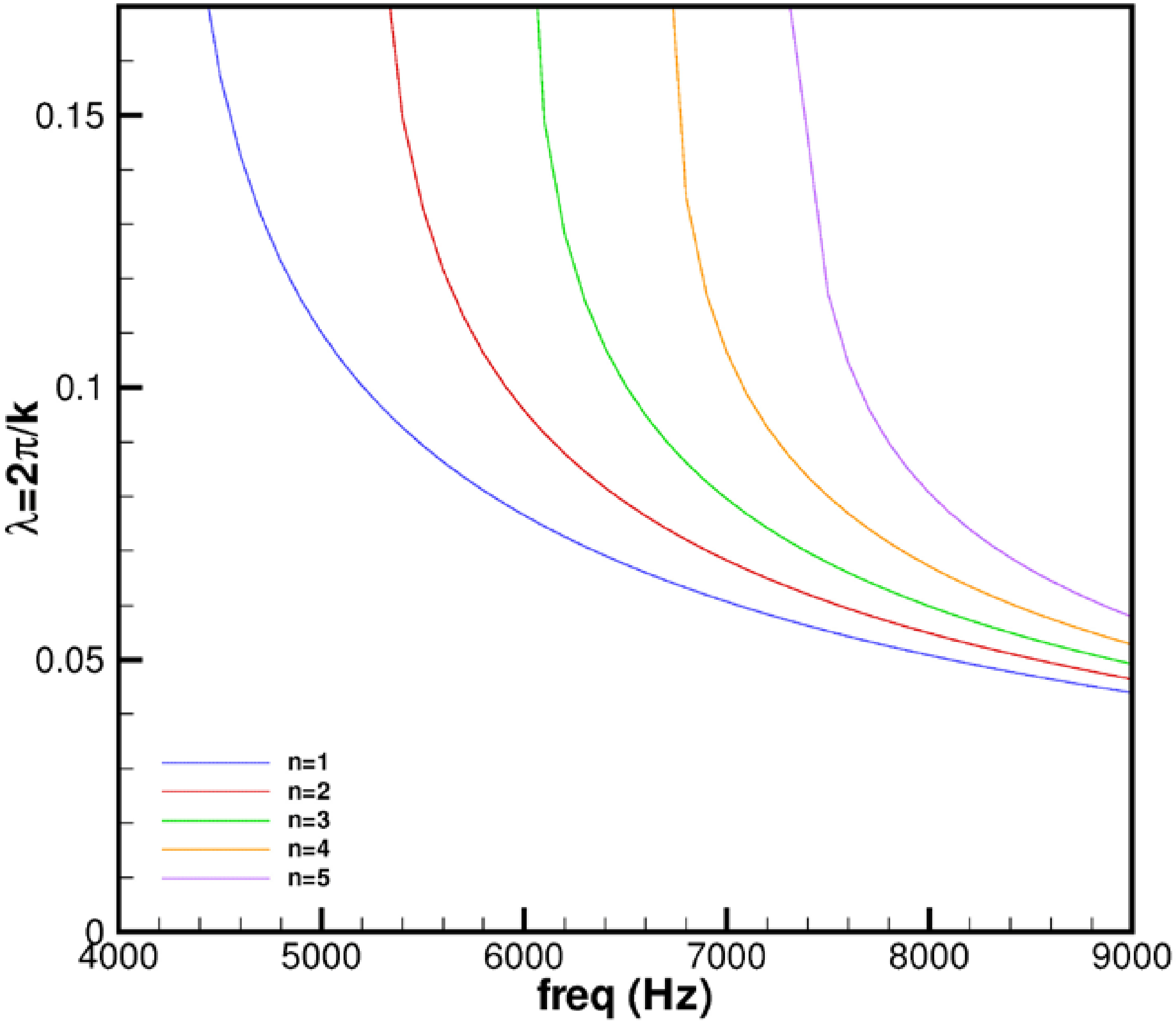


M_{fan}=0.4, M_{flight}=0.0, m=22, n=1

No forward flight

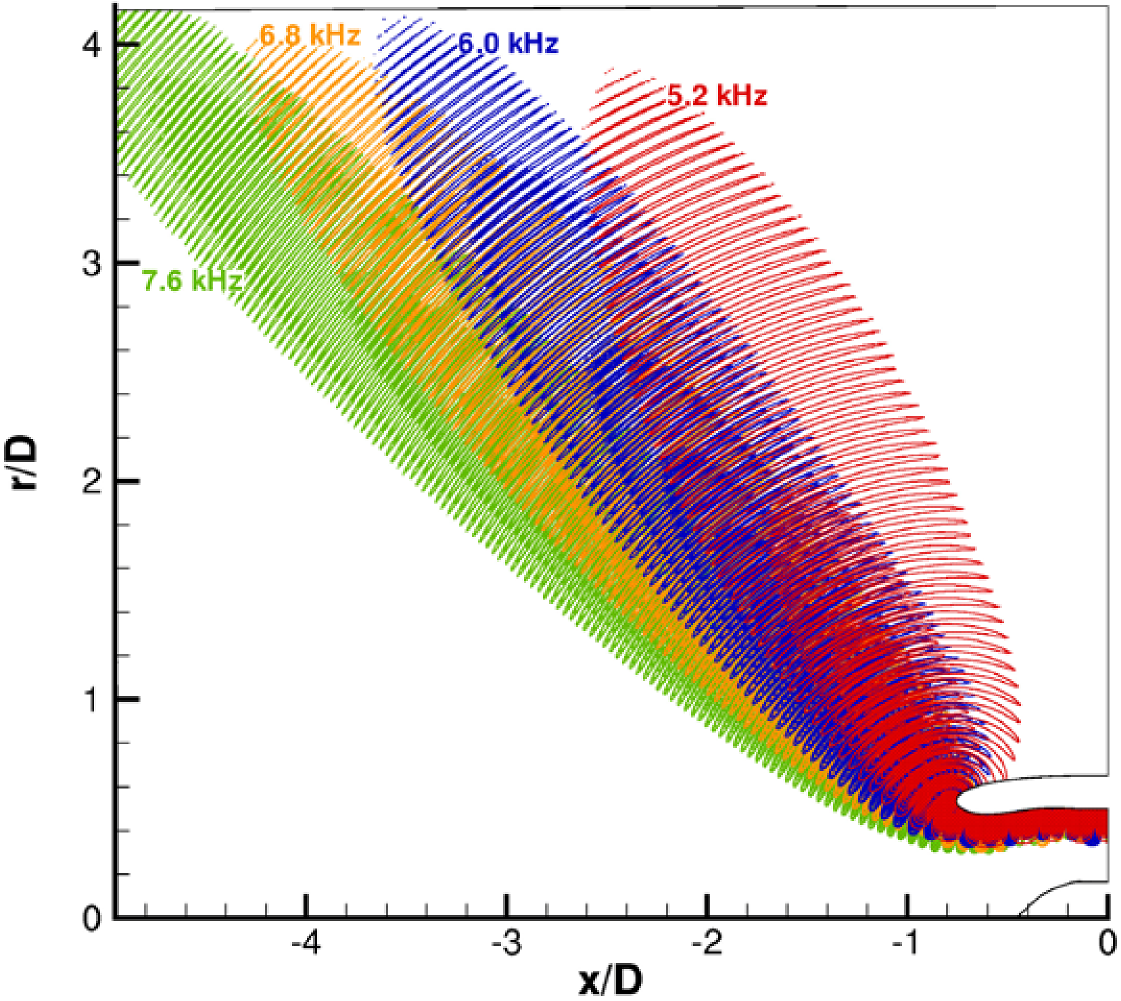
Frequency variation

Variation of duct mode wavelength with frequency



M_{fan}=0.4, M_{flight}=0.0, m=22

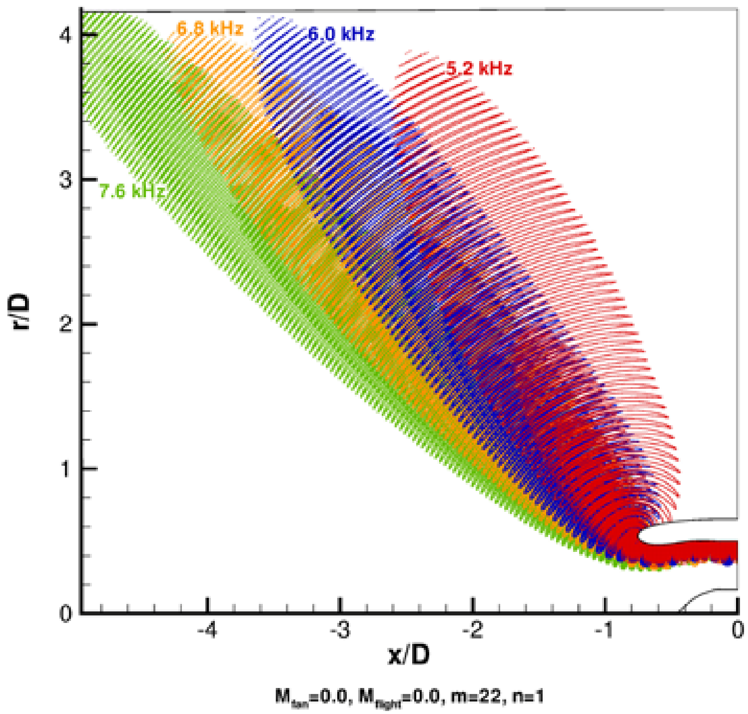
Dependence of effect of diffraction on wavelength/frequency



