Building a new subgrid characteristic length for LES

F.Xavier Trias*, Andrey Gorobets*,*, Alexey Duben*, Assensi Oliva*

*Heat and Mass Transfer Technological Center, Technical University of Catalonia
*Keldysh Institute of Applied Mathematics of RAS, Russia
Building a new subgrid characteristic length for LES/DES

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DNS of turbulent incompressible flows

Main features of the DNS code:

- Structured staggered grids
- High-order symmetry-preserving schemes
- Fully-explicit second-order time-integration method
- Poisson solver for 2.5D problems: FFT + PCG
- Hybrid MPI+OpenMP parallelization
- OpenCL-based extension for its use on GPGPU

Air-filled differentially heated cavity at $Ra = 10^{11}$ (111M grid points), 2008

Plane impingement jet at $Re = 20000$ (102M grid points), 2011
DNS of turbulent incompressible flows

Square duct at $Re_\tau = 1200$ (172M grid points), 2013

Air-filled differentially heated cavity at $Ra = 10^{11}$ (111M grid points), 2008

Square cylinder at $Re = 22000$ (324M grid points), 2014

Plane impingement jet at $Re = 20000$ (102M grid points), 2011

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DNS of turbulent incompressible flows

Square duct at $Re_{\tau} = 1200$ (172M grid points), 2013

Square cylinder at $Re = 22000$ (324M grid points), 2014

Rayleigh-Bénard convection at $Ra = 10^{10}$ (607M grid points), 2015
Scaling is possible\textsuperscript{1}... but never enough

Building a new subgrid characteristic length for LES

\[ \partial_t \bar{u} + (\bar{u} \cdot \nabla) \bar{u} = \nabla^2 \bar{u} - \nabla \bar{p} - \nabla \cdot \tau(\bar{u}) ; \quad \nabla \cdot \bar{u} = 0 \]

eddy-viscosity \quad \rightarrow \quad \tau(\bar{u}) = -2\nu_e S(\bar{u})

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eddy-viscosity \quad \longrightarrow \quad \tau(\bar{u}) = -2 \nu_e S(\bar{u})

\[ \nu_e = (C_m \delta)^2 D_m(\bar{u}) \]

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\[D_m(\bar{u}) \quad \rightarrow \quad \text{Smagorinsky (1963), WALE (1999), Vreman (2004), QR-model (2011), } \sigma\text{-model (2011), S3PQR}^2 \text{ (2015)...}\]

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\[ C_m \quad \rightarrow \quad \text{Germano’s dynamic model (1991), Lagrangian dynamic (1995), Global dynamic approach (2006)} \]

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\[\delta?\]

In the context of LES, most popular (by far) is:

\[ \delta_{\text{vol}} = (\Delta x \Delta y \Delta z)^{1/3} \]

\[ \delta_{\text{Sco}} = f(a_1, a_2) \delta_{\text{vol}}, \quad \delta_{L^2} = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)/3} \]

Deardorff (1970)
Building a new subgrid characteristic length for LES

- In the context of **LES**, most popular (by far) is:

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\]

- In the context of **DES**:

\[
\delta_{\text{max}} = \max(\Delta x, \Delta y, \Delta z)
\]

Recent flow-dependant definitions

\[
\delta_{\omega} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2}
\]

\[
\tilde{\delta}_{\omega} = \frac{1}{\sqrt{3}} \max_{n,m=1,...,8} |l_n - l_m|
\]

\[
\delta_{\text{SLA}} = \tilde{\delta}_{\omega} F_{KH}(VTM)
\]
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Research question:

- Can we find a simple and robust definition of $\delta$ that minimizes the effect of mesh anisotropies on the performance of subgrid-scale models?
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Research question:

- Can we find a **simple and robust** definition of $\delta$ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

Starting point:

$$G \equiv \nabla \bar{u}$$

physical space

$$G_{\delta} \equiv G \Delta$$

computational space

where for a Cartesian grid $\Delta \equiv \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$
Building a new subgrid characteristic length for LES

