The Aeroacoustics of Swirling Flow

CEAA 2012,
20th September 2012

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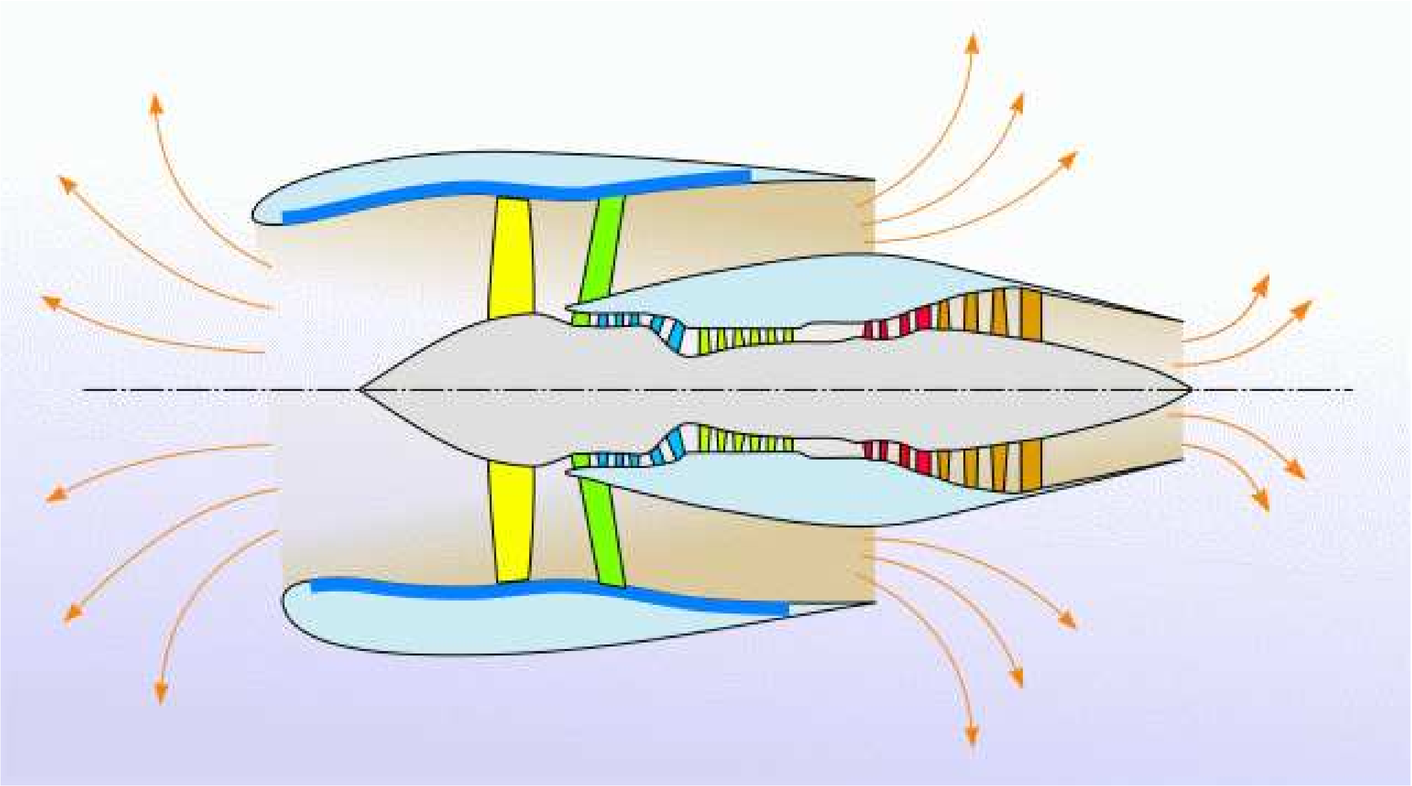
N. Peake & A.B. Parry 2012 Modern Challenges Facing Turbomachinery Aeroacoustics.
Annual Reviews of Fluid Mechanics. **44.**

Typical aeroengine

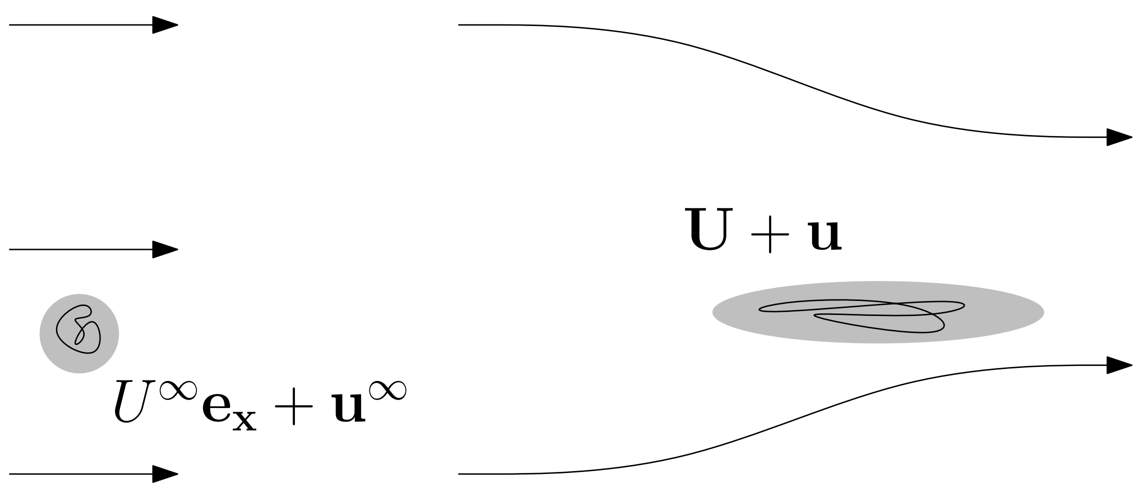


Courtesy of Sjoerd Rienstra

Plan of Talk

- Some basic properties.
- Some asymptotic results.
- Acoustic analogy in swirling flow.

Rapid Distortion Theory (RDT)



- Total velocity decomposed as $\mathbf{U}_0(x, r, \theta) + \mathbf{u}(x, r, \theta, t)$.
- Unsteady perturbation velocity decomposed as $\mathbf{u} = \mathbf{a} + \boldsymbol{\nabla}\phi$.
- Unsteady pressure given by $p = -\rho_0 \frac{D_0 \phi}{Dt}$.

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RDT equations

$$\frac{D\boldsymbol{a}}{Dt} + \boldsymbol{a}\cdot\boldsymbol{\nabla}\boldsymbol{U}_0 = -\boldsymbol{\Omega}_0 \wedge \boldsymbol{\nabla}\phi,$$

$$\frac{D}{Dt}\left(\frac{1}{c_0^2}\frac{D\phi}{Dt}\right)\frac{1}{\rho_0}\boldsymbol{\nabla}\cdot(\rho_0\boldsymbol{\nabla}\phi) = \frac{1}{\rho_0}\boldsymbol{\nabla}\cdot(\rho_0\boldsymbol{a}),$$

- $\boldsymbol{\Omega}_0 = \boldsymbol{\nabla} \wedge \boldsymbol{U}_0$ is the mean vorticity.
- $\boldsymbol{\Omega}_0 = 0$ means that the unsteady vorticity is uncoupled from the unsteady pressure.Classical RDT.