M_{fan}=0.0, M_{flight}=0.0, m=22, n=1

Short waves are less affected by diffraction

Dependence of effect of diffraction on wavelength/frequency

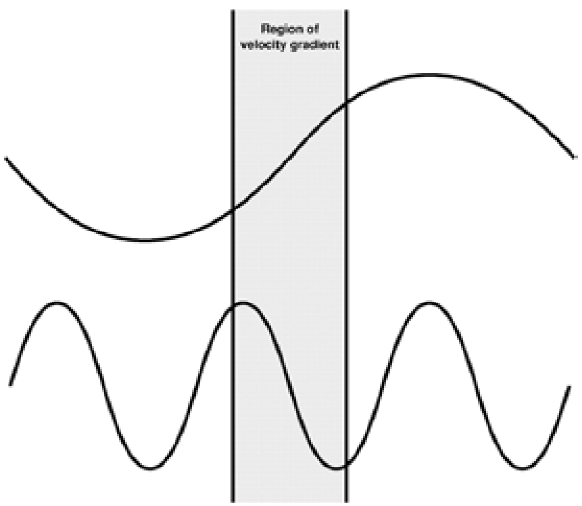


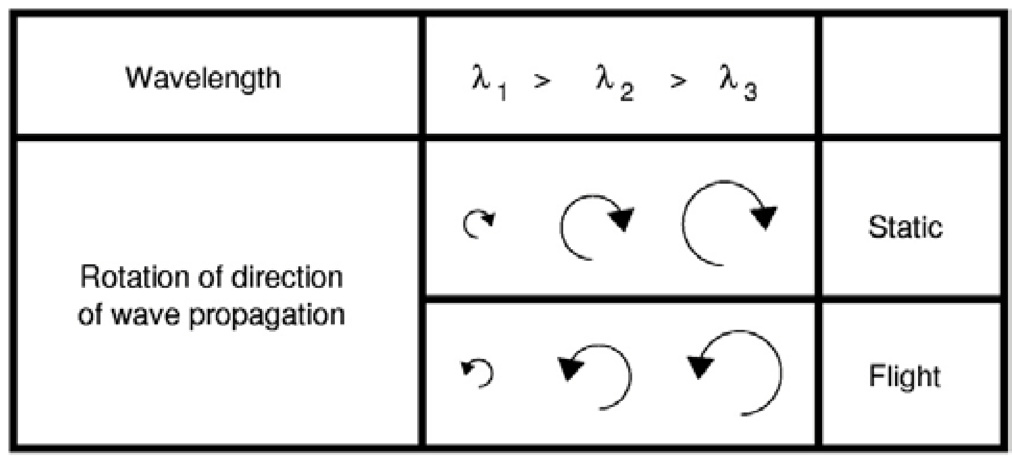
Wavelength	$\lambda_1 > \lambda_2 > \lambda_3$
Rotation of direction of wave propagation	

Dependence of effect of refraction on wavelength/frequency

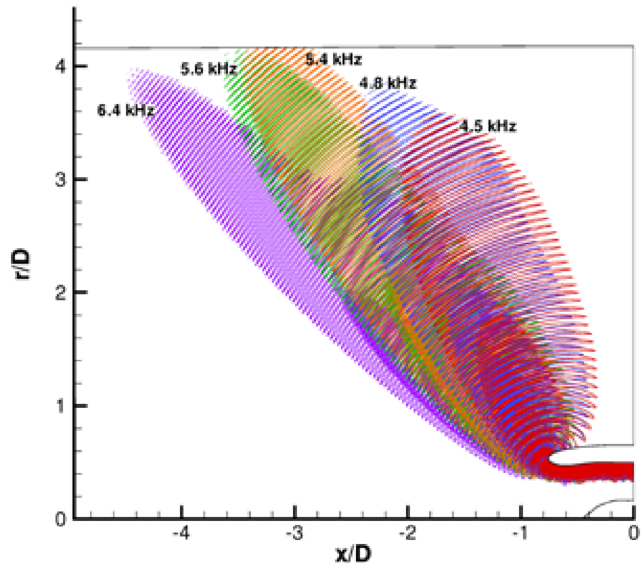
Localized velocity gradients have little influence on the propagation of long waves

Localized velocity gradients have a much large impact on the propagation of short waves

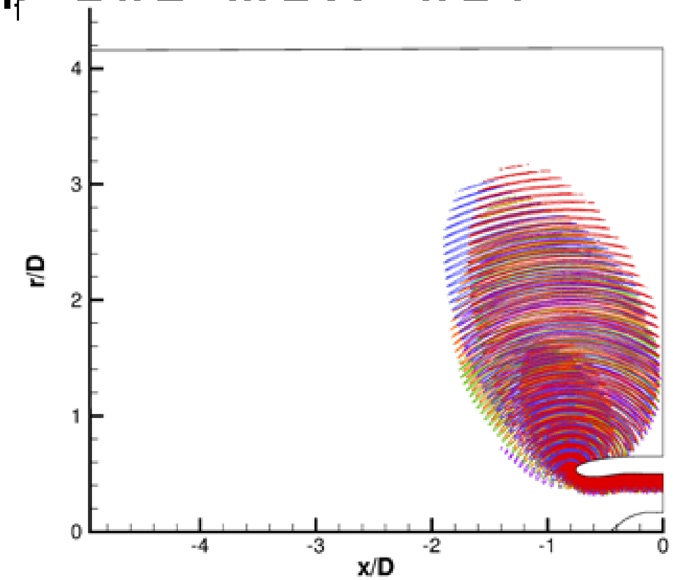




Fan face Mach number, $M_f = 0.4$; $m = 22$; $n = 1$



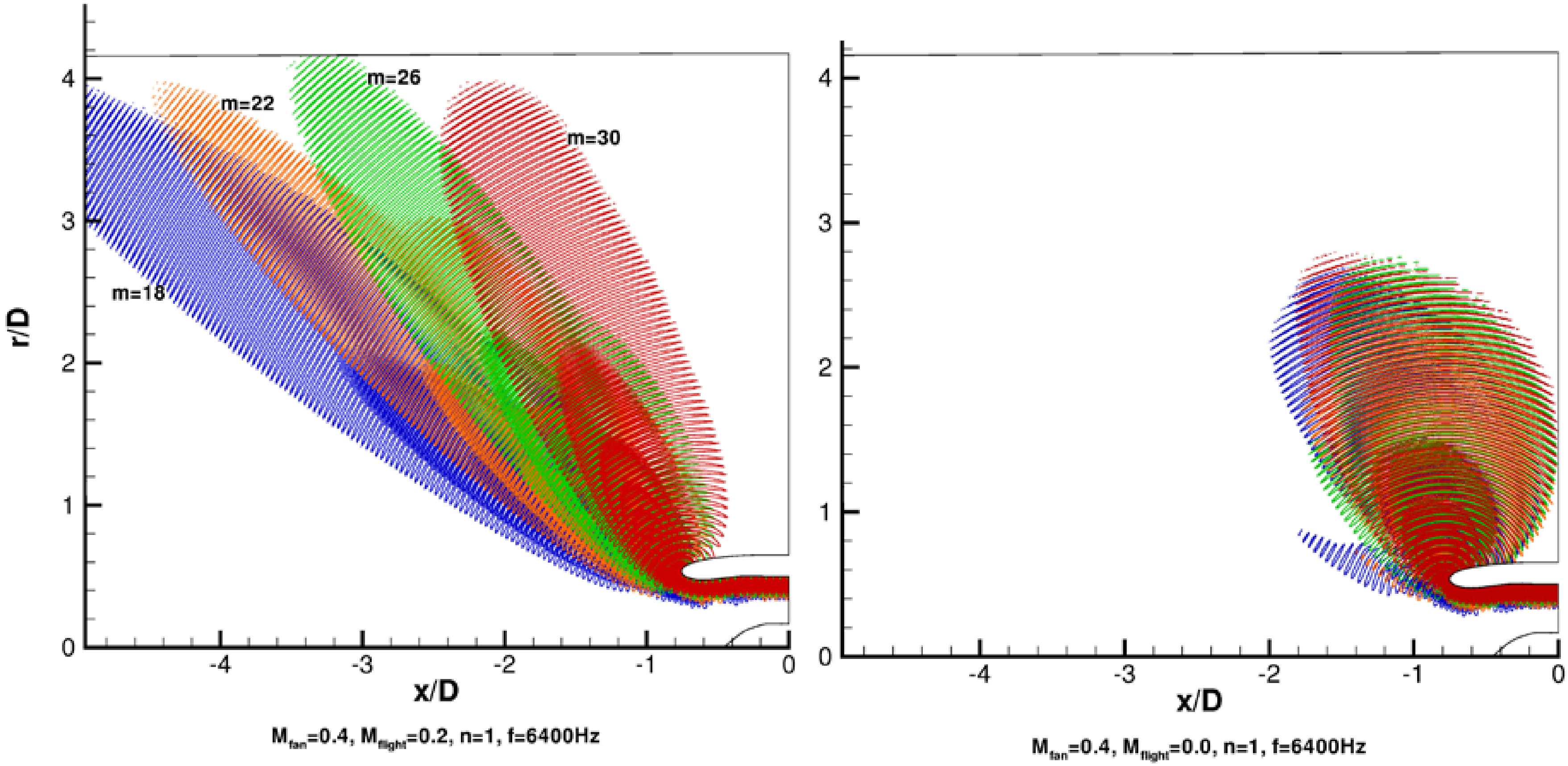
M_{fan}=0.4, M_{flight}=0.2, m=22, n=1



M_{fan}=0.4, M_{flight}=0.0, m=22, n=1

	Static	Flight
Wavelength	$\lambda_1 > \lambda_2 > \lambda_3$	$\lambda_1 > \lambda_2 > \lambda_3$
Diffraction		
Refraction		
Combined		

