Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Building a new subgrid characteristic length for LES

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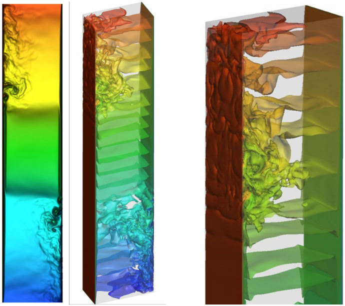
Contents

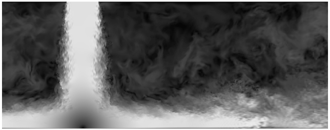
DNS of turbulent incompressible flows

Main features of the DNS code:

- Structured staggered grids
- High-order symmetry-preserving schemes
- Fully-explicit second-order time-integration method
- Poisson solver for 2.5D problems: FFT + PCG
- Hybrid MPI+OpenMP parallelization
- OpenCL-based extension for its use on GPGPU



Air-filled differentially heated cavity at $Ra = 10^{11}$ (111M grid points), **2008**



Plane impingement jet at $Re = 20000$ (102M grid points), **2011**

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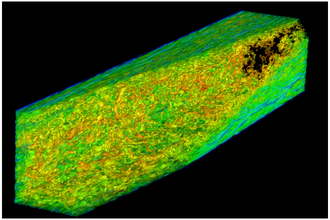
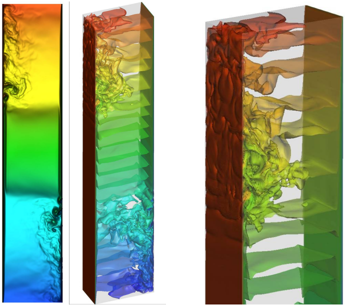


Square duct at $Re_\tau = 1200$ (172M grid points), **2013**



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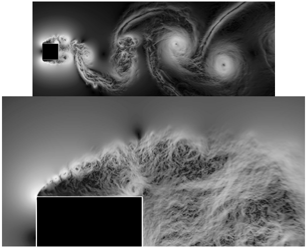
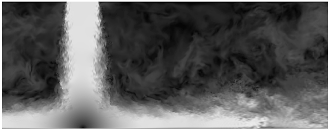


Square cylinder at $Re = 22000$ (324M grid points), **2014**



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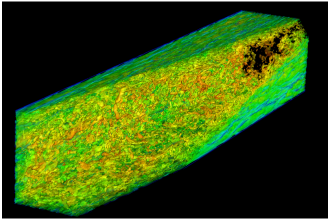

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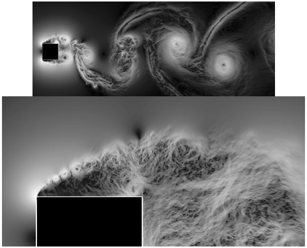

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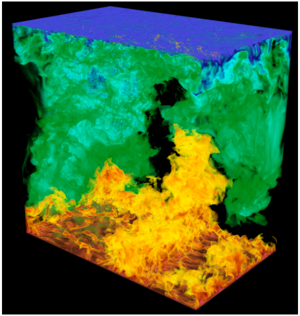
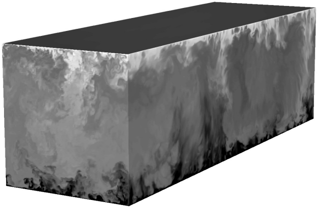
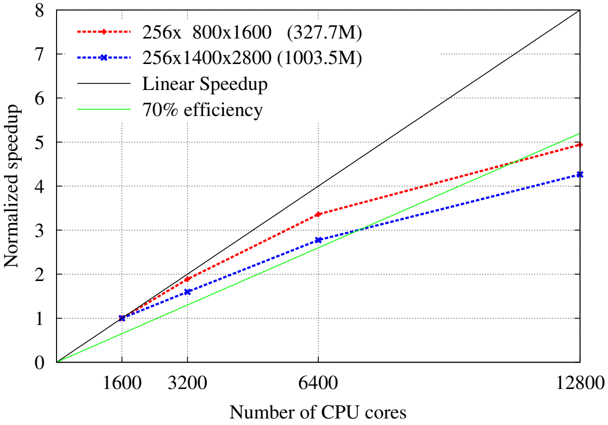




Rayleigh-Bénard convection at $Ra = 10^{10}$ (607M grid points), **2015**

Scaling is possible[1]... but never enough



[1] A.Gorobets, F.X.Trias, A.Oliva. *A parallel MPI+OpenMP+OpenCL algorithm for hybrid supercomputations of incompressible flows*, **Computers&Fluids**, 88:764-772, 2013

Building a new subgrid characteristic length for LES

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nabla^2 \overline{u} - \nabla \overline{p} - \nabla \cdot \tau(\overline{u}) \quad ; \qquad \nabla \cdot \overline{u} = 0$$

$$\text{eddy-viscosity} \quad \longrightarrow \quad \tau\ (\overline{u}) = -2\nu_e S(\overline{u})$$

[2]F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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$D_m(\overline{u}) \longrightarrow$ Smagorinsky (1963), WALE (1999), Vreman (2004), QR-model (2011), σ-model (2011), S3PQR[2] (2015)...

