A FILTERED KINETIC ENERGY PRESERVING METHODOLOGY FOR LARGE EDDY SIMULATIONS OF COMPRESSIBLE FLOWS ON UNSTRUCTURED MESHES.

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Objectives, motivation and justification

Objectives

▶ To solve turbulent high-speed flows, in the presence of discontinuities, by means of Large Eddy Simulations (LES) on unstructured meshes.
▶ To achieve stability and convergence of kinetic energy preserving (KEP) methods by means of filtering (FKEP) in coarse meshes.

Motivation

▶ Shock-vortex and boundary layer shock interaction (SBLI) are important issues in the numerical simulation of compressible flows for transonic and supersonic applications.
▶ Complicated geometries, such as wind turbine blades or complete aircrafts, may require unstructured meshes.
Objectives, motivation and justification

Justification

Figure: NACA0012 at \( \text{AoA} = 5^\circ \) and \( \text{Re} = 5 \times 10^4 \).

- Previous works on NACA0012 [1] concluded that KEP methods are the best for turbulent subsonic compressible flows.
- KEP schemes diverge in discontinuous flows, wiggles appear with coarse meshes.
- To achieve stability, the solution is filtered at each time step.

Three-dimensional Navier-Stokes equations are, in compact form,

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot f_{\text{inv}}(\phi) = \nabla \cdot f_{\text{visc}}(\phi)
\]

with

\[
\phi = \begin{pmatrix} \rho \\ \rho \mathbf{u} \\ E \end{pmatrix} \quad f_{\text{inv}}(\phi) = \begin{pmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} + p \\ (E + p) \mathbf{u} \end{pmatrix} \quad f_{\text{visc}}(\phi) = \begin{pmatrix} 0 \\ \tau \\ \tau \mathbf{u} - \mathbf{q} \end{pmatrix}
\]

For ideal polytropic gas,

\[
E = \frac{p}{\gamma - 1} + \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u} \quad p = \rho RT
\]
Compressible Navier-Stokes Equations

Viscous and heat transfer terms are,

$$
\tau_{ij} = \mu \left[ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]
$$

$$
q = -\kappa \nabla T
$$

The viscosity is given by Sutherland’s law,

$$
\mu = 1.461 \cdot 10^{-6} \frac{T^{3/2}}{T + 110.3}
$$

And the conductivity is,

$$
\kappa = \frac{\mu c_p}{Pr}
$$

For air, \( \gamma = 1.4 \) and \( R = 287 \text{kJ kg}^{-1} \text{K}^{-1} \) and \( Pr = 0.71 \).
Numerical Models

Finite volume method

The compressible Navier-Stokes equations are solved using finite volume (FVM) on a collocated unstructured mesh.

\[
\frac{\partial \phi_o}{\partial t} + \frac{R_o}{V_o} = 0
\]

\[
\sum_{op} F_{op} A_{op} = R_o
\]

Where \( V_o \) is the cell volume, \( F_{op} \) is the total face normal flux, and \( A_{op} \) the face area.

\[
F_{op} = F(\phi_o, \phi_p) \quad F(\phi_o, \phi_o) = f(\phi_o)
\]

LES require the use of KEP numerical schemes.
Numerical Models

KEP Schemes

The inviscid part of the low order numerical fluxes $F_{op}^{inv}(\phi_o, \phi_p)$, with
$\overline{\psi_{op}} = \frac{\psi_o + \psi_p}{2}$ and $\tilde{\psi}_{op} = \sqrt{\psi_o} \sqrt{\psi_p}$.


\[
\begin{align*}
\overline{\rho u \cdot n} & \quad \overline{\rho u \cdot n} & \quad \tilde{\rho} \overline{\rho u \cdot n} \\
\overline{\rho u u \cdot n} + \overline{p n} & \quad \overline{\rho u u \cdot n} + \overline{p n} & \quad \tilde{\rho} \overline{\rho u u \cdot n} + \overline{p n} \\
(\overline{E} + \overline{p})\overline{u \cdot n} & \quad (\overline{E} + \overline{p})\overline{u \cdot n} & \quad (\overline{E} + \overline{p})\tilde{\rho} \overline{u \cdot n}
\end{align*}
\]

We use the recently developed Rozema scheme as it bounds kinetic energy, momentum, ...