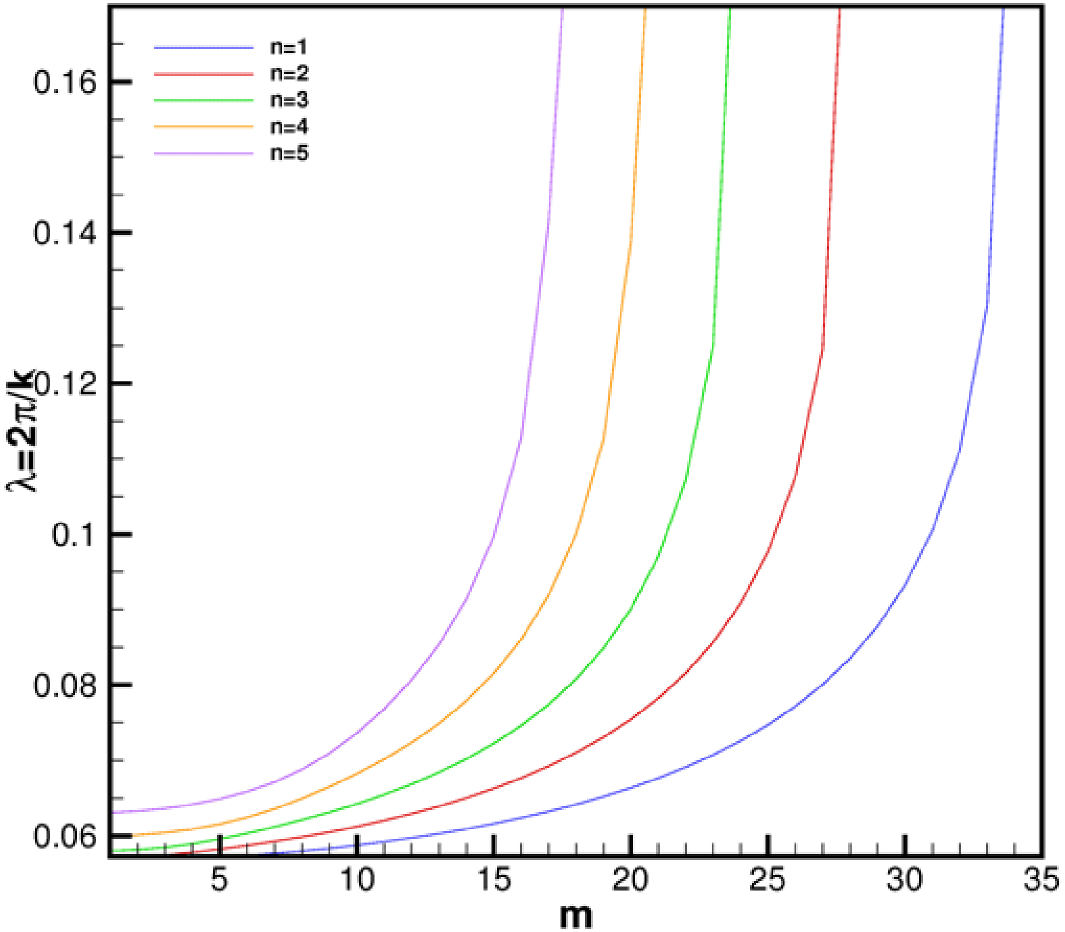
Fan Face Mach number = 0.4; f = 6400 Hz; n = 1



Flight Mach number = 0.2 | **No forward flight**

Azimuthal mode number variation

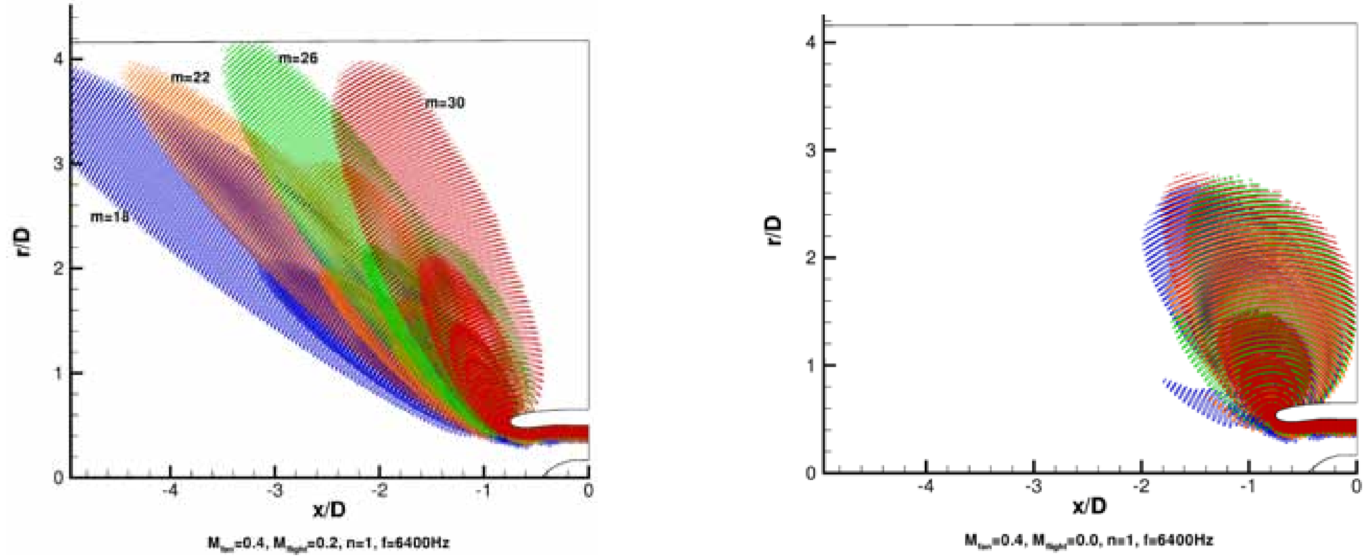
Variation of duct mode wavelength with azimuthal mode number