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$$\delta ?$$

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Building a new subgrid characteristic length for LES

- In the context of **LES**, most popular (by far) is:

$$\boxed{\delta_{\mathrm{vol}} = (\Delta x \Delta y \Delta z)^{1/3}} \Longleftarrow \textbf{Deardorff (1970)}$$

$$\delta_{\mathrm{Sco}} = f(a_1, a_2)\delta_{\mathrm{vol}}, \qquad \delta_{L^2} = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)/3}$$

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- In the context of **DES**:

$$\boxed{\delta_{\mathrm{max}} = \max(\Delta x, \Delta y, \Delta z)} \Longleftarrow \textbf{Sparlart et al. (1997)}$$

Recent flow-dependant definitions

$$\delta_{\omega} = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \Longleftarrow \textbf{Chauvet et al. (2007)}$$

$$\tilde{\delta}_{\omega} = \frac{1}{\sqrt{3}} \max_{n,m=1,\dots,8} |l_n - l_m| \Longleftarrow \textbf{Mockett et al. (2015)}$$

$$\delta_{\mathrm{SLA}} = \tilde{\delta}_{\omega} F_{\mathrm{KH}}(VTM) \Longleftarrow \textbf{Shur et al. (2015)}$$

Building a new subgrid characteristic length for LES

Research question:

- Can we find a **simple and robust** definition of δ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

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Starting point:

$$\underbrace{G \equiv \nabla \overline{u}}_{\text{physical space}} \qquad \underbrace{G_\delta \equiv G\Delta}_{\text{computational space}}$$

$$\text{where for a Cartesian grid } \Delta \equiv \begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}$$

Building a new subgrid characteristic length for LES

Idea: δ, appears in a natural way when we consider the leading term of the Taylor series expansion of the subgrid stress tensor,

$$\underbrace{\tau(\overline{u}) = \frac{\delta^2}{12} GG^T + \mathcal{O}(\delta^4)}_{\text{physical space}} \qquad \underbrace{\tau(\overline{u}) = \frac{1}{12} G_\delta G_\delta^T + \mathcal{O}(\delta^4)}_{\text{computational space}}$$

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A **least-square minimization** leads to

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

Building a new subgrid characteristic length for LES

Properties of new definition

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- Sensitive to flow orientation, *e.g.* for rotating flows ($G = \Omega$)

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{\omega_x^2(\Delta y^2 + \Delta z^2) + \omega_y^2(\Delta x^2 + \Delta z^2) + \omega_z^2(\Delta x^2 + \Delta y^2)}{2|\omega|^2}}$$

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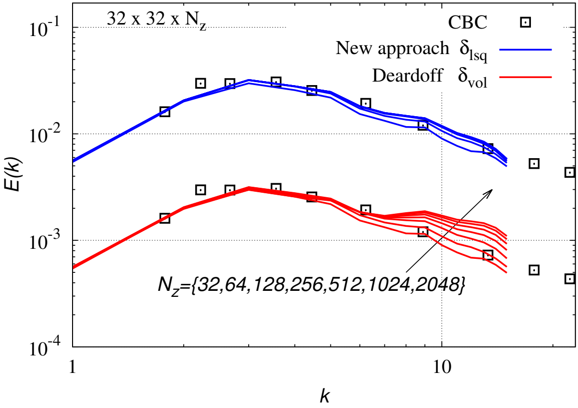
- Applicable to unstructured meshes
- Easy and cheap

Results for LES

Decaying isotropic turbulence

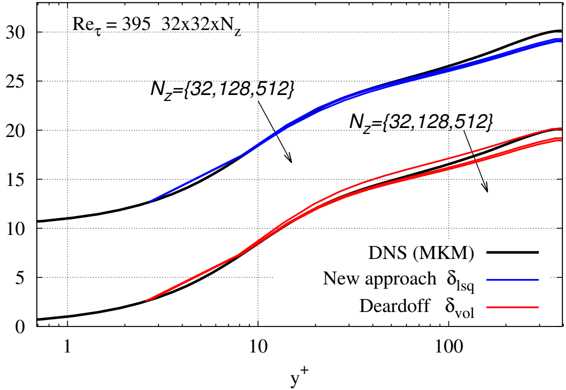
Comparison with classical Comte-Bellot & Corrsin (CBC) experiment



