Acoustic analogies such as Lighthill can be regarded as linear aero-acoustic models which imply that they can be formulated using so called "multi-ports". A multi-port model has a "passive" part describing the scattering and an "active" part describing the sound generation. Multi-port techniques are particularly useful for ducted systems where the acoustic field can be projected on a known basis of acoustic modes [1]. Assuming a time-invariant system a multi-port can in the frequency domain be described by:

\[ \mathbf{p}_+ = \mathbf{S}\mathbf{p}_- + \mathbf{p}_{+s} , \]  

where \( \mathbf{p} \) is a state-vector containing acoustic modal pressure amplitudes, \( \mathbf{S} \) is the scattering matrix ("passive" part), \( \mathbf{p}_{+s} \) is the source vector or "active" part and +/- denotes propagation out from or into the system.

\[ \ ]

**Figure 1** A ducted multi-port with two openings \( a \) and \( b \).

The projection of the field on acoustic modes reduce the effect of turbulence in the data and further improvement is possible by combining data at different cross-sections. Note that the form of Eq. (1) implies that the source data \( (\mathbf{p}_{+s}) \) is reflection-free, i.e., it corresponds to an infinite system. This is very convenient both in experiments and for numerical modelling and facilitates comparison of numerical and experimental data. In principle a multi-port gives a complete acoustic characterization of a ducted component and any duct system with tur-
bo-machines can be reduced to a network of multi-ports. At KTH experimental techniques for multi-port eduction have been developed since the 1990’s [2-5]. More recently multi-port methods have with success also been applied to numerical data and here examples of these efforts will be presented.

The general procedure to determine a multi-port is based on a two-step method. First the system is tested by sending in waves ("modes") and determining the reflected and transmitted waves. If the multi-port is active filtering or correlation methods are applied to eliminate the influence of the source field on the procedure. In the second step the active part is determined by applying Eq. (1) with a known $S$.

Kierkegaard et al. [6] proposed a frequency domain Linearized Navier Stokes Equations method (LNSE) to determine the scattering or "passive" part of multi-ports. The method requires that the background mean flow first is solved, e.g., by a RANS solver and then the LNSE equations can be solved using standard FEM methods. Using a frequency domain approach eliminates absolute instabilities and convective instabilities are controlled by the viscosity. In Ref. [7] Kierkegaard et al. applied the LNSE method to predict whistling for a ducted orifice by computing the complete ("passive") 2-port for the system and applying the Nyquist condition. More recently [8] it has been found that for certain cases, e.g. a T-junction, it is crucial to include eddy viscosity or turbulent dissipation of the acoustic field in the LNSE model.

Alenius et al. [9] applied Large Eddy Simulations (LES) and computed both the "active" and "passive" part of the plane wave 2-port for an orifice plate. The structure of the source part was also studied using so called Dynamic Mode Decomposition ("Koopman modes"). The scattering was computed using harmonic incident waves and turbulence was suppressed by time domain averaging as well as projecting the field on a propagating plane wave mode. Using reflection-free boundaries the source part could be directly computed from $p^+$, i.e., by projecting the pressure on the plane wave mode.

In the most recent KTH work multi-port methods were applied to develop new and more efficient modelling tools for complex duct systems [10]. A key aspect of the work was to apply and test multi-ports also beyond the plane wave range. One example of the results can be found in the paper by Schur et al. [11] analysing an axial fan unit for aircraft climate systems. In the paper the reflection free acoustic source data for the fan is extracted by projecting the field computed using compressible IDDES ("Improved Detached Delayed Eddy Simulation") on the acoustic modes. The results demonstrate the usefulness of the multi-port approach for an adequate comparison of numerical and experimental data. Other results from the work concerns modelling of single and tandem orifice plates, see Sack et al. [12]. The flow modelling is based on a compressible IDDES which is time averaged and used as background flow for a
LNSE model of the scattering. The source or “active” part is then computed from IDDES by applying Eq. (1). Note this type of mixed approach is computationally much more efficient than using IDDES or LES to compute both the “passive” and “active” part. Of course with the risk of a larger error for cases when eddy viscosity effects are important.

References

10. [https://www.idealvent.eu/](https://www.idealvent.eu/)