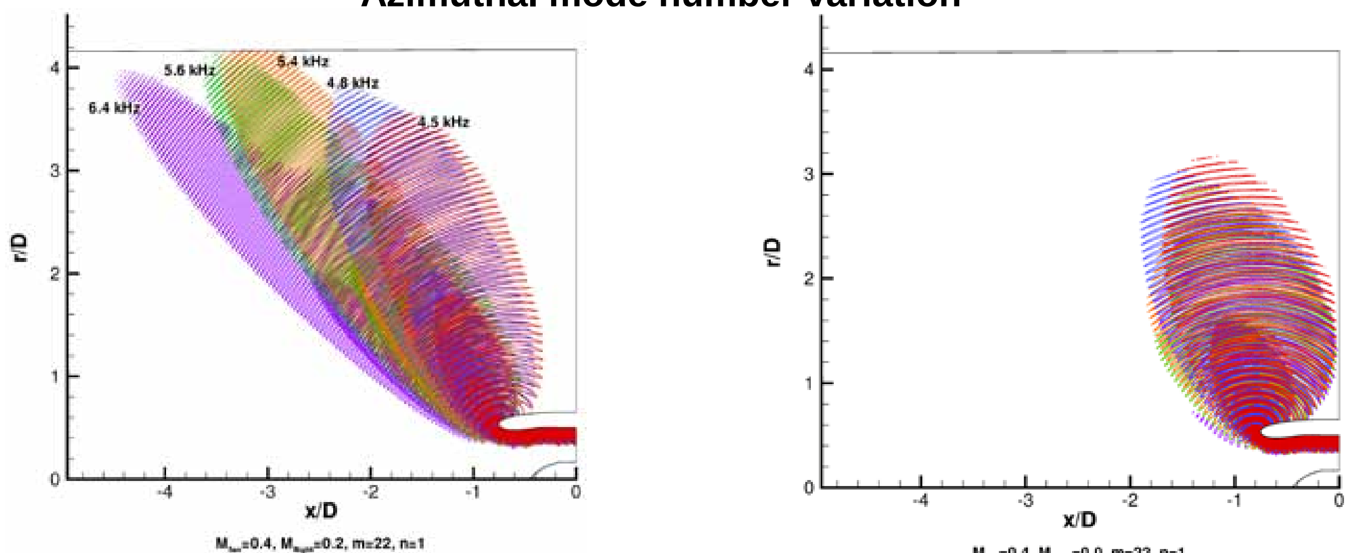
M_{fan}=0.4, M_{flight}=0.0, f=6400Hz

Low azimuthal wave number duct modes have shorter wave lengths

Fan Face Mach number = 0.4; f = 6400 Hz; n = 1



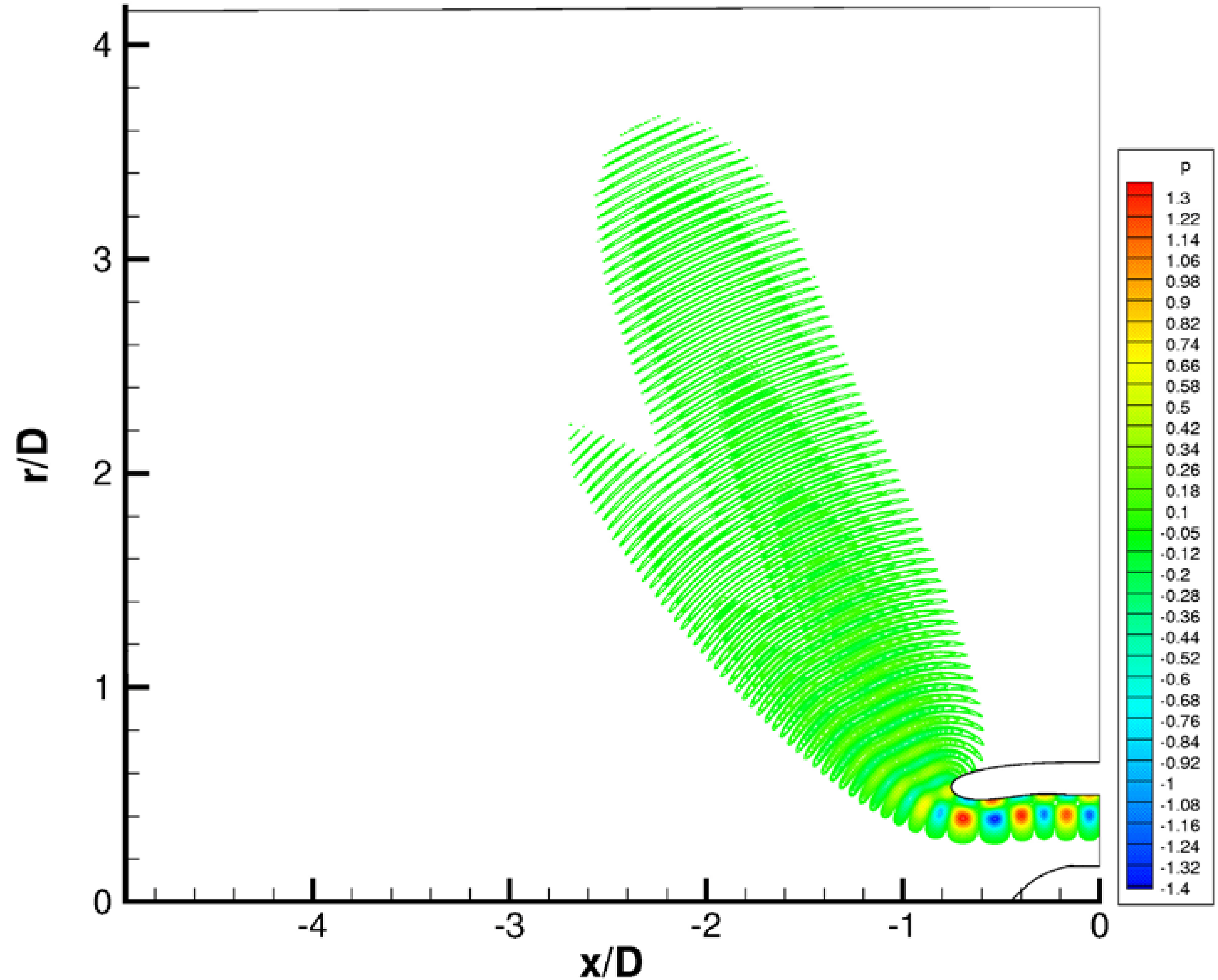
Azimuthal mode number variation



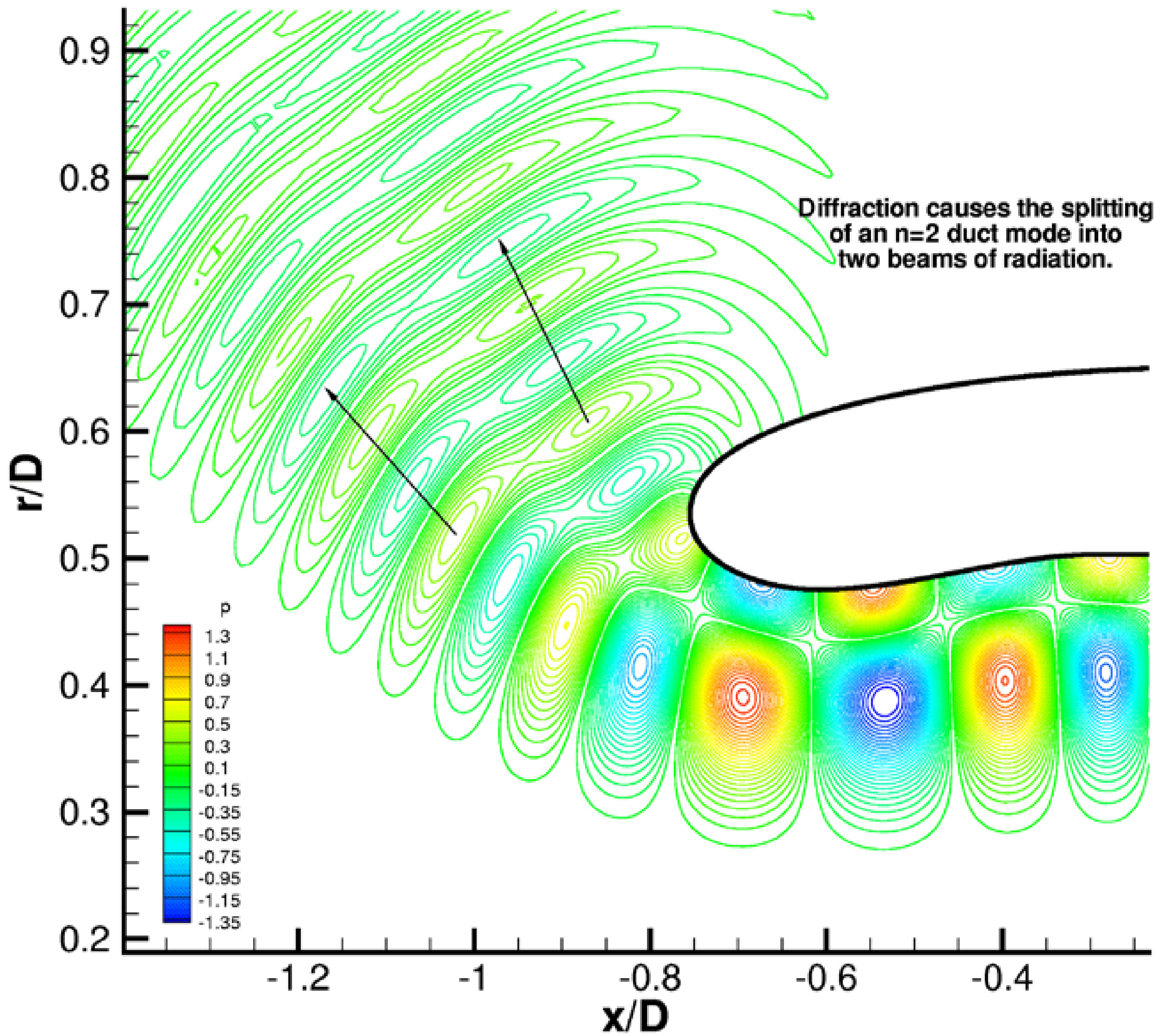
Frequency variation

Flight Mach number = 0.2 **No forward flight**

Higher order radial modes, n = 2 +



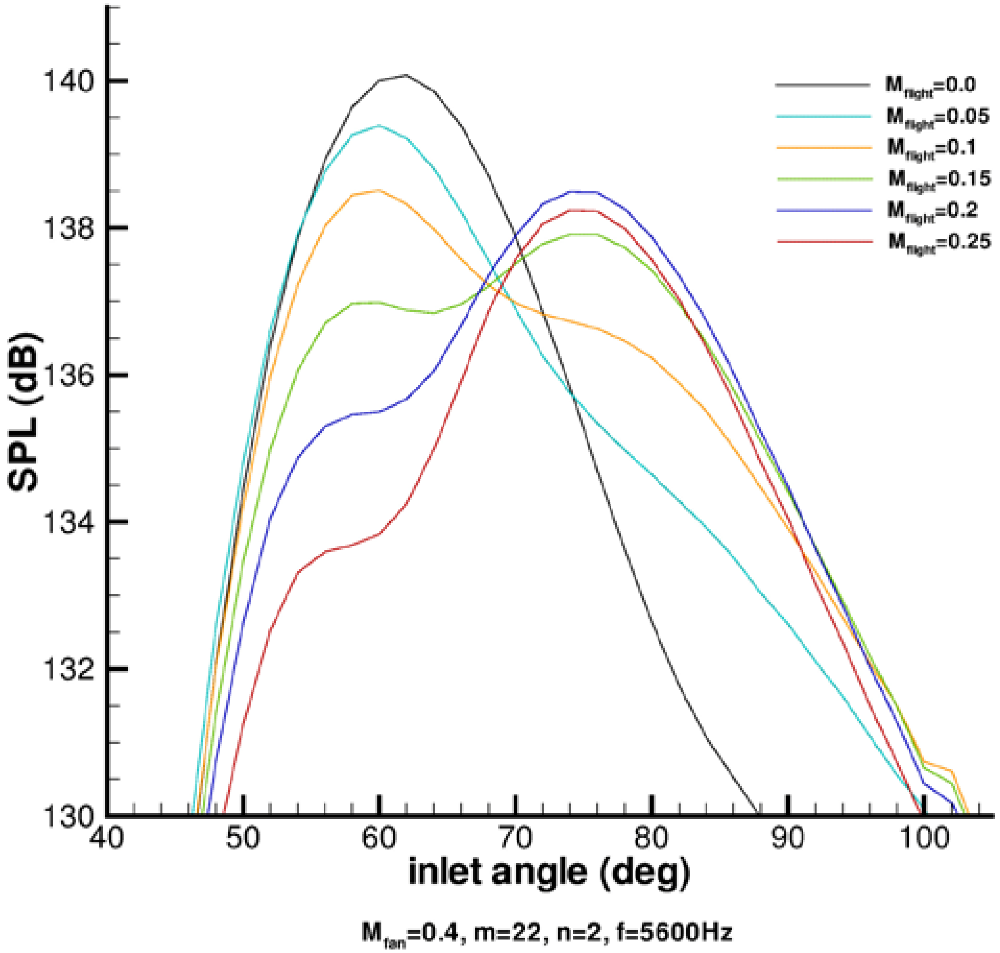
M_{fan}=0.0, M_{flight}=0.0, m=22, n=2, f=6400Hz