Results for LES

Turbulent channel flow

$Re_\tau = 395$ DNS Moser et al. LES $32 \times 32 \times N_z$



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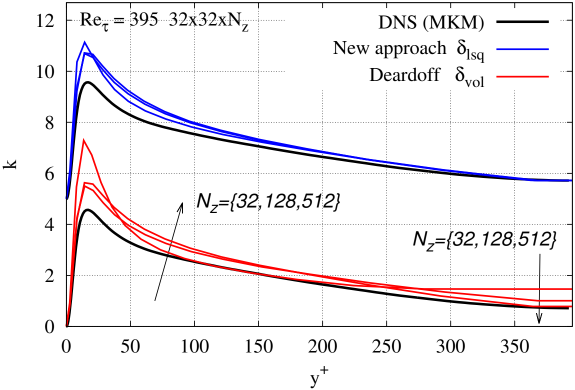
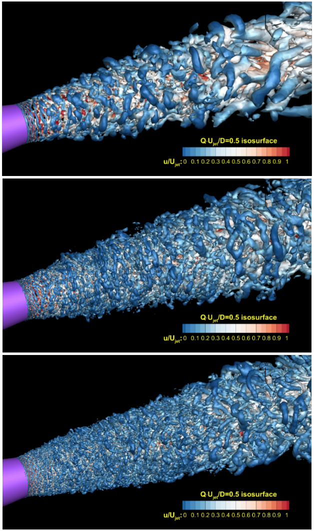


Results for DES

Turbulent round jet

- $Re = 1.1 \times 10^6$
- $M = 0.9$

- DES results using NOISEtte code
- Meshes: **1.52M**, 4.13M and 8.87M
- Comparison of $\tilde{\delta}_\omega$ (Shur et al., 2015, with adaptation for unstructured grids) and δ_{lsq}



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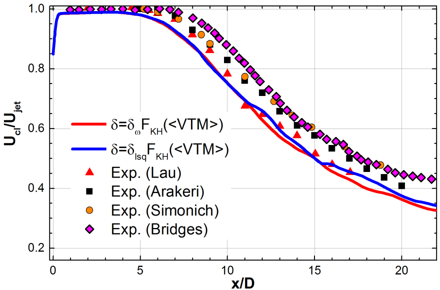

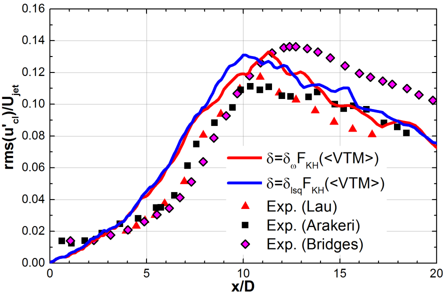

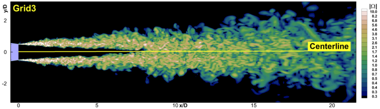

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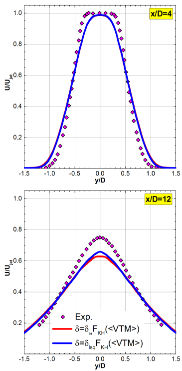

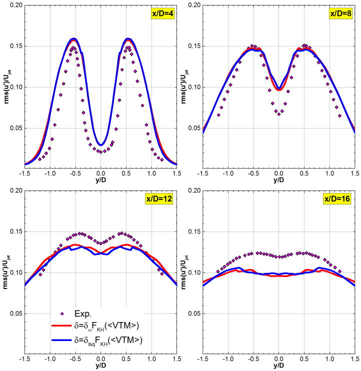

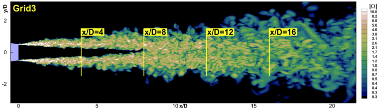

Concluding remarks

- A new definition for δ has been proposed

$$\delta_{\mathrm{lsq}} = \sqrt{\frac{G_\delta G_\delta^T : GG^T}{GG^T : GG^T}}$$

- It is locally defined, well-bounded, cheap and easy to implement.
- LES tests: HIT, turbulent channel flow ✓
- DES tests: turbulent jet ✓

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Takeaway message:

- Definition of δ can have a big effect on simulation results

Thank you for your attention