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A FILTERED KINETIC ENERGY PRESERVING METHODOLOGY FOR LARGE EDDY SIMULATIONS OF COMPRESSIBLE FLOWS ON UNSTRUCTURED MESHES.

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COMPUTATIONAL EXPERIMENT ON AEROACUSTICS

September 24-27th, 2014. Svetlogorsk, Russia.

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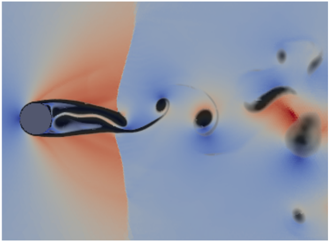

Objectives, motivation and justification

Objectives

- To solve turbulent high-speed flows, in the presence of discontinuities, by means of Large Eddy Simulations (LES) on unstructured meshes.
- To achieve stability and convergence of kinetic energy preserving (KEP) methods by means of filtering (FKEP) in coarse meshes.

Motivation

- Shock-vortex and boundary layer shock interaction (SBLI) are important issues in the numerical simulation of compressible flows for transonic and supersonic applications.
- Complicated geometries, such as wind turbine blades or complete aircrafts, may require unstructured meshes.

Objectives, motivation and justification

Justification

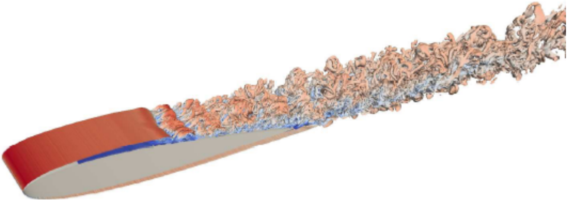

Figure: NACA0012 at $AoA = 5^\circ$ and $Re = 5 \times 10^4$.

- Previous works on NACA0012 [1] concluded that KEP methods are the best for turbulent subsonic compressible flows.
- KEP schemes diverge in discontinuous flows, wiggles appear with coarse meshes.
- To achieve stability, the solution is filtered at each time step.

[1]A. Baez Vidal, J.B. Pedro, O. Lehmkuhl, ECFD IV, 2014

Three-dimensional Navier-Stokes equations are, in compact form,

$$\frac{\partial \phi}{\partial t} + \nabla \cdot f_{inv}(\phi) = \nabla \cdot f_{visc}(\phi)$$

with

$$\phi = \begin{pmatrix} \rho \\ \rho \mathbf{u} \\ E \end{pmatrix} \quad f_{inv}(\phi) = \begin{pmatrix} \rho \mathbf{u} \\ \rho \mathbf{u}\mathbf{u} + p \\ (E+p)\mathbf{u} \end{pmatrix} \quad f_{visc}(\phi) = \begin{pmatrix} 0 \\ \tau \\ \tau \mathbf{u} - \mathbf{q} \end{pmatrix}$$

For ideal polytropic gas,

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho \mathbf{u} \cdot \mathbf{u} \qquad\qquad p = \rho R T$$

Introduction

Compressible Navier-Stokes Equations

Viscous and heat transfer terms are,

$$\tau_{ij} = \mu \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right] \qquad q = -\kappa \nabla T$$

The viscosity is given by Sutherland's law,

$$\mu = 1.461 \cdot 10^{-6} \frac{T^{3/2}}{T + 110.3}$$

And the conductivity is,

$$\kappa = \frac{\mu c_p}{Pr}$$

For air, $\gamma = 1.4$ and $R = 287 kJKg^{-1}K^{-1}$ and $Pr = 0.71$.

Numerical Models

Finite volume method

The compressible Navier-Stokes equations are solved using finite volume (FVM) on a collocated unstructured mesh.

$$\frac{\partial \phi_o}{\partial t} + \frac{R_o}{V_o} = 0$$

$$\sum_{o\,p} F_{o\,p} A_{o\,p} = R_o$$

Where V_o is the cell volume, $F_{o\,p}$ i s the total face normal flux, and $A_{o\,p}$ the face area.

$$F_{o\,p} = F(\phi_o, \phi_p) \quad F(\phi_o, \phi_o) = f(\phi_o)$$

LES require the use of KEP numerical schemes.