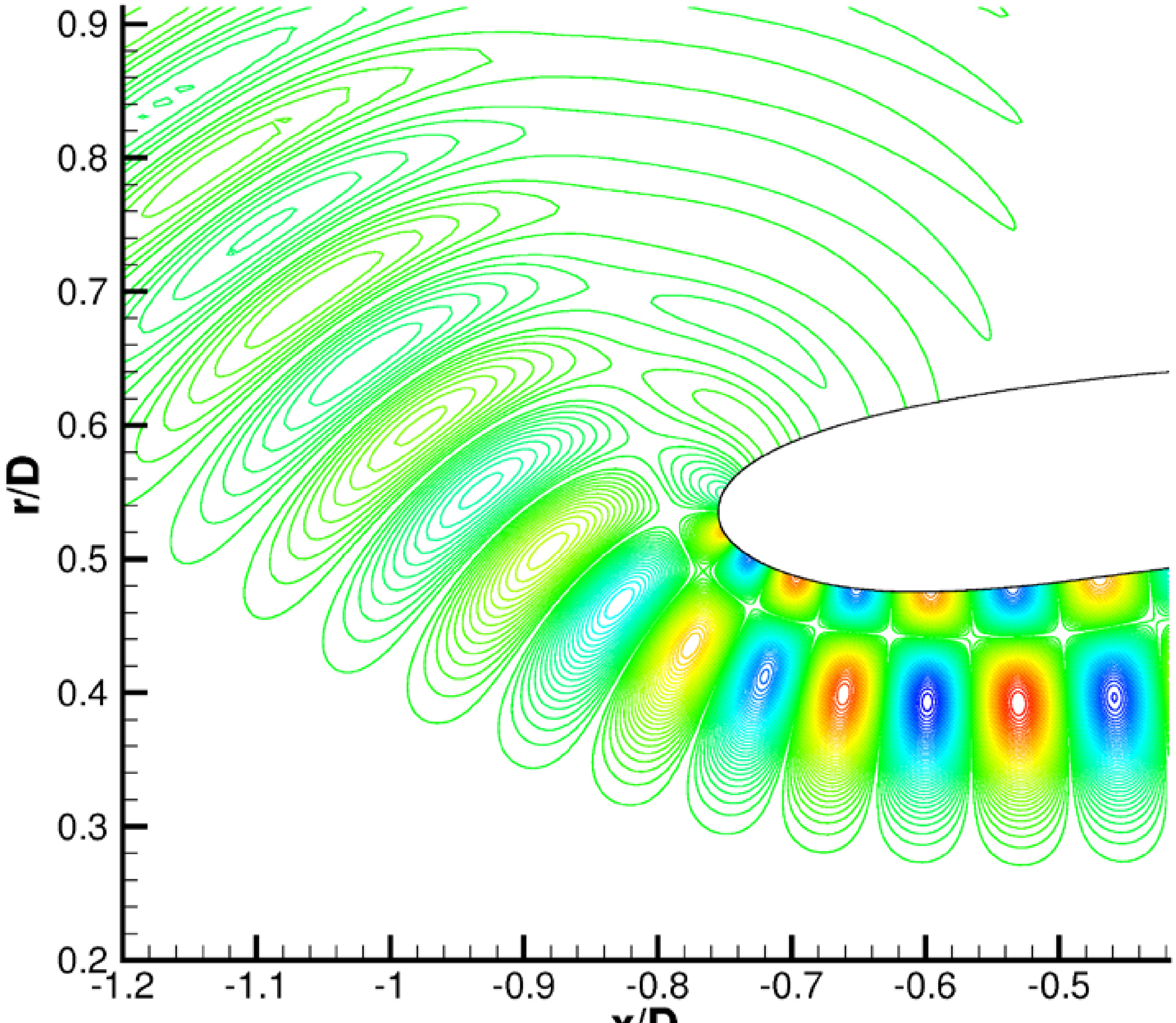


M_{fan}=0.0, M_{flight}=0.0, m=22, n=2, f=6400Hz

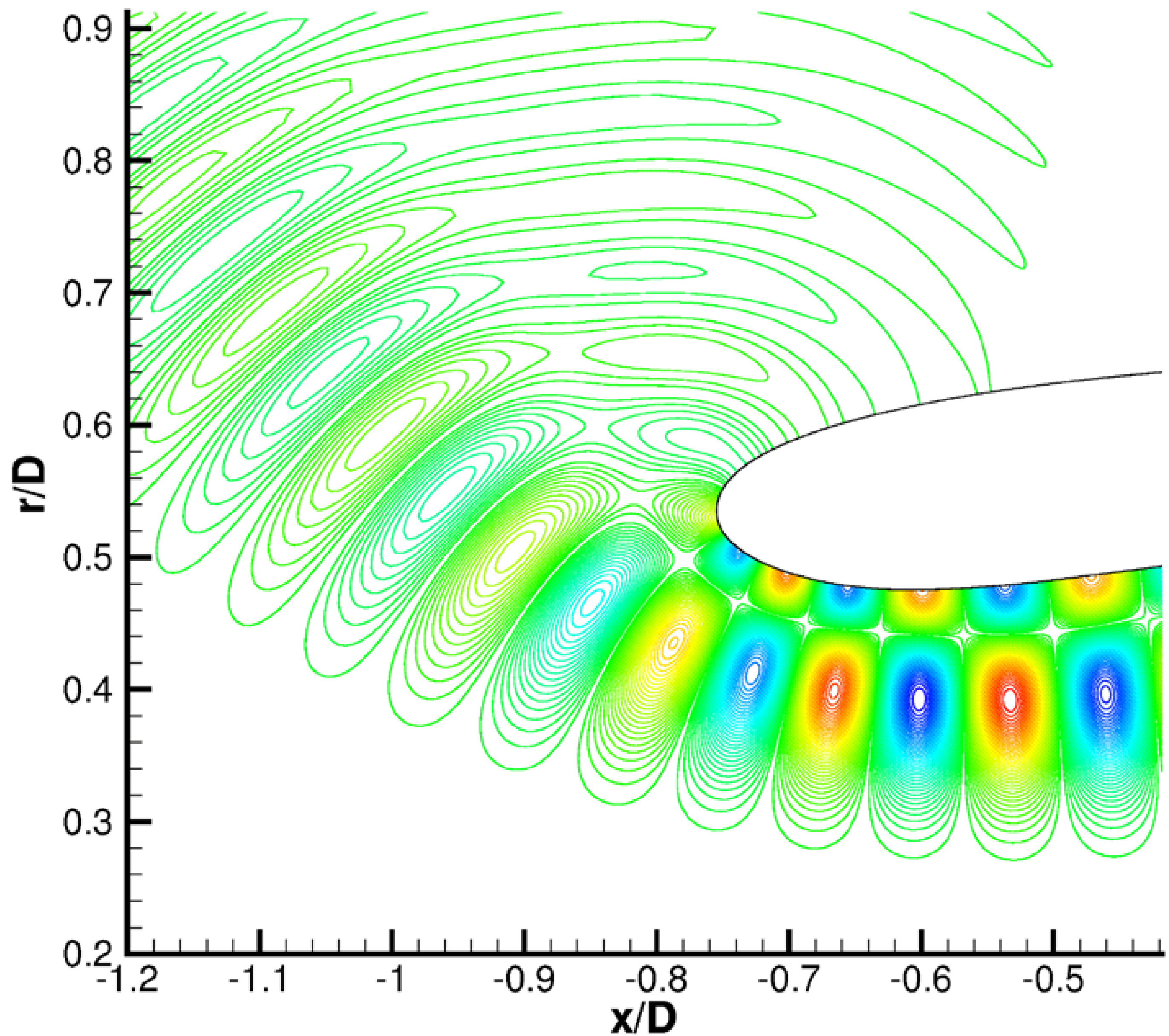
Enlarged contour plot

Relative intensity of the two beams are affected by forward flight.