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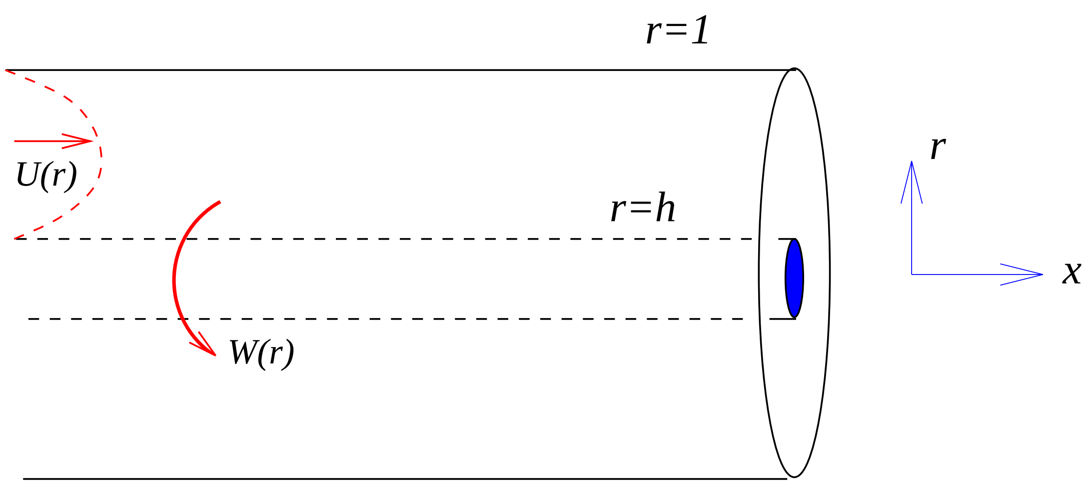
The spectrum

- Consider cylindrical duct $h \le r \le 1$. Maybe $h = 0$.
- Mean flow $\mathbf{U}(r) = U(r)\mathbf{e}_x + W(r)\mathbf{e}_\theta$.
- Assume all unsteady flow quantities proportional to

$$\exp(\mathrm{i}kx + \mathrm{i}m\theta - \mathrm{i}\omega t)$$

- What are the allowed values of k? And the corresponding modes?



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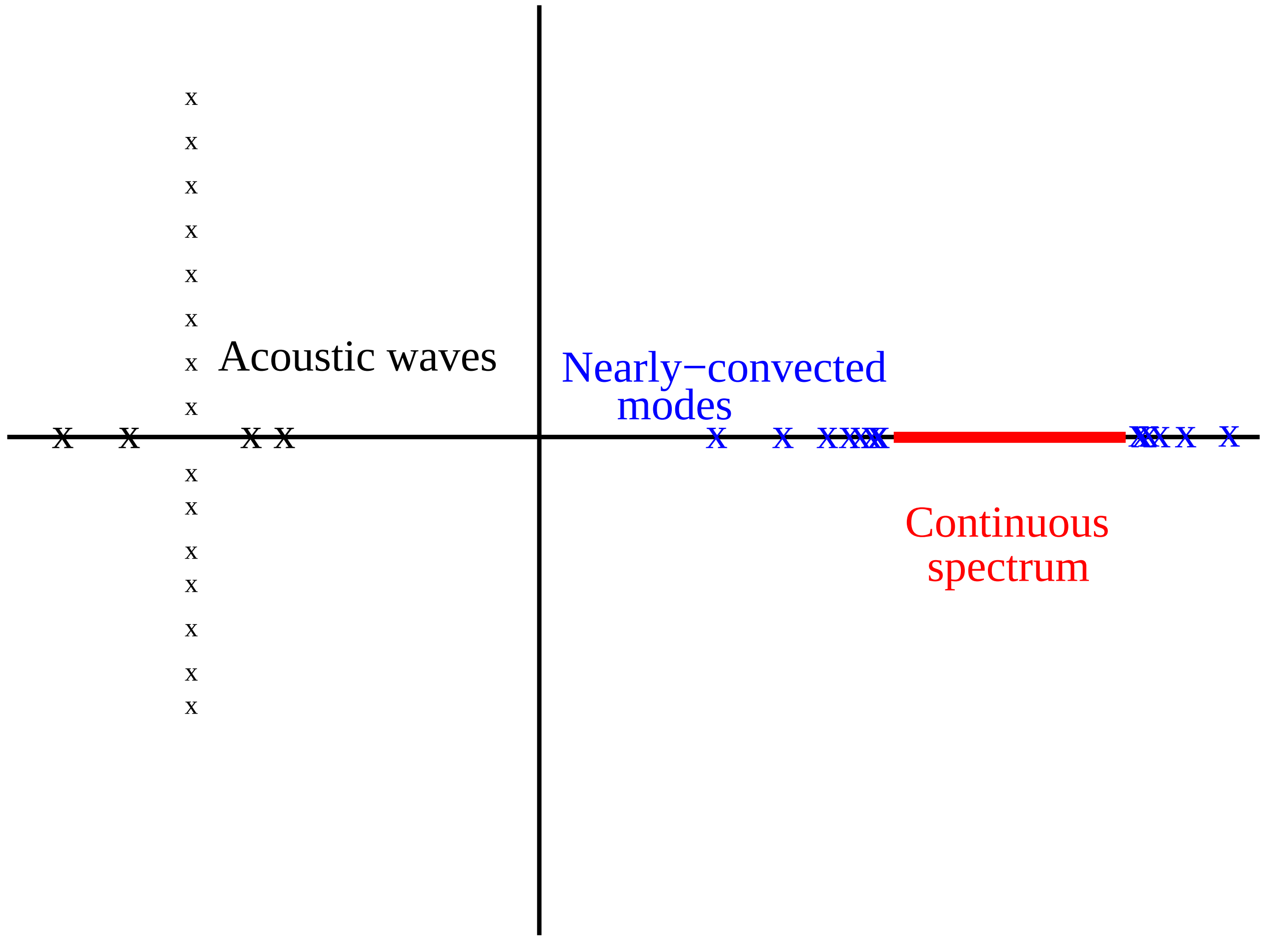
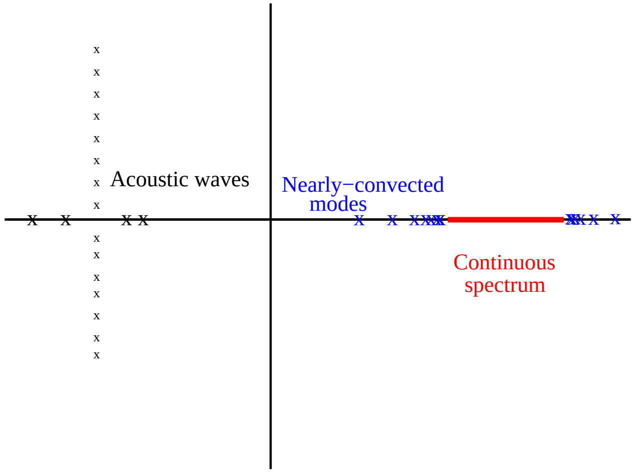
Possible axial spectrum

Eigensolutions in cylindrical duct with rigid walls.