Numerical Models

KEP Schemes

The inviscid part of the low order numerical fluxes $F^{inv}_{o\,p}(\phi_o, \phi_p)$, with $\overline{\psi_{op}} = \frac{\psi_o+\psi_p}{2}$ and $\tilde{\psi_{op}} = \sqrt{\psi_o}\sqrt{\psi_p}$.

Jameson [1]	Jameson* [1]	Rozema[3]
$\overline{\rho\mathbf{u}} \cdot \mathbf{n}$	$\overline{\rho}\,\overline{\mathbf{u}} \cdot \mathbf{n}$	$\tilde{\rho}\,\overline{\mathbf{u}} \cdot \mathbf{n}$
$\overline{\rho\mathbf{u}}\,\overline{\mathbf{u}} \cdot \mathbf{n} + \overline{p}\mathbf{n}$	$\overline{\rho}\,\overline{\mathbf{u}}\,\overline{\mathbf{u}} \cdot \mathbf{n} + \overline{p}\mathbf{n}$	$\tilde{\rho}\,\overline{\mathbf{u}}\,\overline{\mathbf{u}} \cdot \mathbf{n} + \overline{p}\mathbf{n}$
$(\overline{E} + \overline{p})\overline{\mathbf{u}} \cdot \mathbf{n}$	$(\overline{E} + \overline{p})\overline{\mathbf{u}} \cdot \mathbf{n}$	$(\overline{E} + \overline{p})\overline{\mathbf{u}} \cdot \mathbf{n}$

We use the recently developed Rozema scheme as it bounds kinetic energy, momentum, ...

[1] J. Sci. Comput. (2008) 34:188208

[2] J. Turbul. (2014) 15:386-410

Numerical Models

Filtering Methodology I, Basic Concept

Applying explicit s steps Runge Kutta time integration scheme on the semidiscrete NS equations, we write, for each time step,

$$\phi^i = \sum_{j=0}^{j<i} \left(\beta_{ij}\phi^j + \Delta t \alpha_{ij} \mathbf{R}^j\right)$$

$$\mathbf{R}^j = \mathbf{R}(\phi^j) \qquad \phi^0 = \phi^n \qquad \phi^s = \phi^{n+1}$$

- KEP schemes produce wiggles, the computed solution $\hat{\phi}^i$ contains a δ_i numerical perturbation after each integration step $\hat{\phi}^i = \phi_i + \delta_i$
- The perturbation lies in the smallest scales of the mesh.
- We propose to apply a selective filter.

$$\mathcal{F}(\hat{\phi}^i) = \tilde{\phi}^i \simeq \phi_i$$

Numerical Models

Filtering Methodology II, Algorithm

The filter is activated near numerical instabilities that should be identified by an instability detector $\Psi(\phi)$. Hence $\mathcal{F} = \mathcal{F}(\Psi(\phi))$

Filtered KEP algorithm

1. For $1 \leq i \leq s$ steps of the time integration scheme,
   1.1 compute $\mathbf{R}^{i-1}$
   1.2 Compute a RK step , obtaining $\hat{\phi}^i$
   1.3 Detect instabilities $\Psi(\hat{\phi}^i)$ and compute the filter $\mathcal{F}(\Psi(\hat{\phi}^i))$
   1.4 Filter solution $\phi^i \simeq \tilde{\phi}^i = \mathcal{F}(\hat{\phi}^i)$
2. $\phi^{n+1} = \phi^s$

- The method strongly depends on the quality of Ψ and the properties of $\mathcal{F}$

Numerical Models

Filtering Methodology III, Detector and Filter

Detector Properties

- Aim to distinguish between physical solutions and numerical instabilities (hard to do).
- Based on a maxima/minima measuring operator $\sum_{o\,p} |\phi_p - \phi_o|$.
- Combinations of fluid variables p, ρ, *Mach* and dimensionless operators $\tilde{\nabla}(\cdot)$ as test variables ϕ .