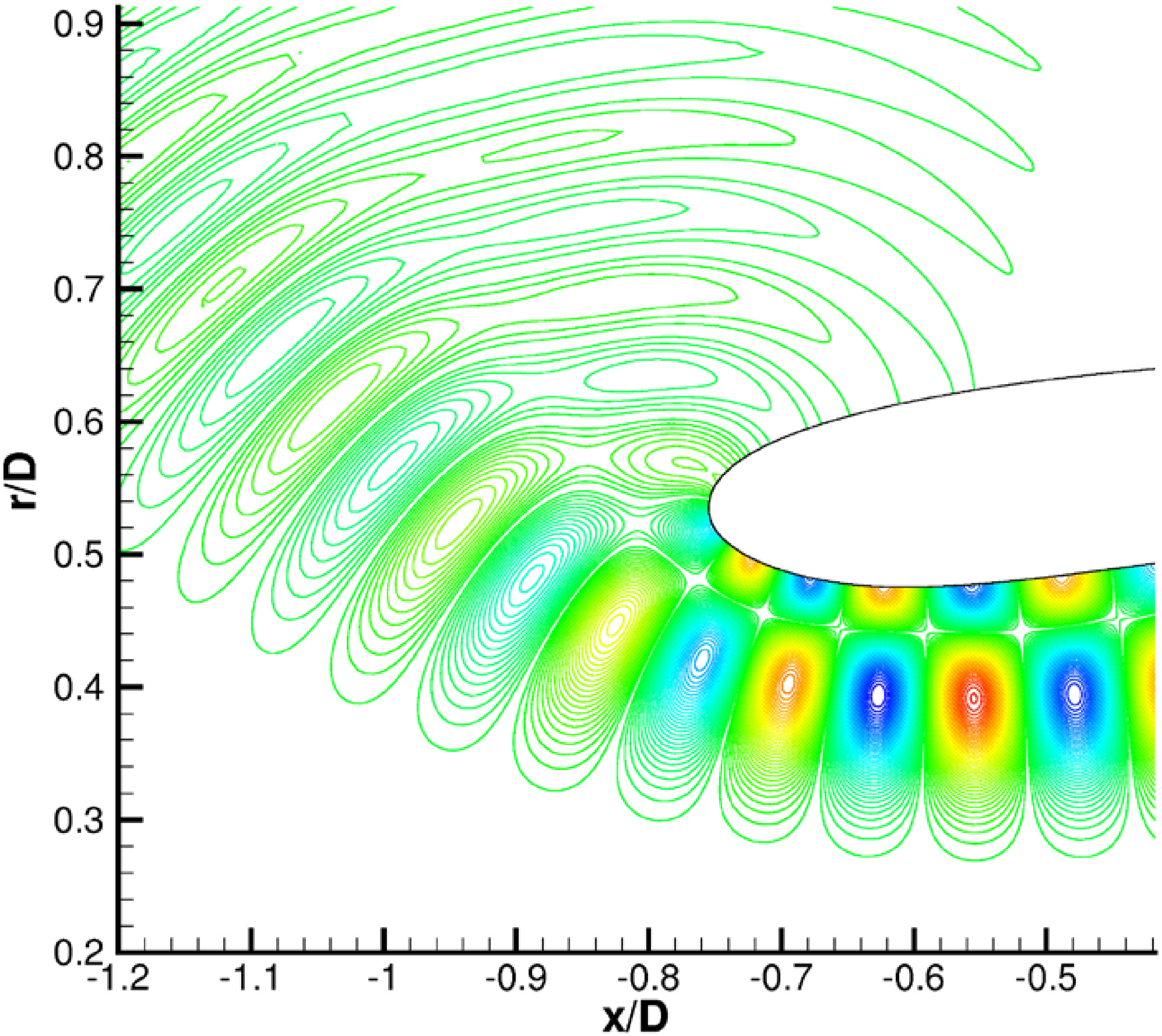




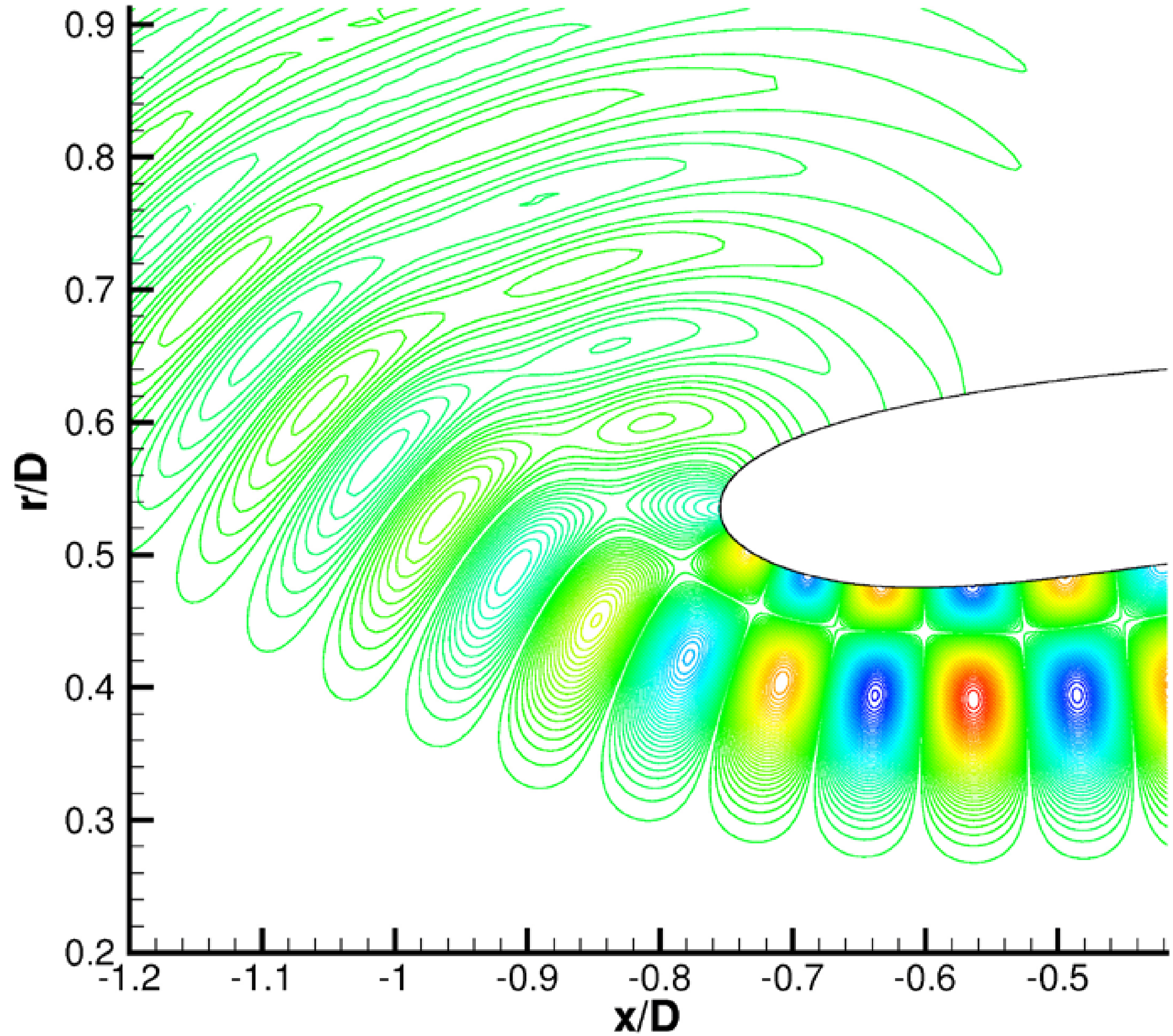
M_{fan}=0.4, M_{flight}=0.0, m=22, n=2, f=5600Hz



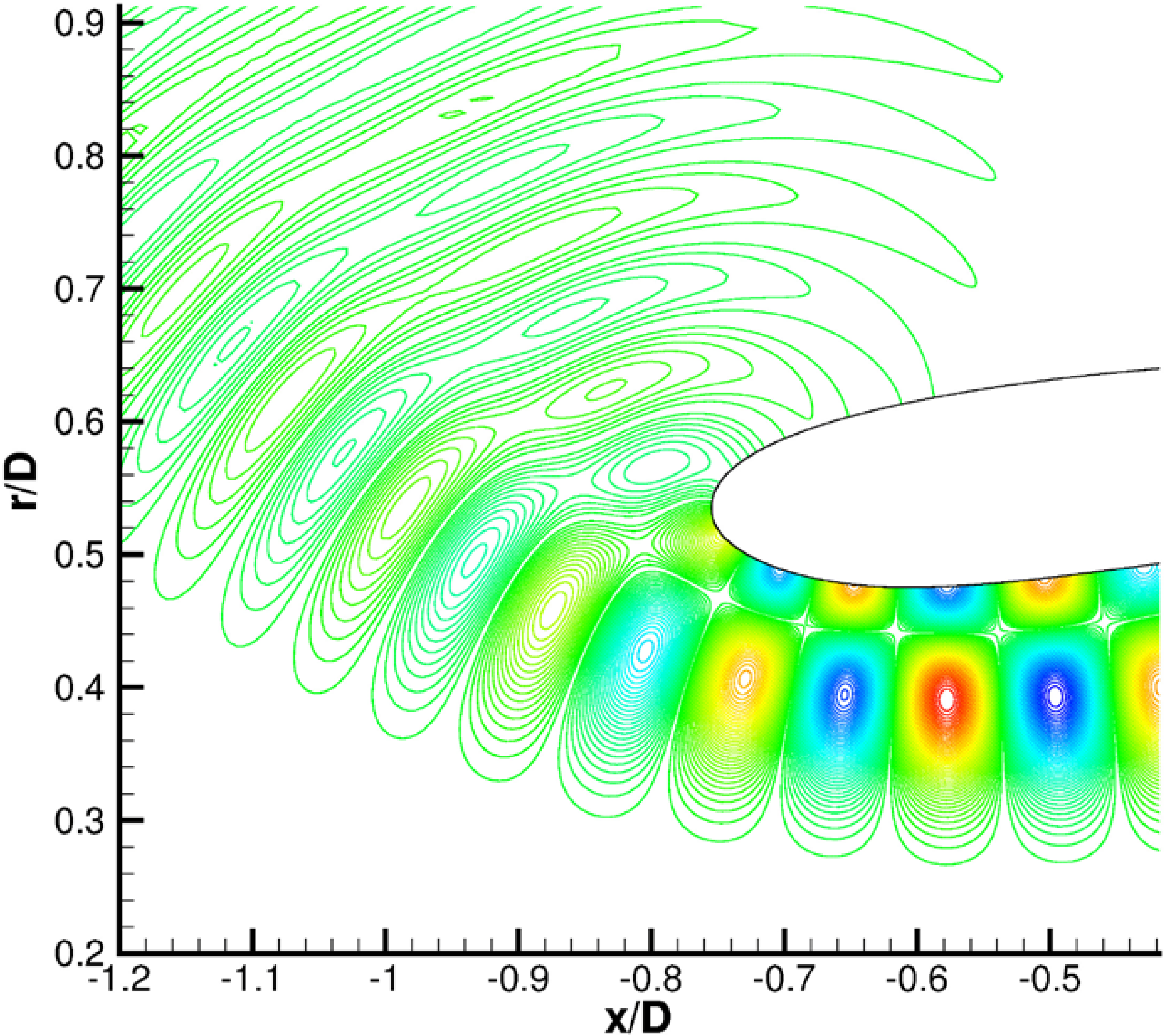
M_{fan}=0.4, M_{flight}=0.05, m=22, n=2, f=5600Hz