Structure of Acoustic Modes



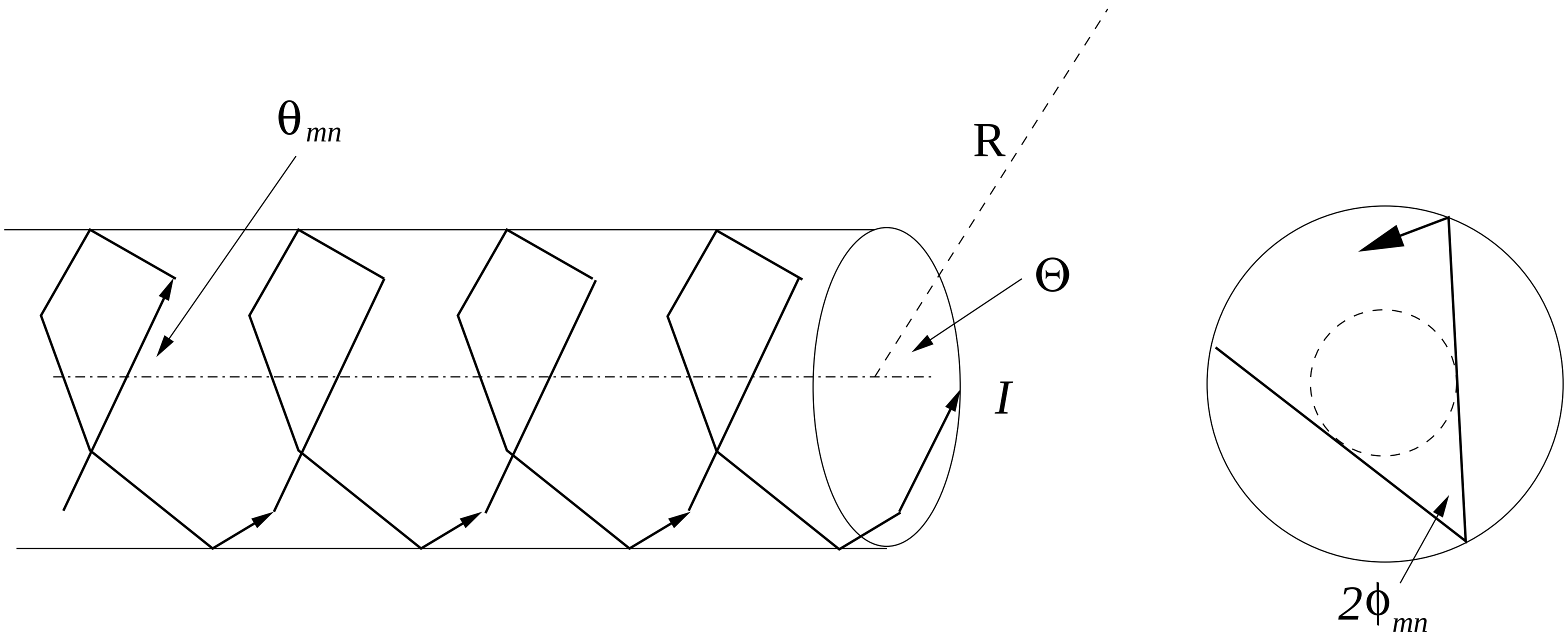
WKB expansion for unsteady pressure satisfies

$$\frac{d^2 p}{\mathrm{d}r^2} = -m^2 f(r) p$$

$$f(r) \equiv \frac{\Lambda^2}{m^2 c_0^2} - \frac{1}{r^2} - k^2$$

$$\Lambda \equiv \omega - kU - \frac{mW}{r} .$$

Ray structure in uniform flow



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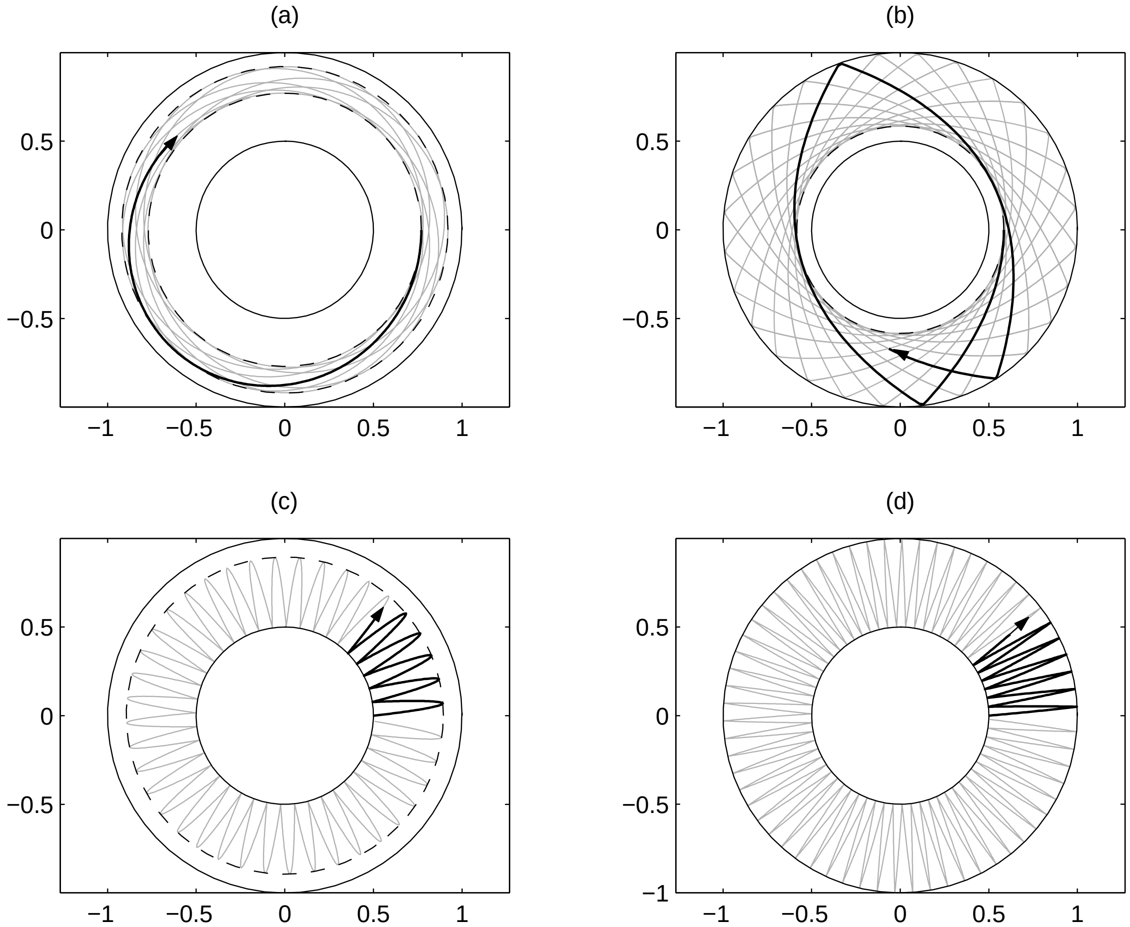
Possible ray structures in swirl.



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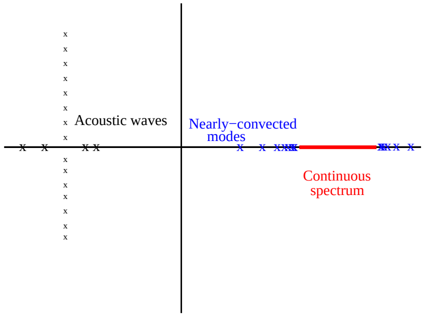
The continuous spectrum



The continuous spectrum is C_k

$$C_k \equiv \{k : \Lambda(k,r) = \omega - kU(r) - mW(r)/r = 0\}$$

For a given $k \in C_k$, write $\phi = a_0(r - r_c)^\sigma$ near critical layer $r = r_c$, and find

$$\sigma = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - A(r)}$$

$$A(r) = \frac{2Wk_c(k_c(Wr)' - mU')}{r^2(m(W/r)' + U'k_c)^2}$$

If $W = 0$, no swirl, $\sigma = -1, 0$.

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Growth of continuous spectrum

- Initial-value problem, given field at $x = 0$ Fourier transform in x, solve for $\overline{\phi}(r, k)$ and invert:

$$\boxed{\phi(r, x) = \sum \text{discrete modes} + \int_{C_k} \overline{\phi}(r, k) \exp(\mathrm{i}kx)\mathrm{d}k}$$

- at $k = k_c(r)$ have singularity $\overline{\phi}(r, k) \sim (k - k_c)^{\sigma(r)}$
- as $x \to \infty$,

$$\phi(r, x) \sim x^{-\sigma(r)-1} \exp(\mathrm{i}k_c(r)x)$$

- condition for algebraic growth is $\sigma < -1$, i.e.

$$\boxed{\frac{2Wk_c}{r^2}\left(k_c(Wr)' - mU'\right) < 0.}$$

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Large blade-number asymptotics

Assume large blade number, so azimuthal order $m \gg 1$.
Unsteady velocities now of form

$$(\mathbf{A}(x,r), \phi(x,r)) \exp\left(\mathrm{i}mK(x,r) + \mathrm{i}m\theta - \mathrm{i}m\Omega t\right)$$

with

$$-\omega + \frac{mW}{r} + K_x U = 0 \,,$$

and

$$\frac{\partial \mathbf{A}}{\partial x} = M(x,r)\mathbf{A} \,,$$

and hydrodynamic boundary-layer, width $O(1/m)$, on walls.