Numerical Models

Filtering Methodology I, Basic Concept

Applying explicit $s$ steps Runge Kutta time integration scheme on the semidiscrete NS equations, we write, for each time step,

$$\phi^i = \sum_{j=0}^{j<i} (\beta_{ij}\phi^j + \Delta t\alpha_{ij} R^j)$$

$$R^j = R(\phi^j) \quad \phi^0 = \phi^n \quad \phi^s = \phi^{n+1}$$

- KEP schemes produce wiggles, the computed solution $\hat{\phi}^i$ contains a $\delta_i$ numerical perturbation after each integration step $\hat{\phi}^i = \phi_i + \delta_i$
- The perturbation lies in the smallest scales of the mesh.
- We propose to apply a selective filter.

$$\mathcal{F}(\hat{\phi}^i) = \tilde{\phi}^i \simeq \phi_i$$
Numerical Models
Filtering Methodology II, Algorithm

The filter is activated near numerical instabilities that should be identified by an instability detector $\Psi(\phi)$. Hence $\mathcal{F} = \mathcal{F}(\Psi(\phi))$

Filtered KEP algorithm

1. For $1 \leq i \leq s$ steps of the time integration scheme,
   1.1 compute $\mathbf{R}^{i-1}$
   1.2 Compute a RK step, obtaining $\hat{\phi}^i$
   1.3 Detect instabilities $\Psi(\hat{\phi}^i)$ and compute the filter $\mathcal{F}(\Psi(\hat{\phi}^i))$
   1.4 Filter solution $\phi^i \simeq \tilde{\phi}^i = \mathcal{F}(\hat{\phi}^i)$

2. $\phi^{n+1} = \phi^s$

- The method strongly depends on the quality of $\Psi$ and the properties of $\mathcal{F}$
Numerical Models
Filtering Methodology III, Detector and Filter

Detector Properties

- Aim to distinguish between physical solutions and numerical instabilities (hard to do).
- Based on a maxima/minima measuring operator
  $$\sum_{op} |\phi_p - \phi_o|.$$ 
- Combinations of fluid variables $p$, $\rho$, Mach and dimensionless operators $\tilde{\nabla}(\cdot)$ as test variables $\phi$.

Filter Properties

- Conservative $||F(\phi)|| = ||\phi||$.
- Linear and explicit.
- Constants lie in filter kernel $F(1) = 1$.
- Local Extrema Diminishing
  $$\sum_{op} |\phi_p - \phi_o| - |\tilde{\phi}_p - \tilde{\phi}_o| \geq 0 \ \forall o.$$ 
- Maxima or Minima don’t change signs.
Numerical Study
Benchmark test

Figure: Up: Mesh. Down: Pressure field (colors) and pressure based instability detector value (greyscale).

- The capability of the method of capturing discontinuities is tested with the one-dimensional Sod’s shock-tube.
- 12000 Tetrahedrons 3D Mesh
- Comparison of a Ducros and a pressure maxima-based instability detectors.
Numerical Study
Benchmark test

Figure: Density and pressure profiles for the Shock-tube.

- Slight differences between detectors.
- KEP diverged for this case.
- Slightly less dissipation than upwind.
- Wiggles are not totally eliminated.
Sajben’s transonic diffuser

Figure: Sajben Transonic Diffuser problem definition.

- Transonic compressible case with experimental\(^3\) and numerical\(^4\) solution for comparison.
- Two cases: weak (steady shock wave, pressure-gradient-induced separation) and strong (transient shock wave and shocklets, shock-induced separation).
- Challenging case: discontinuous transonic turbulent flow, strong SBLI.

\(^3\)Bogar *AIAA* vol. 21 no. 9
\(^4\)http://www.grc.nasa.gov/WWW/wind/valid/transdif/transdif.html
Numerical Study

Results

Figure: FKEP of Sajben’s transonic diffuser strong case, instantaneous Mach number and instability detector.

As vortices reached outlet boundary condition, calculations diverged. No statistical results available.
Conclusions & Future Work

Conclusions

▶ The Filtered KEP can stabilize KEP schemes in transonic flow.
▶ KFEP fits well with unstructured meshes.
▶ Heavy filtering with unstructured meshes can cause instability. With Light filtering, perturbations are not totally eliminated.

Further work

▶ Achieve the validation of the method by resolving the Sajben transonic diffuser.
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