**Idea:** $\delta$, appears in a natural way when we consider the leading term of the Taylor series expansion of the subgrid stress tensor,

$$\tau(\bar{u}) = \frac{\delta^2}{12} GG^T + \mathcal{O}(\delta^4)$$

physical space

$$\tau(\bar{u}) = \frac{1}{12} G_\delta G_\delta^T + \mathcal{O}(\delta^4)$$

computational space
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**physical space**

$$\tau(\bar{u}) = \frac{1}{12} G_\delta G_\delta^T + \mathcal{O}(\delta^4)$$

**computational space**

A least-square minimization leads to

$$\delta_{lsq} = \sqrt{\frac{G_\delta G_\delta^T : G G^T}{G G^T : G G^T}}$$
Building a new subgrid characteristic length for LES

Properties of new definition

\[ \delta_{lsq} = \sqrt{ \frac{G_{\delta} G_{\delta}^T : GG^T}{GG^T : GG^T} } \]
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Properties of new definition

\[ \delta_{lsq} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}} \]

- Locally defined: only \( G \) and \( \Delta \) needed (\( G_\delta \equiv G\Delta \))
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Properties of new definition

\[ \delta_{lsq} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}} \]

- Locally defined: only \( G \) and \( \Delta \) needed (\( G_\delta \equiv G\Delta \))
- Well-bounded: \( \Delta x \leq \delta_{lsq} \leq \Delta z \) (assuming \( \Delta x \leq \Delta y \leq \Delta z \))
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Properties of new definition

\[ \delta_{lsq} = \sqrt{ \frac{G_G \delta G_T^T : G G_T^T}{G G_T^T : G G_T^T}} \]

- Locally defined: only \( G \) and \( \Delta \) needed (\( G_\delta \equiv G \Delta \))
- Well-bounded: \( \Delta x \leq \delta_{lsq} \leq \Delta z \) (assuming \( \Delta x \leq \Delta y \leq \Delta z \))
- Sensitive to flow orientation, e.g. for rotating flows (\( G = \Omega \))

\[ \delta_{lsq} = \sqrt{\frac{\omega_x^2(\Delta y^2 + \Delta z^2) + \omega_y^2(\Delta x^2 + \Delta z^2) + \omega_z^2(\Delta x^2 + \Delta y^2)}{2|\omega|^2}} \]
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- Applicable to unstructured meshes
Building a new subgrid characteristic length for LES

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- Locally defined: only \( G \) and \( \Delta \) needed (\( G_\delta \equiv G\Delta \))
- Well-bounded: \( \Delta x \leq \delta_{lsq} \leq \Delta z \) (assuming \( \Delta x \leq \Delta y \leq \Delta z \))
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- Applicable to unstructured meshes
- Easy and cheap
Results for LES
Decaying isotropic turbulence

Comparison with classical Comte-Bellot & Corrsin (CBC) experiment

![Diagram showing comparison of energy spectra with CBC and new approach]

- **CBC**
- **New approach** $\delta_{lsq}$
- **Deardoff** $\delta_{vol}$

Energy spectrum $E(k)$ versus wavenumber $k$ for various grid sizes $N_z = \{32, 64, 128, 256, 512, 1024, 2048\}$
Results for LES
Turbulent channel flow

\[ \text{Re}_\tau = 395 \quad \text{DNS Moser et al.} \quad \text{LES } 32 \times 32 \times N_z \]
Results for LES
Turbulent channel flow

\[ Re_\tau = 395 \quad \text{DNS Moser et al.} \quad \text{LES } 32 \times 32 \times N_z \]
Results for DES
Turbulent round jet

- \( Re = 1.1 \times 10^6 \)
- \( M = 0.9 \)

- DES results using NOISEtte code
- Meshes: \( 1.52M \), 4.13M and 8.87M
- Comparison of \( \tilde{\delta}_w \) (Shur et al., 2015, with adaptation for unstructured grids) and \( \delta_{lsq} \)
Results for DES

Turbulent round jet

Graphs showing the results for DES, with data comparing different models and experimental results.
Results for DES
Turbulent round jet

Building a new subgrid characteristic length for LES
A new definition for $\delta$ has been proposed

$$\bar{\delta}_{lsq} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

- It is locally defined, well-bounded, cheap and easy to implement.
- LES tests: HIT, turbulent channel flow ✓
- DES tests: turbulent jet ✓
Concluding remarks

- A new definition for $\delta$ has been proposed

\[ \tilde{\delta}_{lsq} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}} \]

- It is locally defined, well-bounded, cheap and easy to implement.
- LES tests: HIT, turbulent channel flow ✓
- DES tests: turbulent jet ✓

Takeaway message:
- Definition of $\delta$ can have a big effect on simulation results
Thank you for your attention