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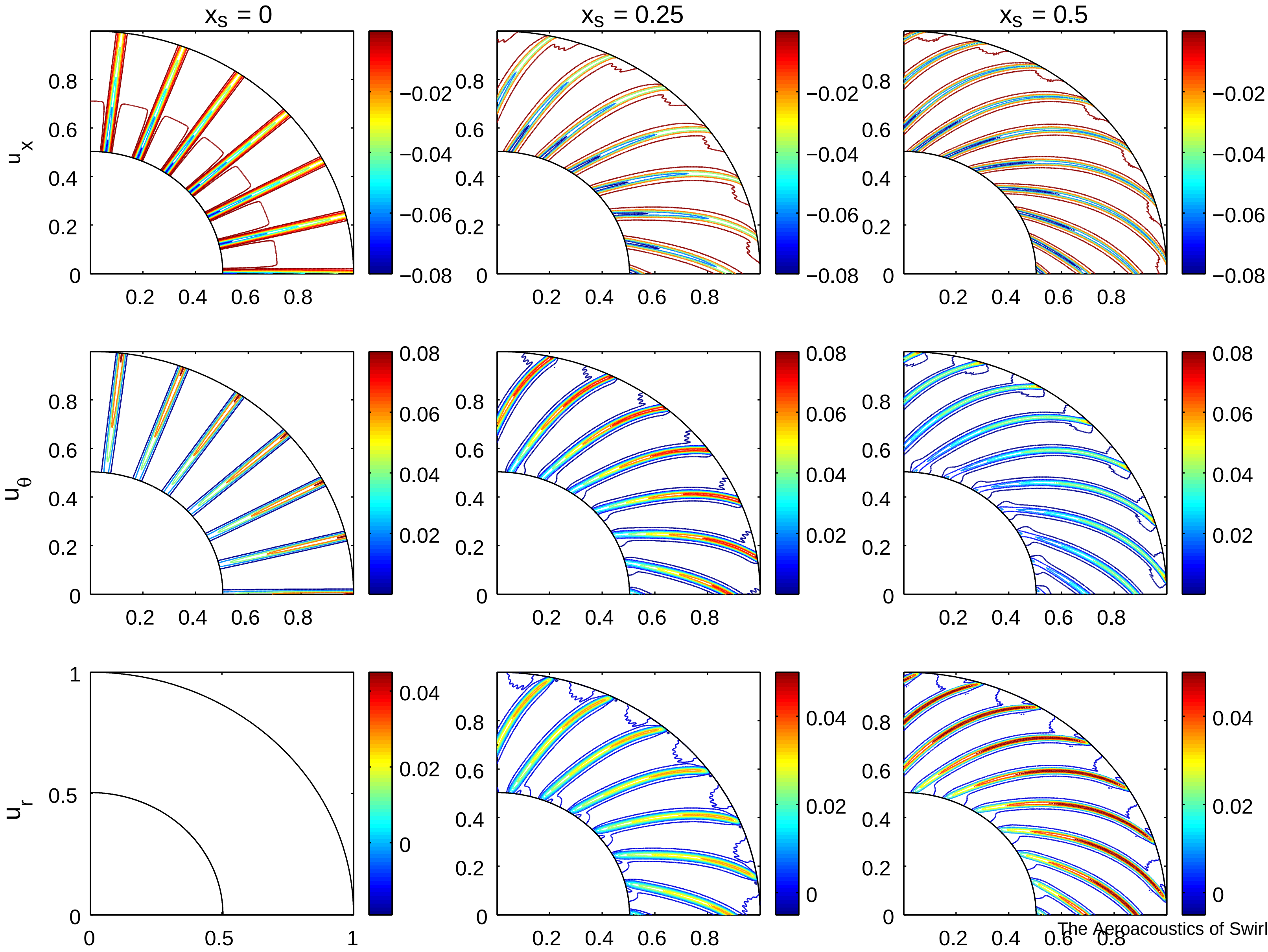

$x_S = 0$
$x_S = 0.25$
$x_S = 0.5$
u_x
u_θ
u_r
−0.02
−0.04
−0.06
−0.08
0.08
0.06
0.04
0.02
0
0.2
0.4
0.6
0.8
0.5
1

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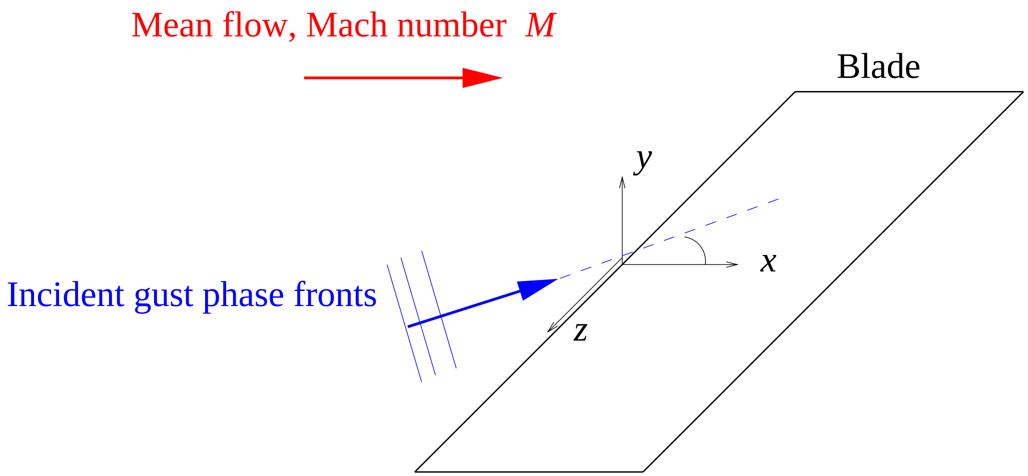


Gust component

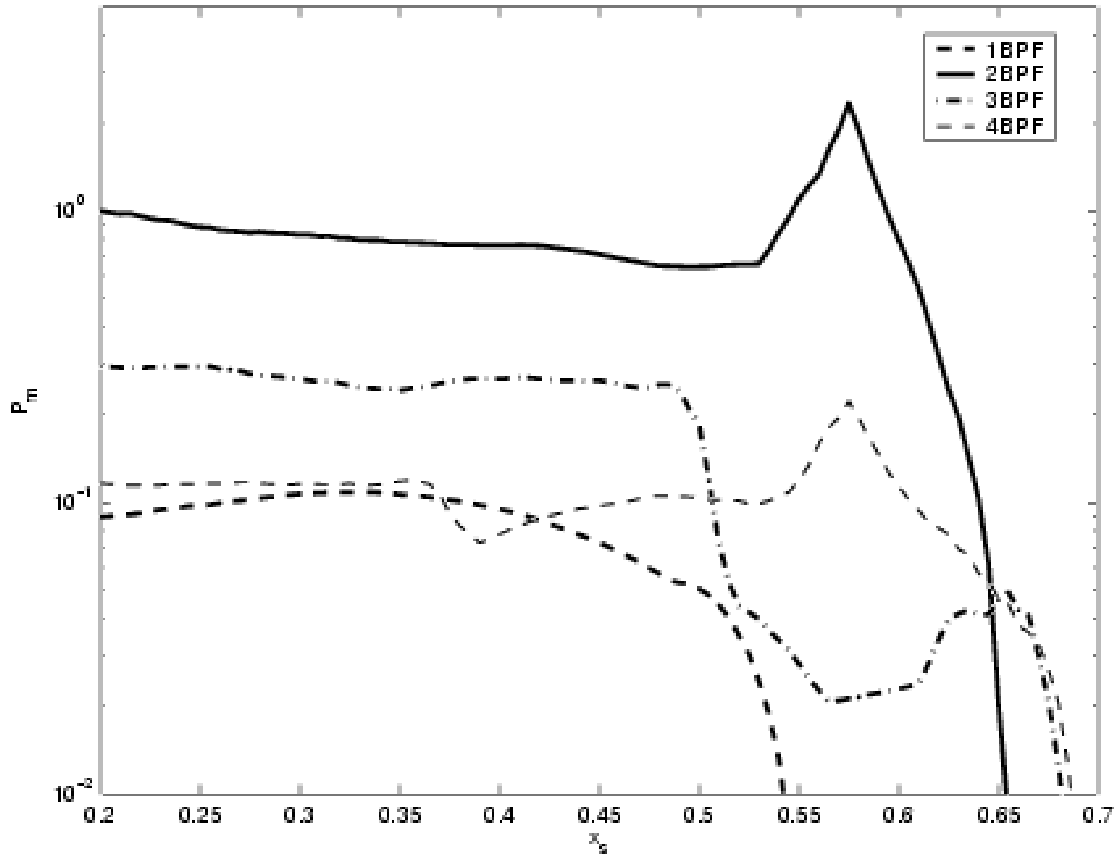
$$\exp(-\mathrm{i}kt + \mathrm{i}k[x + k_y y + k_z z])$$