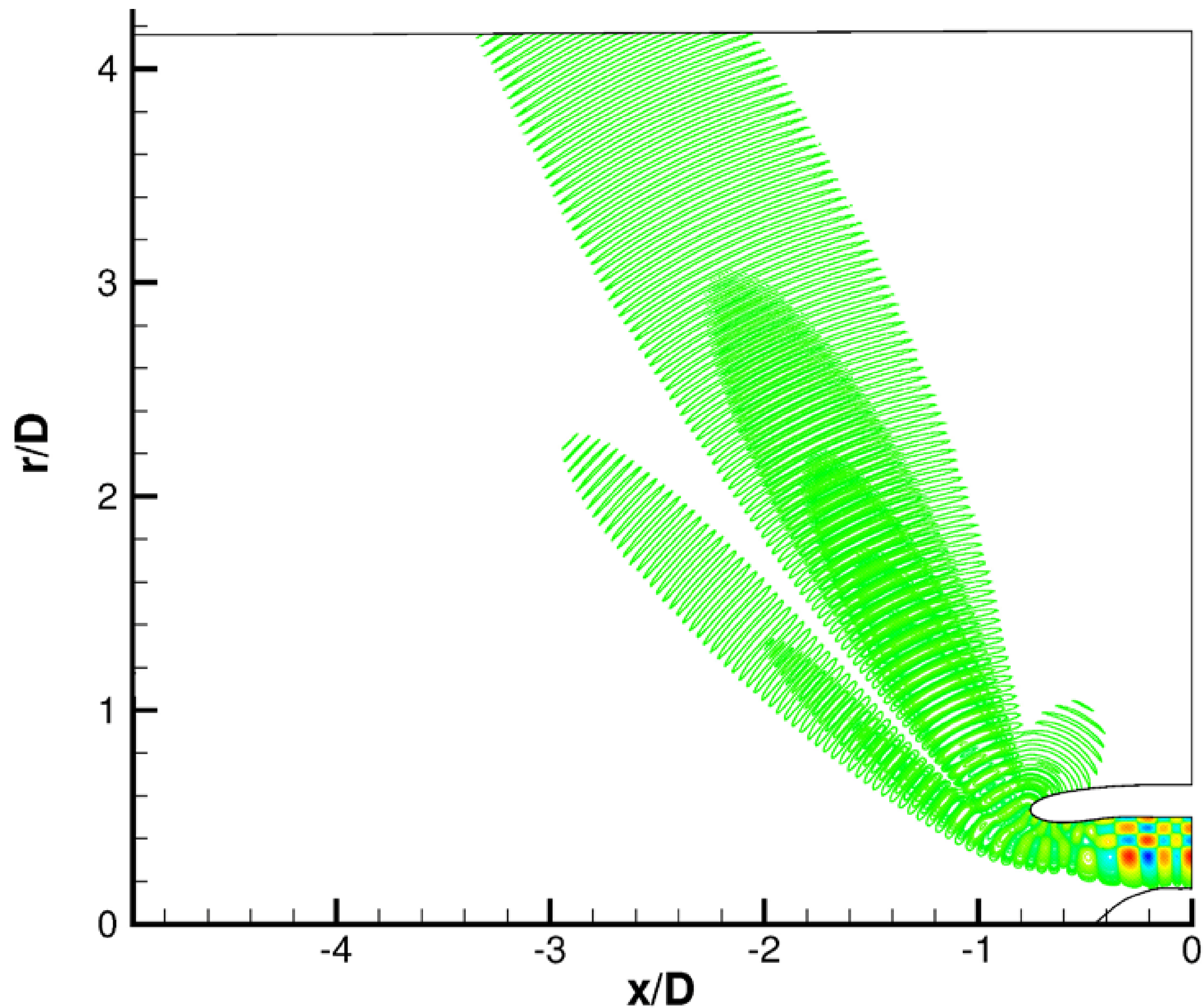
M_{fan}=0.4, M_{flight}=0.1, m=22, n=2, f=5600Hz



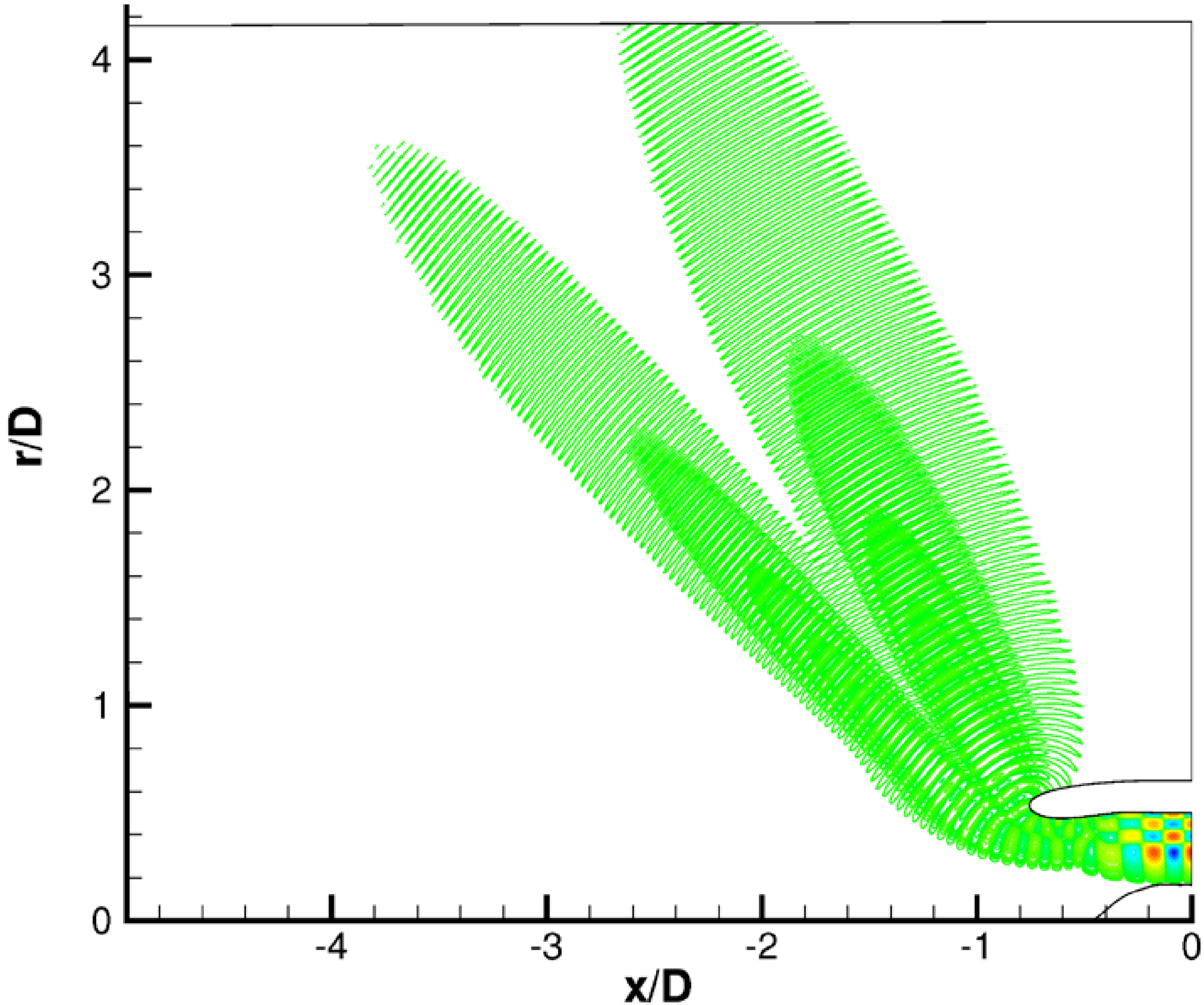
M_{fan}=0.4, M_{flight}=0.15, m=22, n=2, f=5600Hz



M_{fan}=0.4, M_{flight}=0.2, m=22, n=2, f=5600Hz



M_{fan}=0.4, M_{flight}=0.0, m=22, n=4, f=6800Hz



M_{fan}=0.4, M_{flight}=0.2, m=22, n=4, f=6800Hz

Summary and Conclusions

1. Jet engine inlet radiation is controlled largely by two physical processes.

   (a) Diffraction (natural tendency for acoustic waves to follow a solid surface.

   (b) Refraction (the change in direction of radiation due to mean flow velocity gradients).

2. Refraction effect on direction of radiation at static condition is opposite to that when there is forward flight.

3.. Axial acoustic wave number is an important parameter influencing the rotation of the direction of radiation due to diffraction and refraction.

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1. Jet engine inlet radiation is controlled largely by two physical processes.

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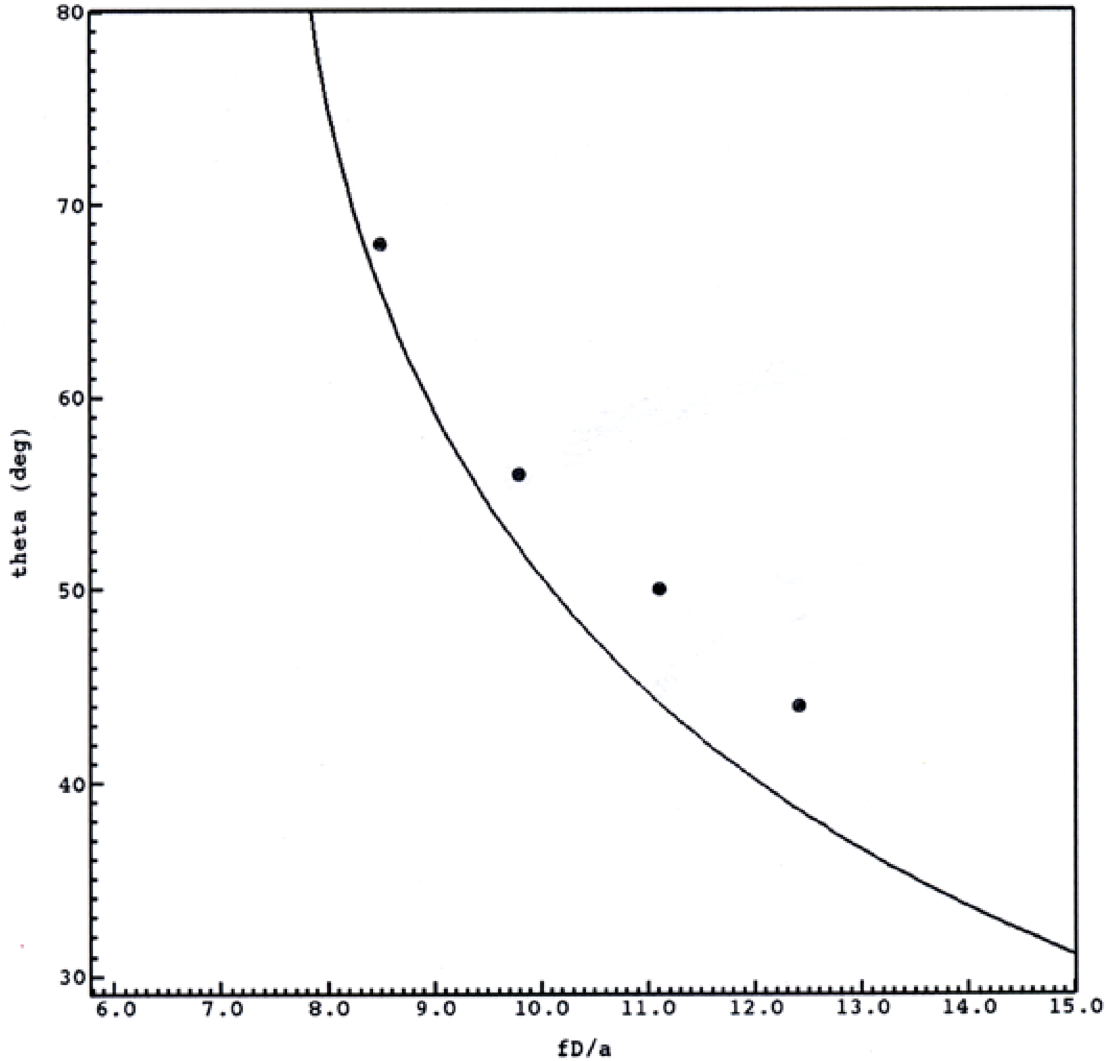
Summary and Conclusions