Filter Properties

- Conservative $||\mathcal{F}(\phi)|| = ||\phi||$.
- Linear and explicit.
- Constants lie in filter kernel $\mathcal{F}(\mathbf{1}) = \mathbf{1}$.
- Local Extrema Diminishing $\sum_{o\,p} |\phi_p - \phi_o| - |\tilde{\phi}_p - \tilde{\phi}_o| \geq 0 \quad \forall o$.
- Maxima or Minima don't change signs.

Numerical Study

Benchmark test

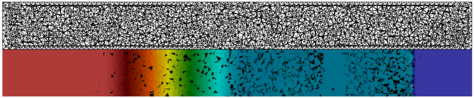

Figure: Up: Mesh. Down:Pressure field (colors) and pressure based instability detector value (greyscale).

- The capability of the method of capturing discontinuities is tested with the one-dimensional Sod's shock-tube.
- 12000 Tetrahedrons 3D Mesh
- Comparison of a Ducros and a pressure maxima-based instability detectors.

Numerical Study

Benchmark test

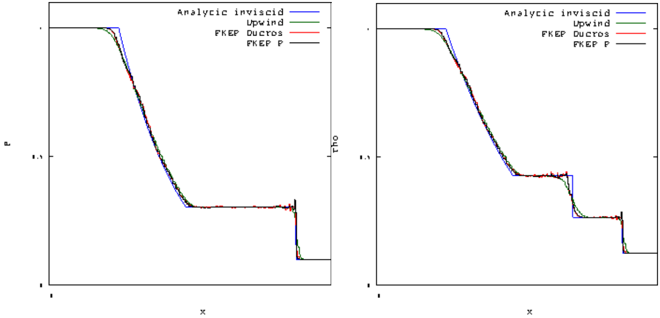

Figure: Density and pressure profiles for the Shock-tube.

- Slight differences between detectors.
- KEP diverged for this case.
- Slightly less dissipation than upwind.
- Wiggles are not totally eliminated.

Numerical Study

Sajben's transonic diffuser

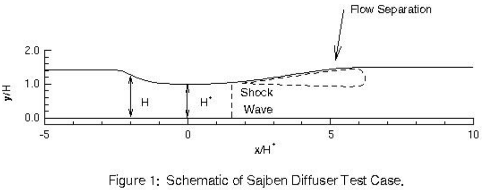

Figure: Sajben Transonic Diffuser problem definition.

- Transonic compressible case with experimental[3] and numerical[4] solution for comparison.
- Two cases: weak (steady shock wave, pressure-gradient-induced separation) and strong (transient shock wave and shocklets, shock-induced separation).
- Challenging case: discontinuous transonic turbulent flow, strong SBLI.

[3] Bogar *AIAA vol. 21 no. 9*

[4] http://www.grc.nasa.gov/WWW/wind/valid/transdif/transdif.html

Numerical Study

Results

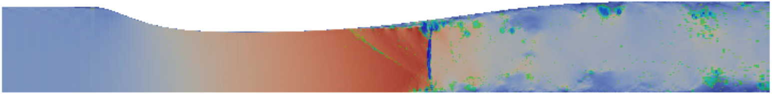

Figure: FKEP of Sajben's transonic diffuser strong case, instantaneous Mach number and instability detector.

As vortices reached outlet boundary condition, calculations diverged. No statistical results available.

Conclusions & Future Work

Conclusions

- The Filtered KEP can stabilize KEP schemes in transonic flow.
- KFEP fits well with unstructured meshes.
- Heavy filtering with unstructured meshes can cause instability. With Light filtering, perturbations are not totally eliminated.

Further work

- Achieve the validation of the method by resolving the Sajben transonic diffuser.

Acknowledgments

This work has been partially granted by the Spanish Secretaría de Estado de Investigación, Desarrollo y Competitividad of the Ministerio de Economía y Competitividad BES-2010-032414 grant for the ENE2009-07689 project.

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