Noise radiated if

$$|k_z| < M/\sqrt{1 - M^2}$$

i.e. AS LONG AS GUST IS NOT ‘TOO’ OBLIQUE.

Rotor-stator gap is increased.



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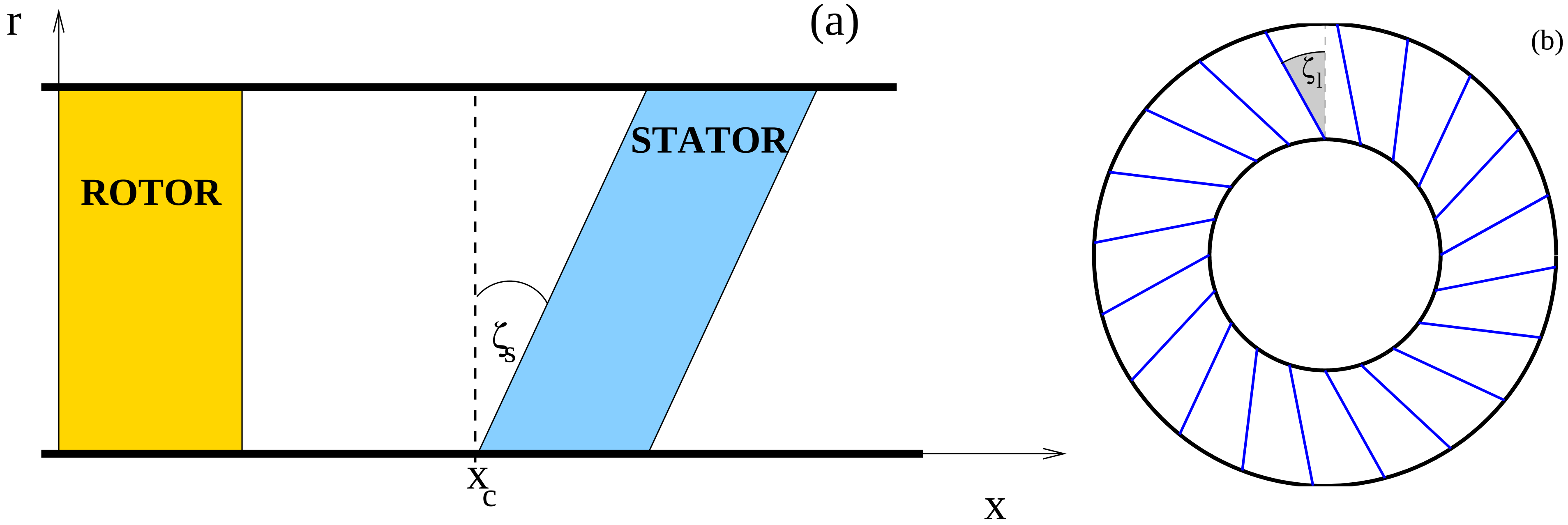
Varying blade geometry



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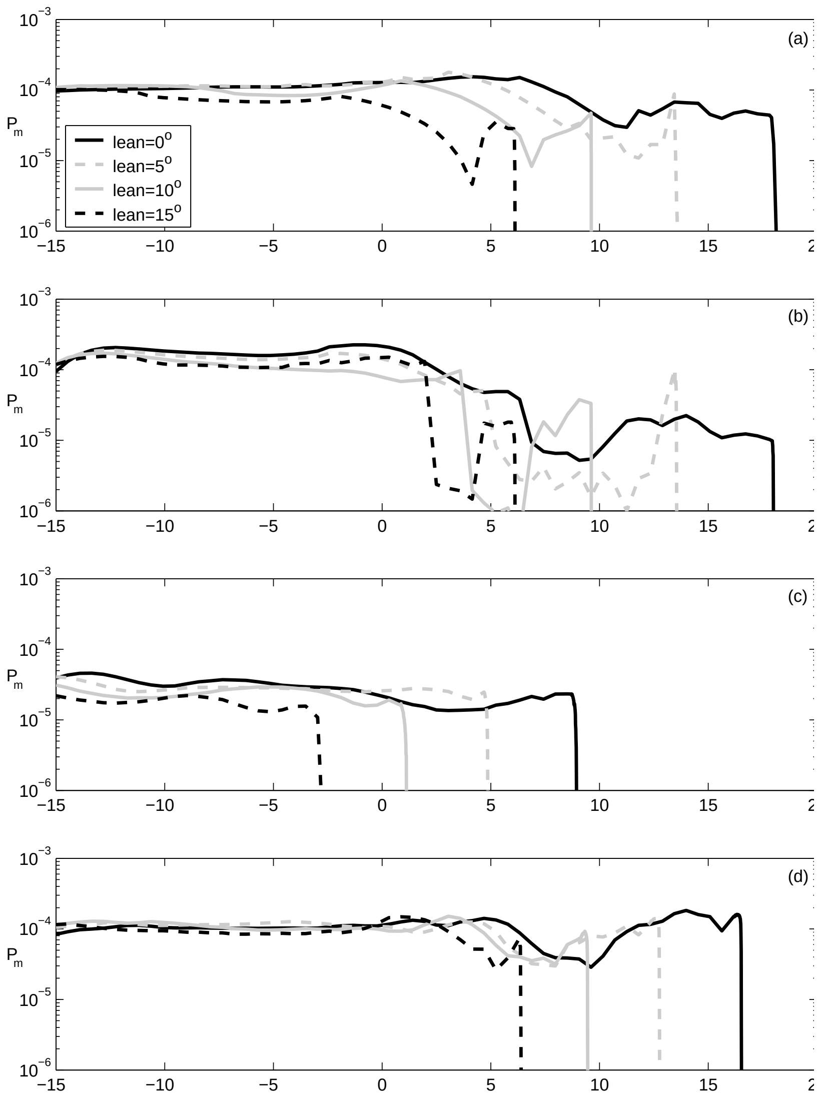
Noise for varying sweep