4. Prediction formulas for principal direction of radiation from engine inlets developed prior to 1990 are generally not applicable to modern jet engines.

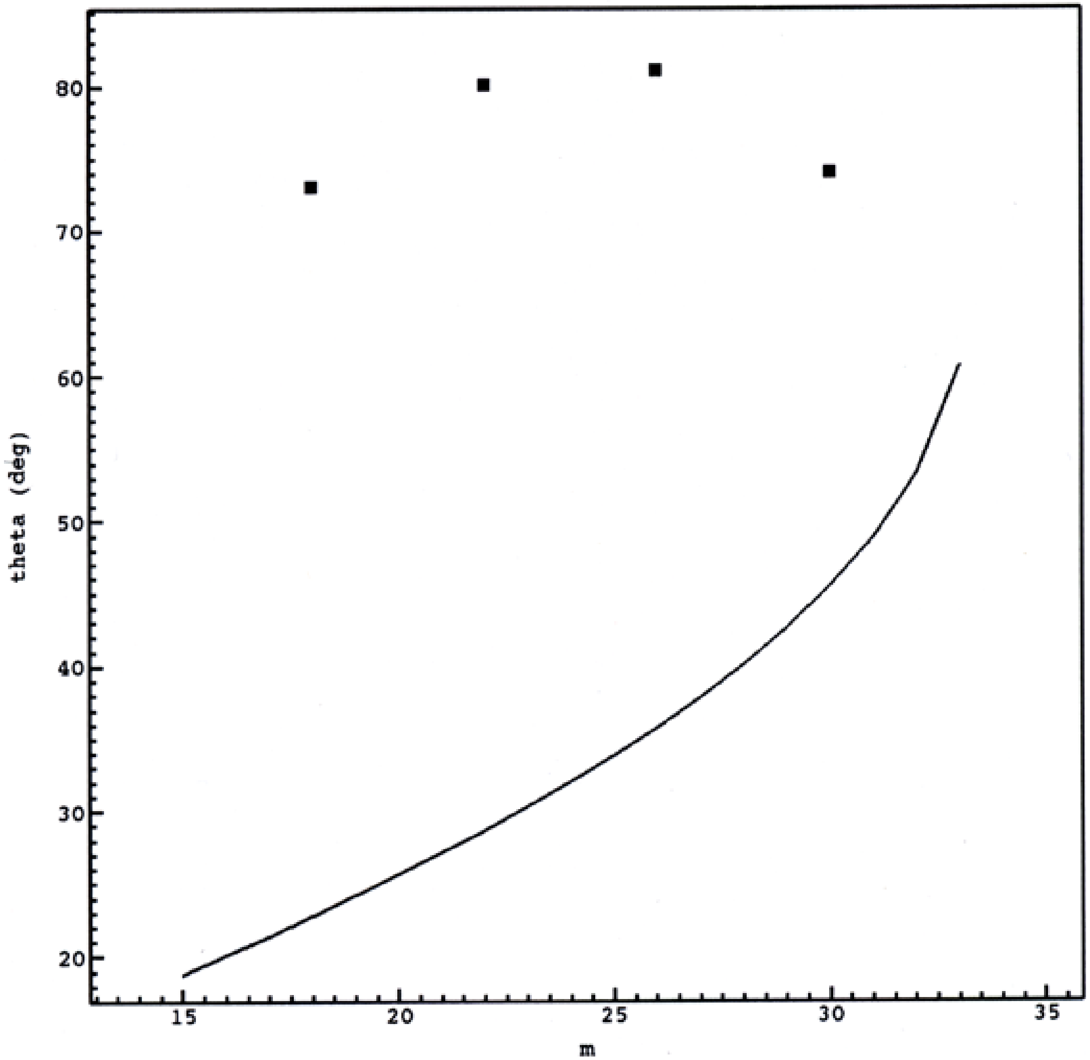
   e.g. Rice, E.J., Heidmann, M.F. & Soffrin, T.G. 1979 “Modal Propagation Angles in a Cylindrical Duct with Flow and their Relation to Sound Radiation,” AIAA Paper 79-0183.

   Based on exact solution of zero thickness cylindrical duct (Wiener-Hopf technique; no refraction)

$$\cos(\psi) = \frac{M_{fan} + \left(1 - \frac{1}{\xi^2}\right)^{\frac{1}{2}}}{1 + M_{fan}\left(1 - \frac{1}{\xi^2}\right)^{\frac{1}{2}}} \qquad \xi = \frac{k}{\alpha_{nm}\left(1 - M_{fan}^2\right)}$$

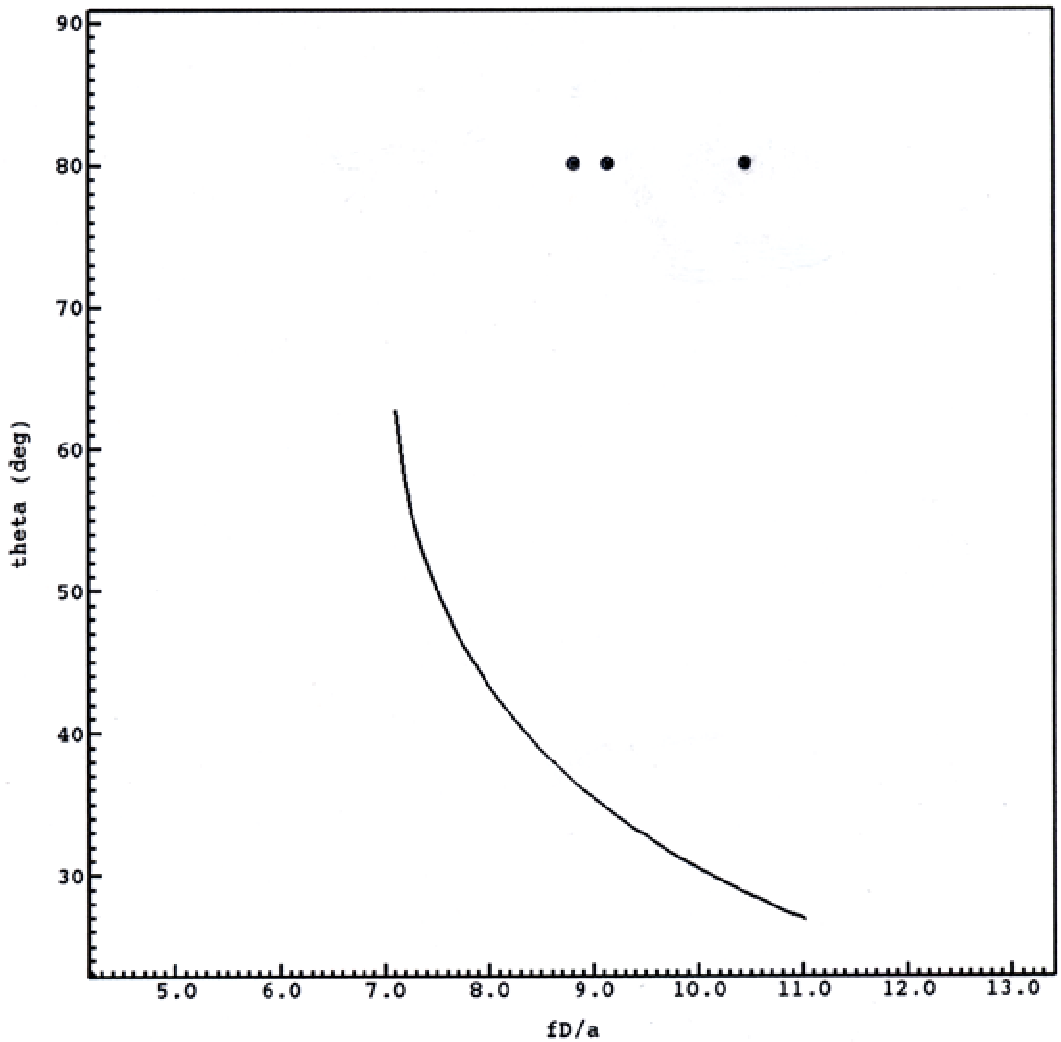


SDT inlet geometry, m = 22, n = 1. Full line = Rice et al formula, black circle = numerical simulation



SDT inlet geometry, f = 6400 Hz, n = 1. Full line = Rice et al formula,

black square = numerical simulation



SDT inlet geometry, m = 22, n = 1. Full line = Rice et al formula,

black circle = numerical simulation

Principal Conclusion

Data from static engine tests cannot be used to predict the directivity and spectra of the engine when in forward flight. This is because the physics involved in the two cases are quite difference.