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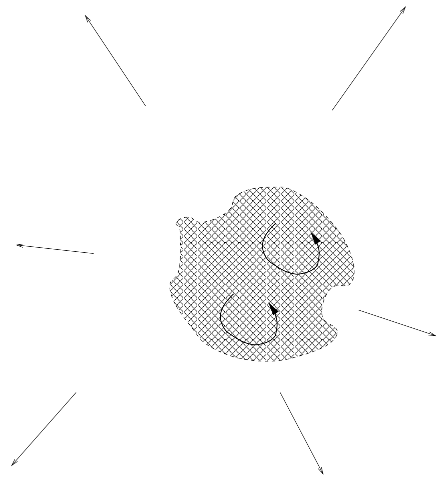
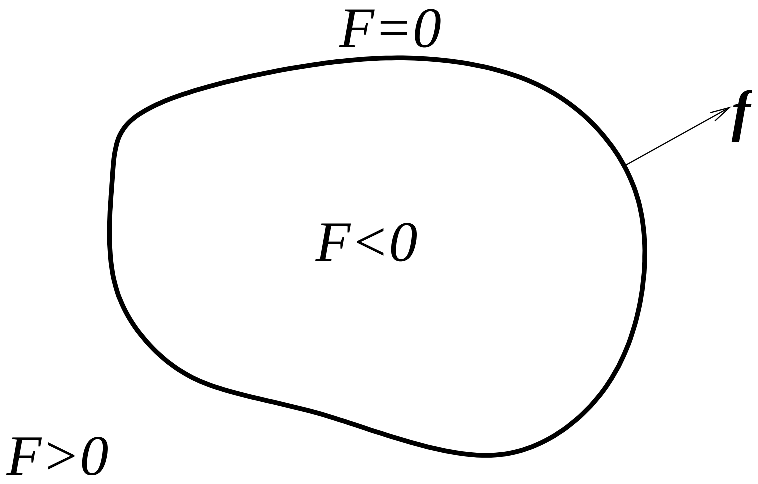
Lighthill's acoustic analogy



$$\left[\frac{\partial^2}{\partial t^2} - c_0^2 \boldsymbol{\nabla}^2\right] \rho' = \frac{\partial^2}{\partial x_i \partial x_j}\left[T_{ij}\right]$$

$$T_{ij} = \rho u_i u_j + (p - c^2 \rho)\delta_{ij} + \tau_{ij}$$

Ffowcs Williams' extension



$$\left[\frac{\partial^2}{\partial t^2} - c_0^2 \boldsymbol{\nabla}^2\right](\rho' \mathrm{H}(F)) = \frac{\partial^2}{\partial x_i \partial x_j}\left[T_{ij}\mathrm{H}(F)\right] + \frac{\partial}{\partial t}\left[\rho_0 V_n \delta(F)\right] - \frac{\partial}{\partial x_i}\left[f_i \delta(F)\right]$$

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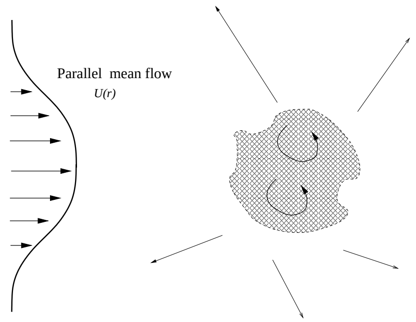
Lilley's equation



Third order partial differential equation in space and time for the pressure fluctuation p:

$$\frac{D}{Dt}\left(\frac{1}{c_0^2}\frac{D^2 p}{Dt^2} - \nabla^2 p\right) + 2\frac{\mathrm{d}U}{\mathrm{d}r}\frac{\partial^2 p}{\partial r \partial x} =$$

$$\frac{D}{Dt}\left(\frac{\partial^2}{\partial x_i \partial x_j}[T_{ij}]\right) - 2\frac{\mathrm{d}U}{\mathrm{d}r}\frac{\partial(\nabla.(\rho \mathbf{u} u_r))}{\partial x}$$

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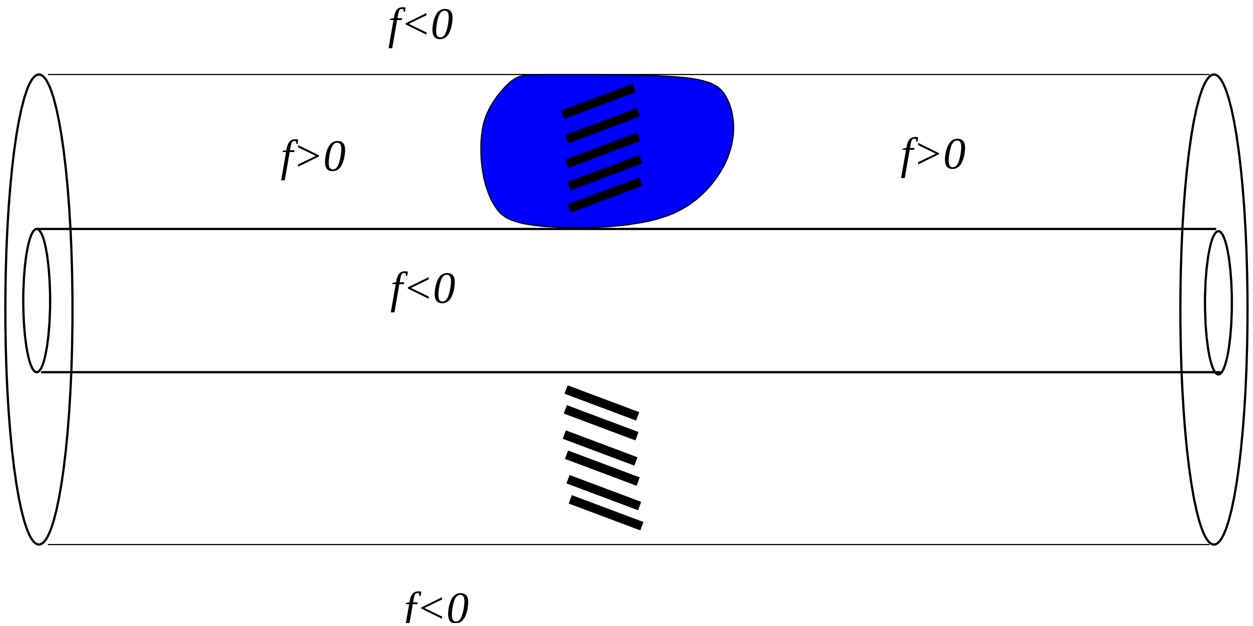

Fluid volume $\mathcal{V}(t)$, surface $\Sigma(t)$, $f > 0$ in $\mathcal{V}(t)$.

$$\mathbf{u}_{to} = \mathbf{U} + \mathbf{u}\ , \quad \rho_{to} = \rho_0 + \rho\ , \quad p_{to} = P_0 + p\ .$$

Mean flow $\mathbf{U} = U_\theta(r)\mathbf{e}_\theta + U_x(r)\mathbf{e}_x$.

Rearrange 4 compressible Navier-Stokes in form

$$\mathcal{L}\left(\mathrm{H}(f)\mathbf{u}, \mathrm{H}(f)\rho, \mathrm{H}(f)p\right) = \begin{pmatrix} S_\rho \\ \mathbf{S} \end{pmatrix} \mathrm{H}(f) + \begin{pmatrix} S_{FWH,\rho} \\ \mathbf{S_{FWH}} \end{pmatrix} \delta(f).$$

- $\mathcal{L}$ is linearised Euler operator based on mean swirling flow,
- $\mathbf{S}$ and S_ρ volume sources including quadratic perturbation terms, viscous and non isentropic terms.
- Surface sources

$$\begin{aligned} \mathbf{S_{FWH}} &= \rho_{to}\mathbf{u}\left(\mathbf{u}_{to} - \mathbf{V}^{\Sigma}\right) \cdot \mathbf{n} + p\mathbf{n} - \overline{\overline{\boldsymbol{\tau}}}_{to} \cdot \mathbf{n} \\ S_{FWH,\rho} &= \rho_{to}\left(\mathbf{u}_{to} - \mathbf{V}^{\Sigma}\right) \cdot \mathbf{n} - \rho_0\left(\mathbf{U} - \mathbf{V}^{\Sigma}\right) \cdot \mathbf{n} \end{aligned}$$

Single equation for p

After a considerable amount of algebra, we find:

$$\mathcal{F}(p) \quad = \quad \mathbb{S} + \mathbb{S}_{FWH} \,,$$

with

$$\begin{aligned}\mathcal{F}(p) \quad = \quad & \mathcal{D}\left(\frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{\partial}{\partial r}\left(\mathcal{M}\left(p\right)\right)\right)\right) + \left\{\left[\frac{1}{r}\frac{\mathrm{D}_0}{\mathrm{D}t} - \frac{\mathrm{d}(rU_\theta)}{r\mathrm{d}r}\frac{\partial}{r\,\partial\theta} - \frac{\mathrm{d}U_x}{\mathrm{d}r}\frac{\partial}{\partial x}\right]\mathcal{D}\right. \\ & \left. +2\frac{\mathrm{D}_0^2}{\mathrm{D}t^2}\left[\frac{\mathrm{d}U_x}{\mathrm{d}r}\frac{\partial}{\partial x} + \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{U_\theta}{r}\right)\frac{\partial}{\partial\theta}\right] + \frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{2\,U_\theta}{r^2}\frac{\mathrm{d}(r\,U_\theta)}{\mathrm{d}r}\right]\frac{\mathrm{D}_0}{\mathrm{D}t}\right\}\mathcal{M}(p) \\ & - \left[\frac{\partial^2}{r^2\partial\theta^2} + \frac{\partial^2}{\partial x^2} - \frac{1}{c_0^2}\frac{\mathrm{D}_0^2}{\mathrm{D}t^2}\right]\mathcal{D}^2\left(p\right) \,,\end{aligned}$$

with

$$\mathcal{M}\left(p\right) = \frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{\partial p}{\partial r}\right) + 2\,\frac{U_\theta}{r^2}\frac{\partial p}{\partial\theta} - \frac{U_\theta^2}{r\,c_0^2}\frac{\mathrm{D}_0 p}{\mathrm{D}t} \quad \text{and} \quad \mathcal{D}\left(u\right) = -\frac{\mathrm{D}_0^2 u}{\mathrm{D}t^2} - 2\,\frac{U_\theta}{r^2}\frac{\mathrm{d}(r\,U_\theta)}{\mathrm{d}r}\,u\,.$$

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Zero swirl, no surfaces

Now

$$\mathcal{F}(p) = \frac{\mathrm{D}_0^3}{\mathrm{D}t^3}\left[\frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{1}{c_0^2}\frac{\mathrm{D}_0^2 p}{\mathrm{D}t^2} - \nabla^2 p\right) + 2\frac{\mathrm{d}U_1}{\mathrm{d}y_2}\frac{\partial^2 p}{\partial y_1\,\partial y_2}\right],$$

and

$$\mathbb{S} = \frac{\mathrm{D}_0^3}{\mathrm{D}t^3}\left\{\frac{\mathrm{D}_0}{\mathrm{D}t}\left[\frac{\partial^2 \rho_{to}\, u_i\, u_j}{\partial y_i\,\partial y_j}\right] - 2\frac{\mathrm{d}U_1}{\mathrm{d}y_2}\frac{\partial^2(\rho_{to} u_2 u_i)}{\partial y_1\,\partial y_i}\right\}.$$

so Lilley's equation is recovered

$$\frac{\mathrm{D}_0^3}{\mathrm{D}t^3}\left\{\frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{1}{c_0^2}\frac{\mathrm{D}_0^2 p}{\mathrm{D}t^2} - \nabla^2 p\right) + 2\frac{\mathrm{d}U_1}{\mathrm{d}y_2}\frac{\partial^2 p}{\partial y_1\,\partial y_2}\right\}$$

$$= \frac{\mathrm{D}_0^3}{\mathrm{D}t^3}\left\{\frac{\mathrm{D}_0}{\mathrm{D}t}\left[\frac{\partial^2 \rho_{to}\, u_i\, u_j}{\partial y_i\,\partial y_j}\right] - 2\frac{\mathrm{d}U_1}{\mathrm{d}y_2}\frac{\partial^2(\rho_{to} u_2 u_i)}{\partial y_1\,\partial y_i}\right\}.$$

Integral form

$$p(\mathbf{x},t) = \int \iiint_V G\left(\mathbf{x},t|\,\mathbf{x_0},t_0\right)\left(\mathbb{S}\left(\mathbf{x_0},t_0\right) + \mathbb{S}_{FWH}\left(\mathbf{x_0},t_0\right)\right) r_0 \mathrm{d}\theta_0 \mathrm{d}r_0 \mathrm{d}x_0 \mathrm{d}t_0\,,$$

$G(\mathbf{x},t|\mathbf{x_0},t_0)$ Green's function in duct with swirl.

Move derivatives onto G:

$$p(\mathbf{x},t) = T_V(\mathbf{x},t) + T_S(\mathbf{x},t)$$

with e.g.

$$T_S(\mathbf{x},t) = +\int \iint_{\Sigma_B(t_0)} \mathbf{S_{FWH}} \cdot \boldsymbol{\nabla}\left(\mathcal{D}_0^2\left(G\right)\right) - S_{FWH,\rho}\frac{\mathrm{D}_0}{\mathrm{D}t_0}\left(\mathcal{D}_0^2\;(G)\right)\mathrm{d}\Sigma_0(t_0)\mathrm{d}t_0$$

$$+\int \iint_{\Sigma_B(t_0)} 2S_{FWH,\theta}\frac{U_\theta}{r_0}\mathcal{R}_{0,1}\left(G\right)\mathrm{d}\Sigma_0(t_0)\mathrm{d}t_0 + \int \iint_{\Sigma_B(t_0)} S_{FWH,r}\,\mathcal{R}_{0,2}\left(G\right)\mathrm{d}\Sigma_0(t_0)\mathrm{d}t$$

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Test case

Simple blades:

$$p(\mathbf{x},t) = \iiint_{\bigcup_j \Sigma_{B,j}(t_0)} p(\mathbf{x_0},t_0)\mathcal{T}_0(G(\mathbf{x},t|\mathbf{x_0},t_0))\mathrm{d}\Sigma_0(t_0)\mathrm{d}t_0\ ,$$

with

$$\mathcal{T}_0(G) = \left[n_{x,j}\mathcal{D}_0^2\frac{\partial G}{\partial x_0} + n_{\theta,j}\left(\frac{\mathcal{D}_0^2}{r_0}\frac{\partial G}{\partial \theta_0} + 2\frac{U_\theta}{r_0}\mathcal{R}_{0,1}(G)\right)\right]\ .$$

Rotor steady distortion:

$$\mathbf{u}_{inc}(\mathbf{x}) = U_x^\infty\left(1-\alpha\ \mathrm{e}^{-\ln 2\left(\frac{\theta}{\theta_0}\right)^2}\right)\mathbf{e}_x = \sum_{q\in\mathbb{Z}} u_q\ \mathrm{e}^{\mathrm{i}\ q\ \theta}\ \mathbf{e}_x$$

Blade forcing: flat-plate response theory.

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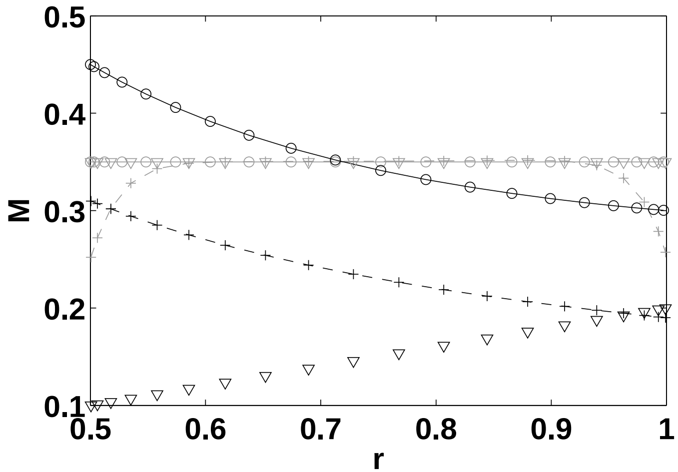
Parameters

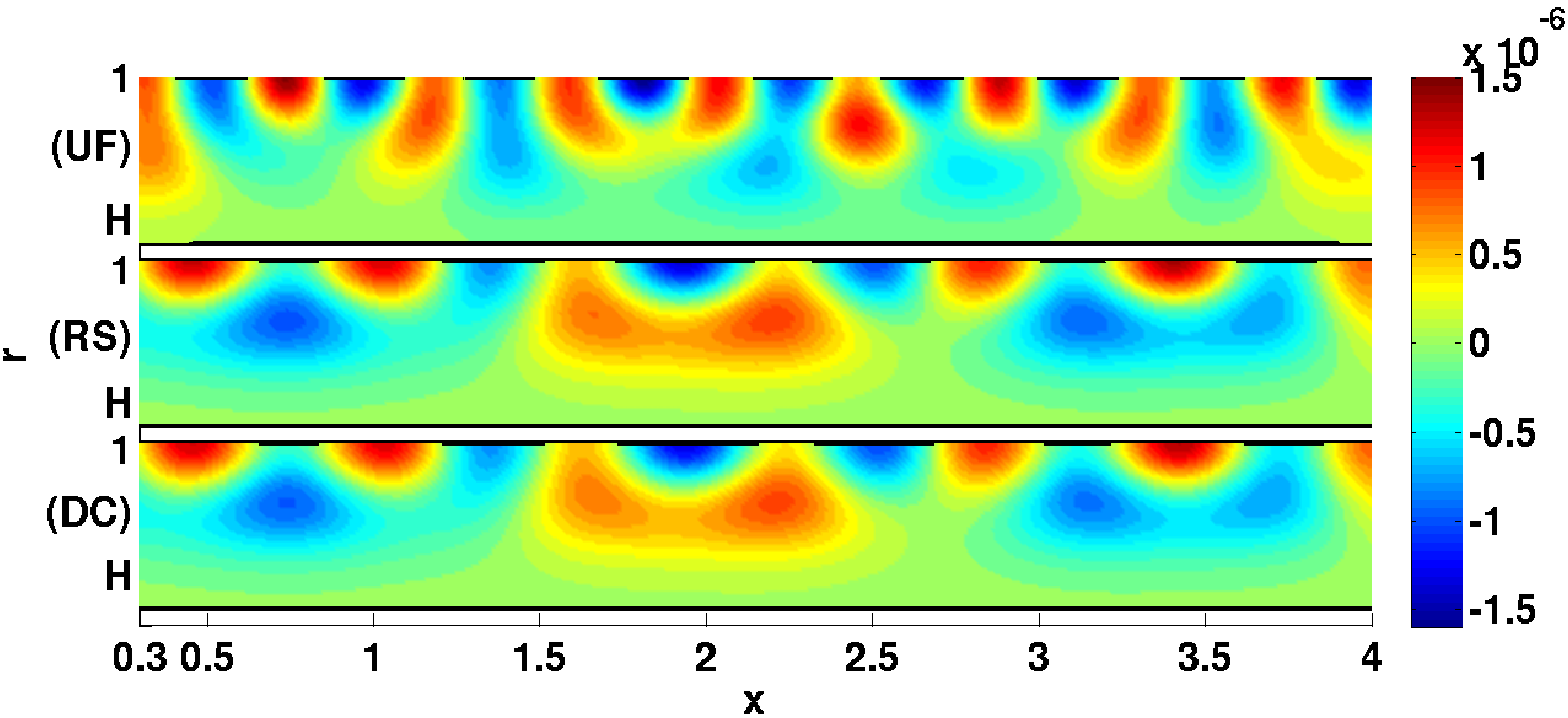
Inspired by 22-in NASA fan SDT, Hughes et al 2002.



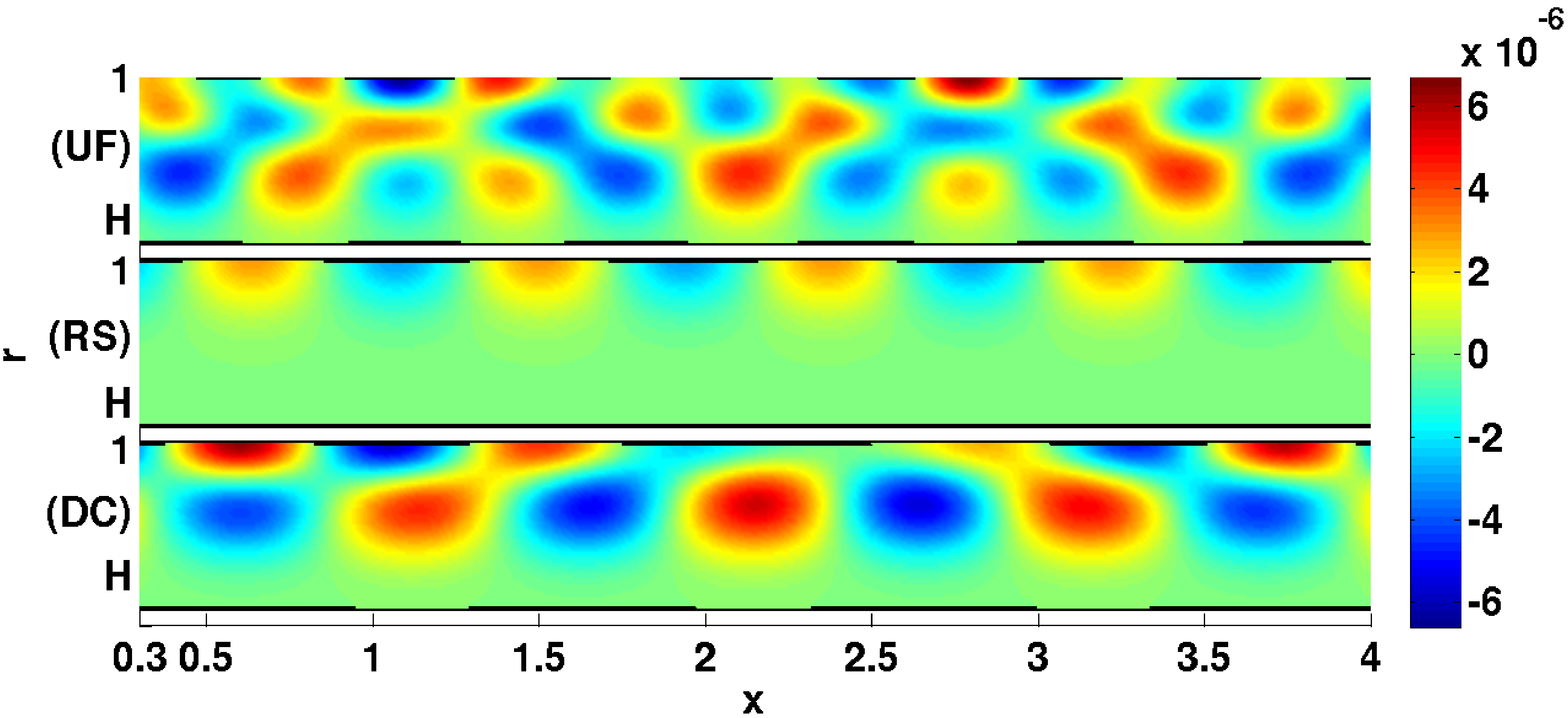
$+$ symbol realistic swirl

$\bigtriangledown$ symbol solid-body swirl plus uniform axial flow

Solid body swirl and uniform axial



Realistic mean flow



Ongoing work

- Rotor broadband self noise in swirling flow.
- Non isentropic swirling flow turbine noise.